Electron loss from hydrogenlike, heliumlike, and lithiumlike uranium ions in collisions with atoms at low relativistic impact energies

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We consider the (single) electron loss from hydrogenlike, heliumlike, and lithiumlike uranium ions in collisions with neutral atoms in the domain of the low-relativistic impact energies where the collision velocity is already a substantial fraction of the speed of light but still does not exceed the typical electron velocities in the K shell of the uranium ions. In collisions with many-electron atoms at these impact energies the presence of the atomic electrons is of minor importance for the electron loss process which occurs predominantly via the interaction with the unscreened atomic nucleus. This interaction can be effectively very strong if the atoms have large atomic numbers which leads to a tremendous failure of the first Born approximation. We show that experimental data for the loss cross sections can be well described using an eikonal amplitude proposed recently [Phys. Rev. A **75**, 062716 (2007)].

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I. INTRODUCTION

During the last two decades there have been substantial experimental and theoretical efforts to investigate the process of the electron loss from highly charged ions occurring in relativistic collisions with atomic targets (see, e.g., Refs. [1–10], and references therein). In particular, the very considerable amount of results have been accumulated for the total loss cross sections.

In experiments the electron loss has been explored in relativistic collisions between different ionic projectiles and solid and gas targets. The experiments have also covered the very broad interval of the projectile impact energies ranging from rather low relativistic values (0.1–0.2 GeV/u, see, e.g., Refs. [1–4]) to extreme relativistic ones (10 and 160 GeV/u [5–7]), where the velocity v of the projectile differs just fractionally from the speed of light c in vacuum ($c \approx 137$ a.u.).

At extreme relativistic impact energies, even in the case of collisions with very heavy targets, the field of the atomic target acting on the electron of the projectile is effectively not very strong. Therefore, at such energies the cross sections for the single electron loss, summed over all possible final states of the target atom, can be reasonably well described within the first order approximation in the interaction between the electron of the projectile and the target atom [10–12].

In the present paper we consider the electron loss from hydrogenlike, heliumlike, and lithiumlike ions of uranium occurring in collisions with targets ranging from beryllium to gold in the range of impact energies in which the collision velocity already reaches quite a substantial fraction of the speed of light but still does not exceed the typical velocity of the electron(s) in the K shell of the uranium ions. In such collisions the loss process is characterized by two main features. First, at these relatively low impact energies, if the target atom is sufficiently heavy, the field of the atom acting on the projectile electron becomes quite strong. Therefore, the first order perturbation theory is expected to lead to reasonable results only in collisions with light target atoms. Secondly, in the case of the uranium ions the accuracy of the semirelativistic approximation, which is very widely used in the field of relativistic collisions (see, e.g., Ref. [13], and references therein) and in which the bound and continuum states of the electron of the ion are described by the Darwin and Sommerfeld-Maue-Furry wave functions, respectively, becomes quite questionable and one has to use the fully relativistic electron description. Atomic units are used throughout except where otherwise stated.

II. GENERAL

The collision between an ion carrying an electron and a multielectron or many-electron atom at relativistic impact energies represents a complex many-body problem whose exact solution is still very far from being reachable. Therefore, our consideration of the electron loss process will be based on two simplified models. Both these basic models treat the electron loss as an effectively three-body process which involves the (active) electron of the projectile ion and the nuclei of the ion and the target atom. These models, thus, ignore the presence of the electrons of the atom. The influence of the atomic electrons on the loss process (the so-called screening and antiscreening effects, see, e.g., Refs. [10,14]) will be discussed in the next section.

Because of extremely large differences between the mass of the electron and those of the nuclei one can use the semiclassical approximation. Within the latter the active electron of the ion is regarded as the only particle having the dynamical degrees of freedom described by the (Dirac) wave equation. The nuclei of the ion and the atom are described as classical particles which move along given (straight-line) trajectories and just represent the sources of the external electromagnetic field acting on the electron of the ion.

It is convenient to treat the electron loss process using a reference frame K in which the nucleus of the ion is at rest. We take the position of the nucleus as the origin and assume that in the frame K the nucleus of the atom moves along a straight-line classical trajectory $\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t$, where $\mathbf{b}=(b_x,b_y,0)$ is the impact parameter, $\mathbf{v}=(0,0,v)$ is the collision velocity, and t is the time.

The Dirac equation for the electron of the ion moving in the fields of the nuclei of the ion and atom reads

$$i\frac{\partial\Psi}{\partial t}=[\hat{H}_0+\hat{W}(t)]\Psi, \tag{1}$$

where

$$\hat{H}_0=c\alpha\cdot\hat{\mathbf{p}}-\frac{Z_p}{r}+\beta c^2 \tag{2}$$

is the electronic Hamiltonian for the undistorted ion. Further,

$$\hat{W}(t)=-\Phi(\mathbf{r},t)+\alpha\cdot\mathbf{A}(\mathbf{r},t) \tag{3}$$

is the interaction between the electron of the ion and the nucleus of the atom, where Φ and $\mathbf{A}$ are the scalar and vector potentials of the electromagnetic field generated by the atomic nucleus. In the above equations $\alpha=(\alpha_x,\alpha_y,\alpha_z)$ and β are the Dirac matrices, Z_p is the charge of the nucleus of the ion, and $\mathbf{r}=(x,y,z)$ are the electron coordinates with respect to the nucleus of the ion.

One can show (see Ref. [15]) that within the first order approximation in the interaction between the electron of the ion and the nucleus of the atom the amplitude for the electron to undergo a transition can be written as

$$S_{fi}^{(1)}(\mathbf{q})=\frac{2iZ_tc}{v^2}\frac{1}{q'^2q_z}\left(\langle\psi_f|\exp(i\mathbf{q}\cdot\mathbf{r})(q_x\alpha_x+q_y\alpha_y)|\psi_i\rangle\right.$$
$$\left.+\frac{1}{\gamma^2}\langle\psi_f|\exp(i\mathbf{q}\cdot\mathbf{r})q_z\alpha_z|\psi_i\rangle\right). \tag{4}$$

Here, ψ_i and ψ_f are the initial and final undistorted (Dirac) states of the electron in the field of the nucleus of the ion. These states have energies ε_i and ε_f, respectively. Further, Z_t is the charge of the nucleus of the atom, the momentum transfer $\mathbf{q}$ to the electron in the collision with this nucleus is given by

$$\mathbf{q}=(q_x,q_y,q_z)=(\mathbf{Q};q_z),$$

$$q_z=\frac{\varepsilon_f-\varepsilon_i}{v}, \tag{5}$$

where $\mathbf{Q}=(q_x,q_y)$ is the transverse part of the momentum which is perpendicular to the collision velocity ($\mathbf{Q}\cdot\mathbf{v}=0$) and

$$\mathbf{q}'=(q_x,q_y,q_z/\gamma)=(\mathbf{Q};q_z/\gamma), \tag{6}$$

where $\gamma=1/\sqrt{1-v^2/c^2}$ is the collisional Lorentz factor.

It has been suggested very recently in Ref. [15] that the symmetric eikonal model, which has been known for quite a while as a very useful approximation to study different processes occurring in nonrelativistic ion-atom collisions, may represent a relatively simple and quite useful tool to evaluate cross sections for the excitation of very heavy ions in collisions at relativistic impact energies.

Unlike the first order approximation, the symmetric eikonal model takes into account the distortion of the initial and final electron states by the field of the atomic nucleus and, therefore, is expected to be a better approximation when the field of the atomic nucleus is effectvely strong in the collision. Within this model the initial and final (distorted) states of the electron of the ion in our case are approximated by

$$\chi_i(t)=\psi_i(\mathbf{r})(vs+\mathbf{v}\cdot\mathbf{s})^{-i\nu_t}\exp(-i\varepsilon_it),$$

$$\chi_f(t)=\psi_f(\mathbf{r})(vs-\mathbf{v}\cdot\mathbf{s})^{i\nu_t}\exp(-i\varepsilon_ft), \tag{7}$$

where $\nu_t=Z_t/v$. Based on these distorted states the following transition amplitude was obtained in Ref. [15]

$$S_{fi}^{\mathrm{eik}}(\mathbf{Q})=\frac{2iZ_tc}{v^2}\frac{1}{q'^2q_z}\left(\frac{q'}{2}\right)^{2i\nu_t}\Gamma^2(1-i\nu_t)(1-i\nu_t)_2$$
$$\times F_1(1-i\nu_t,i\nu_t;2;Q^2/q'^2)\left(\langle\psi_f|\exp(i\mathbf{q}\cdot\mathbf{r})\right.$$
$$\left.\times(q_x\alpha_x+q_y\alpha_y)|\psi_i\rangle+\frac{1}{\gamma^2}\langle\psi_f|\exp(i\mathbf{q}\cdot\mathbf{r})q_z\alpha_z|\psi_i\rangle\right), \tag{8}$$

where $\Gamma(z_1)$ and ${}_2F_1(a,b;c;z_2)$ are the gamma function and hypergeometric function, respectively (see, e.g., Ref. [16]). Taking into account that $\lim_{\nu_t\to0}\Gamma(1-i\nu_t)=1$ and $\lim_{\nu_t\to0}\,{}_2F_1(1-i\nu_t,i\nu_t;m;z)=1$ (see Ref. [16]) we find that in the limit of weak perturbations $\nu_t\to0$ ($\nu_t\ll1$) the eikonal amplitude (8) reduces exactly to the first order amplitude (4).

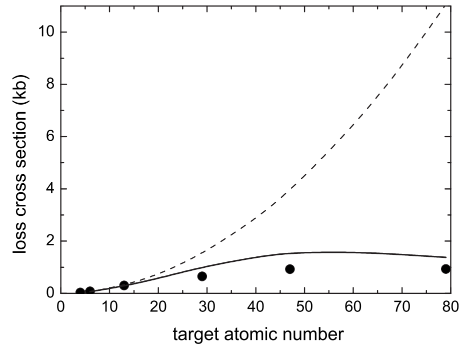

FIG. 1. The total cross section (per electron) for the single electron loss from 105 MeV/u $U^{90+}(1s^2)$ ions colliding with different targets. Circles show experimental results for the loss in collisions with solid state targets of beryllium, carbon, aluminum, copper, silver, and gold which were measured in Ref. [1]. Dash and solid curves display results of calculations with the amplitudes (4) and (8), respectively.

III. RESULTS AND DISCUSSION

A. Electron loss from hydrogenlike and heliumlike uranium ions

Our results for the total cross section for the single loss from 105 MeV/u $U^{90+}(1s^2)$ ions in collisions with atomic targets are presented in Fig. 1. In this figure they are also compared with experimental data for the loss cross section measured in Ref. [1] for collisions of 105 MeV/u $U^{90+}(1s^2)$ with solid state targets of beryllium, carbon, aluminum, copper, silver, and gold [17]. In our calculation we considered the electron loss as occurring from the ground state of a hydrogenlike ion whose effective nuclear charge was determined from the binding energy of the electrons in $U^{90+}(1s^2)$.

At an impact energy of 105 MeV/u the typical minimum momentum transfer to the electron of the ion, which is necessary to get the electron out of the ion, is estimated to be $q_{\min} \sim 100-160$ a.u. Even in collisions with atoms of quite heavy elements, such as, e.g., gold, the magnitude of the momentum transfer is much larger than the typical momenta of the most electrons in the target atom. This means that the screening effect of the atomic electrons has a negligible impact on the electron loss process. The complementary way to arrive at this conclusion is to notice that the typical values of the impact parameters in the collisions resulting to the electron loss from the uranium ion are so small that the atomic electrons at such impact parameters are not really able to screen the field of the atomic nucleus acting on the projectile electron.

Further, the effective energy threshold for the antiscreening mode in the electron loss process from $U^{90+}(1s^2)$ is about 240 MeV/u. As a result, the antiscreening effect of the atomic electrons in the collisions under consideration is also very weak and can safely be ignored. Thus, the process of the electron loss can be regarded as an effectively three-body problem and we may apply the three-body approximations discussed in the previous section.

It is seen in Fig. 1 that in collisions with atomic targets having not very large atomic numbers, for which one has $Z_t/v \ll 1$, both theoretical models yield very close loss cross section values. However, when the ratio Z_t/v increases the difference between the results of the first order and eikonal models rapidly grows and reaches about a factor of 10 for collisions with atoms of gold. A comparison with the experimental data clearly shows a strong failure of the first order approximation. This approximation predicting the dependence $\sim Z_t^2$ for the loss cross sections does not reproduce the saturation clearly visible in the experimental loss cross at $Z_t \gtrsim 30$ and overestimates the data by more than ten times in collisions with the gold target. At the same time, the eikonal model seems to do a rather good job clearly showing the saturation in the loss cross section with increase in Z_t.

Figure 2 displays results of our calculations for the total cross section from 220 MeV/u $U^{91+}(1s)$ ions. Impact energy of 220 MeV/u is not yet sufficiently large to make the screening effect of the atomic electrons to be of importance for the loss process. In addition, this impact energy is still below the effective threshold of 240 MeV/u for the antiscreening collision mode. Therefore, as in the previous case, both the screening and antiscreening effects of the atomic electrons are weak (we estimate their total effect in the loss cross section as not exceeding a few percent) and can be neglected. As a result, the projectile-electron loss process can again be considered as a three-body problem.

In Fig. 2 the results yielded by the first order and eikonal models are displayed by dash and solid curves, respectively. Similarly to the previous case, in collisions with targets having low atomic numbers ($Z_t/v \ll 1$) the results of both the models practically coincide. When the atomic number of the target increases the difference between the first order and eikonal results rapidly increases and reaches about a factor of 4 for collisions with the gold target.

Taking into account that the experimental data shown in Figs. 1 and 2 (as well as in Fig. 3) may possess a possible

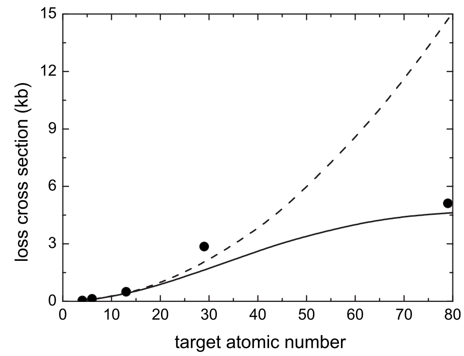

FIG. 2. The total cross section for the electron loss from 220 MeV/u $U^{91+}(1s)$ ions colliding with different targets. Circles show experimental results measured in Ref. [1] for the loss in collisions with solid state targets of beryllium, carbon, aluminum, copper and gold. Dash and solid curves display results of calculations with the amplitudes (4) and (8), respectively.

systematic error of up to 20% (see the book of Eichler and Meyerhof in Ref. [13]), one can say that, for both the collision systems considered, the results of the eikonal model are in reasonable agreement with the experiment.

B. Electron loss from $U^{89+}(1s^2 2s)$

Our results for the total cross section for the single electron loss from 105 MeV/u $U^{89+}(1s^2 2s)$ are shown in Fig. 3 where they are also compared with the experimental data from Ref. [1]. As the experimental data show, at this impact energy there is a very large difference between the cross sections for the electron loss from the K and L shells. Therefore, in our calculation it was assumed that the loss occurs only from the L shell. The $2s$ electron in the initial and final states of the undistorted ion was described by considering this electron as moving in the Coulomb field of the ionic core (the nucleus plus the two K-shell electrons) whose effective charge was determined from the binding energy of the $2s$ electron in $U^{89+}(1s^2\,2s)$.

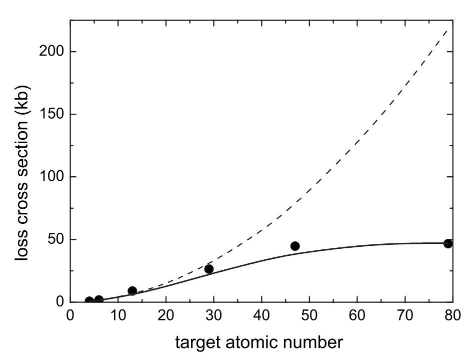

FIG. 3. The total cross section for the single electron loss from 105 MeV/u $U^{89+}(1s^2\,2s)$ ions. Circles show experimental results from Ref. [1] for the loss in collisions with beryllium, carbon, aluminum, copper, and gold. Dash and solid curves display results of calculations with the amplitudes (4) and (8), respectively. (These results were corrected to account for the antiscreening effect of the atomic electrons.)

Compared to the K-shell electrons, the $2s$ electron of the uranium ion is substantially less tightly bound. However, due to the relatively low value of the impact energy, the typical minimum momentum transfer to this electron necessary to remove it from the ion, is still very large on the atomic scale. Therefore, as our calculation show, although the screening effect is now substantially larger than in the case of the electron loss from the K shell, it nevertheless still remains quite modest and can be neglected (the reduction of the first order cross section caused by the screening effect reaches about 11% in collisions with atoms of gold which is to be compared with the fact that the eikonal result for this case is more than 4 times smaller compared to the first order one).

The effective threshold for the antiscreening mode for the loss from lithiumlike uranium ions is about 60 MeV/u and, thus, this mode is now open. We have evaluated its contribution by treating this mode within the first order perturbation theory. Note that since the relative contribution of this mode to the loss cross section scales approximately as $1/Z_t$ the antiscreening effect has to be taken into account in collisions with light targets, such as beryllium and carbon, but may be simply neglected in collisions with atoms of silver and gold.

It is seen in Fig. 3 that, similarly to the case with 105 MeV/u $U^{90+}(1s^2)$ ions, both the first order and eikonal calculation yield very close cross section values for collisions with atomic targets having low atomic numbers. When the ratio Z_t/v increases the difference between the results of these models starts to grow rapidly. However, compared to the case of the electron loss from 105 MeV/u $U^{90+}(1s^2)$, this difference is substantially smaller (reaching "merely" a factor of 4 for collisions with atoms of gold). This reflects the fact that the loss from the L shell occurs in collisions with smaller momentum transfers corresponding to larger impact parameters where the interaction between the electron of the ion and the nucleus of the atom is weaker. A comparison with the experimental data favors the eikonal model whose results are in reasonable agreement with the experiment.

C. Relativistic and semirelativistic electron descriptions

Results of calculation shown in Figs. 1–3 were obtained by using the relativistic (Dirac) wave functions to describe the initial ψ_i and final ψ_f states of the "active" electron in the undistorted ion. In order to get an idea about the importance of such a fully relativistic electron description, we have calculated the electron loss from $U^{91+}(1s)$ colliding with atoms of gold at impact energies ranging between 0.1 and 1 GeV/u by using both the relativistic and semirelativistic descriptions (see Fig. 4). In the latter the initial and final undistorted states of the ion are approximated by the Darwin and Sommerfeld-Maue-Furry wave functions, respectively.

It is seen in Fig. 4 that the application of the semirelativistic approximation to the uranium ion leads to a considerable overestimation of the loss cross section. The semirelativistic results are about a factor 1.5 larger compared to those obtained by employing the Dirac states and this ratio remains basically a constant for the whole range of impact energies considered in the figure.

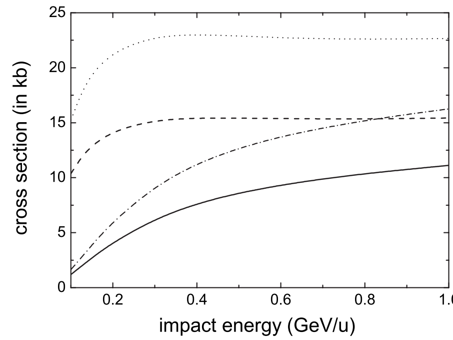

FIG. 4. The total cross sections for the electron loss from $U^{91+}(1s)$ ions in collisions with atoms of gold given as a function of the impact energy. Dash (solid) and dot (dash-dot) curves display results of the first order (eikonal) calculations which employ the relativistic and semirelativistic wave functions, respectively, to describe the initial and final states of the undistorted ion. In all these calculations the presence of the atomic electrons was ignored.

It also follows from Fig. 4 that the difference between the first order and eikonal results decreases, as expected, when the impact energy increases. However, even at 1 GeV/u the first order-to-eikonal cross section ratio still amounts to a factor of about 1.4. This ratio is to be compared with the effect of the screening of the nucleus of the atom by the atomic electrons. This effect increases with the atomic number of the target atom and the impact energy, yet, even in collisions with gold atoms at 1 GeV/u, it reduces the first order result for the loss from $U^{91+}(1s)$, obtained for collisions with the unscreened atomic nucleus, just by about 5%. Note also that because of the very large number of electrons in gold atoms their antiscreening effect always remains negligible. Therefore, one can conclude that for a proper description of the electron loss from the uranium ions in collisions with heavy atoms at impact energies below 1 GeV/u it is much more important to treat with a necessary care the interaction of the electron of the ion with the bare atomic nucleus and to fully account for the relativistic effects in the motion of this electron than to pay attention to the presence of the atomic electrons.

IV. CONCLUSIONS

We have considered the total (single) electron loss from hydrogenlike, heliumlike, and lithiumlike uranium ions colliding with different targets in the domain of low-relativistic impact energies where the collision velocity does not exceed the typical velocities of the electron(s) in the K shell of the ions. In collisions with heavy atoms at these impact energies the effect of the atomic electrons on the electron loss from the uranium projectiles is very weak and can be neglected. At the same time, in collisions with such atoms the ratio Z_t/v can reach values close to 1 which means that the field of the atomic nucleus acting on the electron of the ion in these collisions is effectively quite strong.

Our consideration employed two different amplitudes describing transitions of the electron of the ion in the ion-atom collisions. The amplitude (4) describes the electron transition in the first order approximation in the interaction between the electron of the ion and the nucleus of the atom. Being applied to treat the electron loss in collisions with very heavy targets this amplitude yielded strongly overestimated results for the electron loss cross section.

The second amplitude used in our consideration was the eikonal amplitude (8). From the point of view of computation, this amplitude is almost as simple as the first order one (4) and does not noticeably increase the computing time. However, unlike the amplitude (4), the eikonal amplitude enabled us to describe the effect of the saturation of the loss cross section when the atomic number of the target Z_t increases at fixed impact energy and to get a reasonable agreement with the experimental data.

Finally we note that, despite the experimental data for the single electron loss from the uranium ions occurring in collisions with heavy targets at the low-relativistic domain of impact energies were obtained more than 20 years ago, it is, to our knowledge, the first time when a calculation was able to reproduce these data reasonably well.

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