Quantum vortices in optical lattices

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A vortex in a superfluid gas inside an optical lattice can behave as a massive particle moving in a periodic potential and exhibiting quantum properties. In this paper we discuss these properties and show that the excitation of vortex dynamics in a two-dimensional lattice can lead to striking measurable changes in its dynamic response. It would be possible by means of Bragg spectroscopy to carry out the first direct measurement of the effective vortex mass. In addition, the experiments proposed here provide an alternative way to study the pinning to the underlying lattice and the dissipative damping.

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I. INTRODUCTION

The understanding of the static and dynamical behavior of vortices has been crucial to describe numerous different situations in superfluids ranging from liquid helium to high-temperature superconductors [1,2]. These defects can be created by means of an applied magnetic field in superconductors and by putting the sample into rotation in superfluid helium, or they can be thermally excited in low-dimensional systems where the unbinding of vortex-antivortex pairs is at the core of the Berezinskii-Kosterlitz-Thouless transition. Low-dimensional superconductors and, in particular, Josephson junction arrays (JJAs) have been for many years a natural playground for studying classical and quantum properties of vortices [3]. A vortex in a JJA behaves as a massive particle moving in a periodic potential and subject to dissipation [4], and under appropriate conditions vortices can show quantum properties such as interference or tunneling. Among the most interesting experiments performed with vortices in JJAs we mention the observation of ballistic motion [5], the measurement of the Aharonov-Casher effect for a vortex going around a charge [6], and the Mott-Anderson insulator of vortices [7].

Optical lattices for atomic gases, which currently are under intense investigation [8–10], can behave as tunneling junction arrays. Experimental evidences of the analogy between cold atoms and Josephson junctions were the observation of an oscillating atomic current in a one-dimensional Bose-Einstein condensate array [11], and, more recently, the vortex pinning in a rotating Bose-Einstein condensate in a square lattice [12]. Theoretically the equilibrium properties of a vortex in a rotating optical lattice was studied near the superfluid-Mott insulator transition by Wu and collaborators [13].

In this paper we analyze vortex excitations in an optical lattice in the superfluid regime and show that a superfluid gas in an optical lattice offers a unique opportunity for a *direct measurement of vortex properties* (such as the mass, the coupling to its environment, or the pinning potential) *via* a Bragg spectroscopy experiment. This is in contrast to the JJA case, where only indirect measurements based on transport properties are available. The Bragg spectroscopy technique [14–16] has been appealed to for a variety of experiments on ultracold atomic gases, as, for instance, to probe the dynamic structure factor of a Lieb-Liniger gas [17], the effect of a confining harmonic potential on a one-dimensional Bose gas [18], or to detect the presence of Cooper pairs in ultracold Fermi gases [19]. In optical lattices Bragg spectroscopy has been considered for a measurement of the excitation spectrum of a Bose gas in the Mott-insulator phase [20], and of its coexistence with a superfluid phase in a dishomogeneous cloud [21].

We consider a Bose gas in the superfluid phase inside a lattice [22,23], in a regime where the hopping and the on-site repulsion between the bosons are competitive. Quantum fluctuations due to the interplay of the local repulsions and of the hopping have dramatic consequences for vortex dynamics. In this case a vortex behaves as a macroscopic quantum particle, moving in a periodic potential with a mass that we evaluate and show to be directly measurable by Bragg spectroscopy. At variance from other recent studies of vortices in frustrated optical lattices [24–29], we discuss the *dynamical* properties of an *individual* vortex. In order to achieve this regime one can either apply a very low frustration by means of a rotation of the lattice [12] generating only a few and very weakly interacting vortices, or create a vortex excitation by means of phase inprinting [30,31]. Of particular relevance is the very recent observation of vortex pinning in corotating optical lattices by Tung *et al.* [12], which indicates that what we propose here is within reach of experimental verification.

II. MODEL

We consider a Bose gas at zero temperature inside a square lattice with lattice constant a and N_s lattice sites. We assume that the system can be described by a single-band Bose-Hubbard Hamiltonian [32]

$$H = -\frac{J}{2}\sum_{\langle ij\rangle} \hat{b}_i^\dagger \hat{b}_j + \text{H.c.} + U\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu\sum_i \hat{n}_i, \quad (1)$$

where $\hat{b}_i^\dagger$ and $\hat{b}_i$ are the creation and annihilation operators for a boson on the ith site and $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$ is the number operator. The coupling constant U describes the local interaction between bosons, μ is the chemical potential, and J is the matrix element for hopping between nearest-neighbors sites. The on-site interaction energy and the hopping energy are given by $U=g\int d\mathbf{r}|w_0(\mathbf{r}-\mathbf{R}_i)|^4$, $J=-(\hbar^2/2m)^{-1}\int d\mathbf{r}\, w_0^*(\mathbf{r}-\mathbf{R}_i)\nabla^2 w_0(\mathbf{r}-\mathbf{R}_j)$, in terms of the Wannier function $w_0(\mathbf{r})$ ($\mathbf{R}_i$ is the coordinate of the ith site). Here $g=4\pi\hbar^2 a_{sc}/(\sqrt{2\pi} m l_\perp)$ is the repulsive interaction strength in the case where the transverse size $l_\perp$ of the lattice is larger than the scattering length a_{sc}.

If the average number of bosons per site $\bar{n}$ is much larger than one, the system is equivalent to an array of superfluid islands. In this regime, which can be easily achieved in the actual experiments ($\bar{n}\simeq 10^4$ in Ref. [12]), the field operators can be approximated as $\hat{b}_i \simeq \sqrt{\bar{n}}\exp(i\hat{\phi}_i)$, with $\hat{\phi}_i$ being the phase operator on the ith site. The Bose-Hubbard model can be recast into the quantum phase Hamiltonian

$$\hat{H} = -J\bar{n}\sum_{\langle i,j\rangle}\cos(\hat{\phi}_i-\hat{\phi}_j) + U\sum_i \delta\hat{n}_i^2 - \tilde{\mu}\sum_i \delta\hat{n}_i, \qquad (2)$$

where $\tilde{\mu}=2U-\mu-1$. The number operator has been expressed in terms of the fluctuations around its average value $\bar{n}$, $\hat{n}_i=\bar{n}+\delta\hat{n}_i$. The number fluctuation operator and the phase are canonically conjugate variables, $[\delta\hat{n}_i, e^{\pm i\phi_j}]=\delta_{ij}e^{\pm i\phi_j}$. The regime that we consider throughout this work is $J\bar{n}\gg U$: the system is deep in the superfluid region, but quantum fluctuations are present and play a crucial role in the vortex dynamics.

III. VORTEX PROPERTIES

The presence of a static vortex inside the lattice can be described to a good approximation by a phase distribution of the boson field given by

$$\phi_i = \arctan\left(\frac{y_i-y}{x_i-x}\right), \qquad (3)$$

where x,y are the coordinates of the center of the vortex. Deep in the superfluid regime and at temperatures much lower than $J\bar{n}/K_B$, phase rigidity ensures that again to a good approximation, a moving vortex can still be described by Eq. (3) but with a time-dependent position of the vortex center. The existence of a vortex mass can be understood qualitatively by noting that if a vortex moves of a distance of the order of a in a time $\delta t=a/v$, v being its velocity, the phase difference $\delta\phi_{ij}$ at each bond changes in time as $\delta\phi_{ij}=\phi_{ij}(t+a/v)-\phi_{ij}(t)$. Due to the commutation relation between the number and phase operators, a time-dependent phase leads to a contribution to the energy, which is quadratic in the vortex velocity [see the second term of the right-hand side of Eq. (2)]. The problem of calculating the vortex mass can then be reduced to find the phase differences across junctions at times t and $t+a/v$.

An effective action for a vortex in a lattice can then be obtained by inserting Eq. (3) in the Bose-Hubbard model in Eq. (2) and then expressing the resulting action in terms of the vortex coordinates $\mathbf{r}(t)=\{x(t),y(t)\}$ [4]. The on-site repulsion term provides a kinetic energy term $T=(M_v/2)\dot{\mathbf{r}}^2$, where the vortex mass in a lattice of size L is

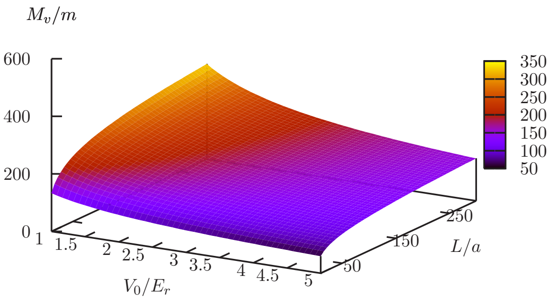

FIG. 1. (Color online) The vortex mass M_v (in units of the boson mass m) as a function of V_0/E_r and L/a, for the case $a=426$ nm, $l_\perp=5$ μm, and $a_{sc}=5.5$ nm.

$$M_v = \frac{\sqrt{2\pi} l_\perp w^2}{4a_{sc}a^2} m \ln(L/a). \qquad (4)$$

The vortex mass thus scales linearly with the boson mass, increases with the width w of the Wannier function, and decreases with the scattering length. Therefore the vortex mass for the case of bosons in an optical lattice can be easily tuned either by varying the thickness of the pancake-shaped external confinement or by exploiting Feshbach resonances to control the scattering length. This versatility of such a system is another advantage over Josephson junctions.

Let us consider for an illustration the ^{87}Rb lattice realized by Greiner and co-workers [23], in the case $V_0=4E_r$, where V_0 and E_r are the well depth of the optical lattice and the recoil energy, respectively. For such a system in 2D we have $a=426$ nm, $w\simeq 96$ nm, $l_\perp=5$ μm, $a_{sc}=5.5$ nm, and $L=75$ μm, and the vortex mass is

$$M_v \simeq 29m\ln(L/a) \simeq 150m \simeq 2.2\times 10^{-20}\ \text{gr}. \qquad (5)$$

The behavior of the vortex mass as a function of the potential well depth (which affects w) and of the size of the lattice is depicted in Fig. 1.

Let us point out that usually in the JJAs, the main interaction term entering in the vortex mass is that between adjacent sites. Unlike the case of a Bose gas characterized by short-range interactions, where only on-site interactions are relevant, in a JJA, intrasite capacities are greater than those on site. If this condition is fulfilled, the vortex mass does not depend on the lattice size [4], on the opposite of Eq. (4). A superfluid atomic gas with dipole-dipole interactions would be closer to a JJA experimental setup.

The effective potential seen by the vortex has been numerically evaluated in the context of JJAs and, in the case of a vortex moving in the x direction inside a large two-dimensional array the effective potential is periodic [33],

$$U_v(x) = 0.1J\bar{n}[\cos(2\pi x/a)-1]. \qquad (6)$$

The potential depth for the vortex is directly proportional to the hopping term for atoms as a consequence of the fact that the number fluctuation operator and the phase are canonically conjugate variables.

In the presence of a vortex the whole array thus behaves as a macroscopic particle of mass M_v moving in a periodic potential. For such a macroscopic object one has to also take into account the interaction with the environment, and the main source of damping is due to the interaction with the long-wavelength phase modes that are excited during vortex motion. This damping has been analyzed in detail in the context of JJAs (see, for example, [34,35]). In this paper for simplicity we shall just comment on how our results are modified in accounting for dissipation.

IV. DYNAMIC STRUCTURE FACTOR

We turn to a calculation of the dynamical response of the Bose gas inside a lattice, both in the absence and in the presence of a vortex. The central quantity of interest is the dynamic structure factor, which in a tight-binding scheme takes the form

$$S(\mathbf{q},\omega) = \int dt e^{i\omega t} \sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{R}_i-\mathbf{R}_j)} \langle \delta\hat{n}_i(t)\delta\hat{n}_j(0)\rangle. \qquad (7)$$

This spectrum can be measured in experiments of Bragg spectroscopy [14,15]: two probe laser beams, with frequencies ω_1 and $\omega_2=\omega_1+\omega$ and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2=\mathbf{k}_1+\mathbf{q}$, scatter on the boson gas and the spectrum measures the probability of momentum transfer $\hbar\mathbf{q}$ at energy $\hbar\omega$ [16]. The f-sum rule gives the first spectral moment $M_1(\mathbf{q})$ as $M_1(\mathbf{q}) \equiv \int S(\mathbf{q},\omega)\omega\, d\omega = \frac{1}{2\hbar}\langle 0|[\delta\hat{n}_{\mathbf{q}}, [\hat{H},\delta\hat{n}^\dagger_{\mathbf{q}}]|0\rangle$, $\delta\hat{n}_{\mathbf{q}}$ being the Fourier transform of $\delta\hat{n}_i$.

We first consider the case in which no vortex is present. Inside the superfluid regime ($J\bar{n} \gg U$), it is enough to consider long-wavelength phase fluctuations, as described by expansion of the cosine in the phase Hamiltonian up to second order. In this limit the Hamiltonian is easily diagonalized in Fourier space by means of the transformations $\hat{\phi}_{\mathbf{k}} = [UN_s/(\hbar\Omega_{\mathbf{k}})]^{1/2}(\hat{a}_{\mathbf{k}}+\hat{a}^\dagger_{-\mathbf{k}})/\sqrt{2}$ and $\hat{n}_{\mathbf{k}}=(N_s\hbar\Omega_{\mathbf{k}}/U)^{1/2}(\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{-\mathbf{k}})/i\sqrt{2}$, with the result

$$\hat{H} = \sum_{\mathbf{k}\in \mathrm{BZ}} \hbar\Omega_{\mathbf{k}}\left(\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \frac{1}{2}\right), \qquad (8)$$

where $\Omega^2_{\mathbf{k}}=(2J\bar{n}U/\hbar^2)[2-\cos(k_x a)-\cos(k_y a)]$. Here the quasimomentum $\mathbf{k}=(k_x,k_y)$ is inside the first Brillouin zone. By taking into account the time dependence of the particle number fluctuation operator dictated by Eq. (8), it is straightforward to obtain the dynamic structure factor as

$$S(\mathbf{q},\omega) = \frac{\hbar\Omega_{\mathbf{q}}}{2U}\delta(\omega-\Omega_{\mathbf{q}}). \qquad (9)$$

The physical interpretation of Eq. (9) is clear: small-q absorption occurs at a frequency corresponding to the dispersion relation of the Goldstone sound mode in the lattice. In this case $M_1(\mathbf{q})=(J\bar{n}/\hbar)[2-\cos(q_x a)-\cos(q_y a)]$.

The presence of a vortex can induce, besides changes in the sound wave spectrum, specific contributions associated with excitations of vortex motions. Several different situations can be envisaged for the latter, but here we discuss the interesting case in which the hopping parameter J is sufficiently large that the vortex is pinned to a minimum of the periodic potential given by Eq. (6). At variance with the dynamics of a single vortex in the absence of a lattice that undergoes precessional motion around the condensate axis [36], in this case the vortex dynamics is associated with small oscillations around its equilibrium position. Thus it can be described by the harmonic oscillator Hamiltonian

$$H_v = \frac{1}{2}M_v\dot{\mathbf{r}}^2 + \frac{1}{2}M_v\Omega_v^2\mathbf{r}^2, \qquad (10)$$

where we have defined $\Omega_v=(0.1J\bar{n}/M_v)^{1/2}2\pi/a$. By noting that

$$\langle \delta\hat{n}_i(t_1)\delta\hat{n}_j(t_2)\rangle = \frac{U^2}{\hbar^2}\langle \delta\dot{\hat{\phi}}_i(t_1)\delta\dot{\hat{\phi}}_j(t_2)\rangle, \qquad (11)$$

using the expression given in Eq. (3) for the phase distribution and performing the average over the vortex degrees of freedom with the help of Eq. (10),

$$\langle \ldots \rangle = \frac{\int e^{-\int_0^\beta H_v d\tau}\cdots \mathcal{D}_{\mathrm{path}}}{\int e^{-\int_0^\beta H_v d\tau}\mathcal{D}_{\mathrm{path}}}, \qquad (12)$$

where τ is the imaginary time, it is possible to write the contribution of the vortex to the dynamic structure factor as

$$S_v(\mathbf{q},\omega) = \frac{\hbar^2\Omega_v}{U^2}\frac{4\pi^2\hbar}{M_v a^4 q^2}\delta(\omega-\Omega_v). \qquad (13)$$

Instead of a $\mathbf{q}$-dependent resonance as in Eq. (9), the Bragg scattering acquires a resonance at a well defined frequency Ω_v indicating that the whole lattice responds collectively having the properties of a single macroscopic particle, the vortex. This is the main result of this paper. Under the conditions specified above, the presence of a vortex induces a resonance at a frequency that allows access to the vortex mass. Let us remark that this resonant behavior is related to the presence of the lattice *and* to the existence of quantum fluctuations originating from the local repulsion. The Bragg spectrum of a vortex in a Bose-Einstein condensate is otherwise determined by a dispersion relation [16]. The peculiar dependence of the spectral strength in Eq. (13) on the transferred momentum q is due to the coupling between the exciting radiation and the lattice: at low momentum all phases in the lattice are excited and the dynamic response is enhanced. We finally should comment on the fact that Eq. (13) does not fulfill the f-sum rule: this should come as no surprise, as this expression is valid only at low energy.

The coupling to long-wavelength phase fluctuations provides the main dissipation mechanism for the vortex motions [34,35,37]. To a first approximation this results in Ohmic damping on the vortex. In the presence of dissipation the delta function in the dynamical response is smeared and acquires a finite width proportional to the dissipation strength.

V. DISCUSSION AND CONCLUSIONS

In summary, in this paper we have discussed some main aspects of quantum dynamics of a vortex in an atomic superfluid gas inside an optical lattice. This physical system is similar to the condensed-matter standard one of a vortex in JJAs. The main differences between atomic superfluid gases in optical lattices and JJAs are (i) the role of the on-site interaction energy in the vortex mass; (ii) the control of this interaction term, and thus of the vortex mass, *via* Feshbach resonances and/or the strength of the transverse confinement; (iii) the access to the direct measurement of vortex properties, such as the mass, the coupling to its environment, or the pinning potential.

We have specifically considered the situation in which the vortex is pinned by the lattice potential and only executes small oscillations around its energy minimum. We have shown that the dynamical response of this system has a resonance frequency, which is $\mathbf{q}$-independent, in contrast with the case without a vortex. The resonance frequency in the presence of a vortex, instead, depends on the vortex mass, and thus a Bragg spectroscopy experiment should allow a direct measurement of the vortex mass itself.

One can envisage other situations in which to study vortex dynamic: the regime $J\bar{n}\sim U$ where vortex tunneling is important, in the presence of several vortices including vortex-vortex interactions, or the vortex dynamic in the presence of defects. Experiments on quantum tunneling or coherence of vortices seem to be within experimental reach.

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[1] G. E. Volovik, *The Universe in a Helium Droplet* (Clarendon Press, Oxford, 2003).
[2] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[3] R. Fazio and H. S. J. van der Zant, Phys. Rep. **355**, 235 (2001).
[4] U. Eckern and A. Schmid, Phys. Rev. B **39**, 6441 (1989).
[5] H. S. J. van der Zant, F. C. Fritschy, T. P. Orlando, and J. E. Mooij, Phys. Rev. Lett. **66**, 2531 (1991).
[6] W. J. Elion, J. J. Wachters, L. L. Sohn, and J. E. Mooij, Phys. Rev. Lett. **71**, 2311 (1993).
[7] A. van Oudenaarden and J. E. Mooij, Phys. Rev. Lett. **76**, 4947 (1996); A. van Oudenaarden, S. J. K. Várdy, and J. E. Mooij, *ibid.* **77**, 4257 (1996).
[8] A. Minguzzi, S. Succi, F. Toschi, M. P. Tosi, and P. Vignolo, Phys. Rep. **395**, 223 (2004).
[9] O. Morsch and M. Oberthaler, Rev. Mod. Phys. **78**, 179 (2006).
[10] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen(De), and U. Sen, Adv. Phys. **56**, 243 (2007).
[11] F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science **293**, 843 (2001).
[12] S. Tung, V. Schweikhard, and E. A. Cornell, Phys. Rev. Lett. **97**, 240402 (2006).
[13] C. Wu, H. Chen, J. Hu, and S. Zhang, Phys. Rev. A **69**, 043609 (2004).
[14] J. Stenger, S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **82**, 4569 (1999).
[15] D. M. Stamper-Kurn, A. P. Chikkatur, A. Gorlitz, S. Inouye, S. Gupta, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **83**, 2876 (1999).
[16] P. B. Blakie, R. J. Ballagh, and C. W. Gardiner, Phys. Rev. A **65**, 033602 (2002).
[17] J.-S. Caux and P. Calabrese, Phys. Rev. A **74**, 031605(R) (2006).
[18] G. E. Astrakharchik and S. Giorgini, Phys. Rev. A **68**, 031602(R) (2003); M. Gattobigio, J. Phys. B **39**, S191 (2006).
[19] J. Ruostekoski, Phys. Rev. A **61**, 041603(R) (2000); G. M. Bruun and G. Baym, *ibid.* **74**, 033623 (2006); K. J. Challis, R. J. Ballagh, and C. W. Gardiner, Phys. Rev. Lett. **98**, 093002 (2007).
[20] A. M. Rey, P. B. Blakie, G. Pupillo, C. J. Williams, and C. W. Clark, Phys. Rev. A **72**, 023407 (2005).
[21] G. G. Batrouni, F. F. Assaad, R. T. Scalettar, and P. J. H. Denteneer, Phys. Rev. A **72**, 031601(R) (2005).
[22] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).
[23] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature (London) **415**, 39 (2002).
[24] D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003).
[25] A. S. Sørensen, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **94**, 086803 (2005).
[26] M. Polini, R. Fazio, A. H. MacDonald, and M. P. Tosi, Phys. Rev. Lett. **95**, 010401 (2005).
[27] E. J. Mueller, Phys. Rev. A **70**, 041603(R) (2004).
[28] R. N. Palmer and D. Jaksch, Phys. Rev. Lett. **96**, 180407 (2006).
[29] K. Osterloh, M. Baig, L. Santos, P. Zoller, and M. Lewenstein, Phys. Rev. Lett. **95**, 010403 (2005).
[30] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **83**, 2498 (1999).
[31] L. Dobrek, M. Gajda, M. Lewenstein, K. Sengstock, G. Birkl, and W. Ertmer, Phys. Rev. A **60**, R3381 (1999).
[32] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B **40**, 546 (1989).
[33] C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983).
[34] U. Geigenmüller, C. J. Lobb, and C. B. Whan, Phys. Rev. B **47**, 348 (1993).
[35] R. Fazio, A. van Otterlo, and G. Schön, Europhys. Lett. **25**, 453 (1994).
[36] B. P. Anderson, P. C. Haljan, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **85**, 2857 (2000).
[37] U. Eckern and E. B. Sonin, Phys. Rev. B **47**, 505 (1993).