Dynamical suppression of decoherence by phase kicks: Master equation approach

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The irreversible time evolution of a quantum system interacting with a large environmental system can be described by a quantum master equation. When an external field is applied to a quantum system, a non-Markovian mater equation is derived in a rigorous way, where the relaxation terms in the quantum master equation include the effects of the external field. It is shown that, when the external field is a sequence of phase-modulation pulses, the decoherence of the quantum system can be suppressed under certain conditions. To see the effects of phase-modulation pulses, the irreversible time evolutions of qubit and photon systems are investigated in detail.

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I. INTRODUCTION

Quantum-information processing [1,2] has recently attracted much attention in quantum physics and information science since it provides novel information technology such as quantum cryptography, quantum communication, and quantum computation as well as insights into the principles of quantum mechanics. A quantum system used in quantum-information processing inevitably interacts with a surrounding environmental system (or a thermal reservoir). Since a quantum system in thermal equilibrium cannot process any information, quantum-information processing must use nonequilibrium states. Hence quantum-information processing is within the scope of nonequilibrium statistical mechanics [3–5]. The various methods developed in nonequilibrium statistical mechanics can be used for investigating quantum-information processing. One of the most useful methods is the quantum master equation [6–10].

The entanglement and nonclassicality of quantum states, which are the essential resources in the quantum-information processing, are fragile under the influence of environmental systems. Hence it is a very important task to suppress the decoherence to realize quantum-information processing in the real world. Thus far, a variety of methods to suppress decoherence have been proposed, such as quantum error-correcting codes [11,12], decoherence-free subspaces [13,14], and dynamical decoupling [15,16]. The dynamical decoupling is performed by means of very fast π pulses. Pulse control methods have been applied to a variety of decoherence models [17–20].

In this paper, using the non-Markovian quantum master equation, we formulate the pulse-control method for suppressing the decoherence of a quantum system, where we do not assume that π pulses are applied. We will find that the pulse-control method works well for suppressing decoherence, even if the pulse area is not equal to π. In Sec. II, we derive the non-Markovian quantum master equation of a quantum system interacting with a large environment under the influence of phase-modulation pulses in a rigorous way [21,22]. In Sec. III, to see explicitly the pulse-control effects on decoherence, we obtain the irreversible time evolution of a qubit (two-level system). The decays of the fidelity, purity, and entanglement of quantum states are investigated. In Sec. IV, we consider a photon system and investigate the decoherence of nonclassicality and entanglement of quantum states. We give concluding remarks in Sec. V.

II. NON-MARKOVIAN QUANTUM MASTER EQUATION INCLUDING PULSE EFFECT

Quantum master equations are useful for describing irreversible processes or decoherence phenomena of quantum systems interacting with thermal reservoirs. To control the relevant systems, we apply external fields to them. In particular, it is well known that the application of π pulses is useful for suppressing decoherence [15–20]. In this section, we investigate how non-π pulses affect the decoherence of the relevant system. We suppose that a quantum system interacting with a thermal reservoir is described by the following Hamiltonian:

$$\hat{H} = \hat{H}_S + \hat{H}_R + \hat{H}_{SR}, \tag{1}$$

where $\hat{H}_S$ and $\hat{H}_R$ are the Hamiltonians of the relevant system and thermal reservoir, and $\hat{H}_{SR}$ is the interaction Hamiltonian between them. The Hamiltonians $\hat{H}_S$ and $\hat{H}_{SR}$ are assumed to be

$$\hat{H}_S = \hbar\omega\hat{Z}, \tag{2}$$

$$\hat{H}_{SR} = \hbar\lambda(\hat{R}^\dagger\hat{X} + \hat{R}\hat{X}^\dagger), \tag{3}$$

where $\hat{R}$ is some operator of the thermal reservoir, and $\hat{Z}$ $(=\hat{Z}^\dagger)$ and $\hat{X}$ are operators of the relevant system, satisfying the commutation relation $[\hat{X},\hat{Z}]=\hat{X}$. We set $(\hat{Z},\hat{X},\hat{X}^\dagger)=(\hat{a}^\dagger\hat{a},\hat{a},\hat{a}^\dagger)$ for a harmonic oscillator (or a photon system) and $(\hat{Z},\hat{X},\hat{X}^\dagger)=(\hat{S}^z,\hat{S}^-,\hat{S}^+)$ with $\hat{S}^\pm=\hat{S}^x\pm i\hat{S}^y$ for a spin system, where $\hat{a}$ is the bosonic annihilation operator and $\hat{S}^\gamma$ $(\gamma=x,y,z)$ is the spin operator. We do not need to specify the reservoir Hamiltonian $\hat{H}_R$ in our treatment. Furthermore, we suppose that an external field is applied to the relevant system, where the interaction Hamiltonian between the relevant system and external field is given by

$$\hat{H}_f(t) = -\hbar\mu\sum_k \delta(t-t_k)\hat{Z}, \tag{4}$$

which implies that the external field modulates or kicks the phase of the relevant system by μ at time t_k ($k=1,2,\ldots,n,\ldots$). In the rest of this paper, we refer to such an external field as a phase-modulation μ pulse. Then the Hamiltonian of the total system becomes

$$H(t) = \hat{H}_S(t) + \hat{H}_R + \hat{H}_{SR}, \tag{5}$$

with $\hat{H}_S(t)=\hat{H}_S+\hat{H}_f(t)$.

The time evolution of the quantum state $\hat{W}(t)$ of the total system is determined by the Liouville–von Neumann equation

$$\frac{\partial}{\partial t}\hat{W}(t) = -\frac{i}{\hbar}[\hat{H}(t),\hat{W}(t)]. \tag{6}$$

To eliminate the information of the thermal reservoir from this equation, we introduce the interaction picture by

$$\hat{W}'(t) = \hat{U}^\dagger(t)\hat{W}(t)\hat{U}(t), \tag{7}$$

$$\hat{H}'_{SR}(t) = \hat{U}^\dagger(t)\hat{H}_{SR}\hat{U}(t), \tag{8}$$

with

$$\hat{U}(t) = T\exp\left(-\frac{i}{\hbar}\int_0^t d\tau[\hat{H}_S(\tau)+\hat{H}_R]\right), \tag{9}$$

where the symbol T means that operators are placed in the chronological order from the right to the left. The unitary operator $\hat{U}(t)$ is calculated to be

$$\hat{U}(t) = e^{-i(\omega t-\mu\theta_t)\hat{Z}}e^{-(i/\hbar)\hat{H}_R t}. \tag{10}$$

In this equation, the parameter θ_t is given by $\theta_t=\Sigma_k\theta(t-t_k)$, where $\theta(x)$ is the usual step function. The interaction Hamiltonian $\hat{H}'_{SR}(t)$ between the relevant system and thermal reservoir becomes

$$\hat{H}'_{SR}(t) = \hbar\lambda[\hat{R}^\dagger(t)\hat{X}e^{-i(\omega t-\mu\theta_t)} + (\text{h.c.})], \tag{11}$$

with $\hat{R}(t)=e^{(i/\hbar)\hat{H}_R t}\hat{R}e^{-(i/\hbar)\hat{H}_R t}$. This result shows that the effect of the phase-modulation μ pulses applied to the relevant system induces the time-dependent phase factor $e^{\pm i\mu\theta_t}$ in the system-reservoir interaction Hamiltonian. Then we obtain in the interaction picture,

$$\frac{\partial}{\partial t}\hat{W}'(t) = -\frac{i}{\hbar}[\hat{H}'_{SR}(t),\hat{W}'(t)]. \tag{12}$$

Using the projection operator method, we eliminate the reservoir information from Eq. (12). Here we assume that the relevant system is initially uncorrelated with the thermal reservoir in the thermal equilibrium state $\hat{\rho}_R=e^{-\hat{H}_R/k_BT}/\mathrm{Tr}_R e^{-\hat{H}_R/k_BT}$, and thus we have $\hat{W}(0)=\hat{\rho}(0)\otimes\hat{\rho}_R$. Furthermore, we use the projection operator $\hat{P}$ defined by $\hat{P}\hat{O}=\hat{\rho}_R\mathrm{Tr}_R\hat{O}$, where Tr_R stands for taking the trace over the Hilbert space of the thermal reservoir. In this case, the equality $\hat{P}\hat{W}(0)=\hat{W}(0)$ holds. Then we can derive the time-convolutionless master equation for the reduced quantum state $\hat{\rho}'(t)=\mathrm{Tr}_R\hat{W}'(t)$ of the relevant system [6–10],

$$\frac{\partial}{\partial t}\hat{\rho}'(t) = \hat{K}_f(t)\hat{\rho}'(t), \tag{13}$$

with

$$\hat{K}_f(t) = \sum_{n=1}^{\infty}\int_0^t d\tau_1\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{n-2}}d\tau_{n-1} \times\langle\hat{L}'_{SR}(t)\hat{L}'_{SR}(\tau_1)\hat{L}'_{SR}(\tau_2)\cdots\hat{L}'_{SR}(\tau_{n-1})\rangle_R^{\text{oc}}, \tag{14}$$

where $\hat{L}'_{SR}(t)$ is the Liouville superoperator defined by

$$\hat{L}'_{SR}(t)\circ = -i\lambda[\hat{R}^\dagger(t)\hat{X}e^{-i(\omega t-\mu\theta_t)} + (\text{h.c.}),\circ], \tag{15}$$

and $\langle\cdots\rangle_R^{\text{oc}}$ represents the time-ordered cumulant [8,9] with respect to the average of the thermal reservoir [8,9]. Using the solution of the quantum master equation (13), we can calculate the average value of any operator of the relevant system,

$$\langle\hat{A}(t)\rangle = \mathrm{Tr}[e^{i(\omega t-\mu\theta_t)\hat{Z}}\hat{A}e^{-i(\omega t-\mu\theta_t)\hat{Z}}\hat{\rho}'(t)]. \tag{16}$$

If the strength of the interaction between the relevant system and thermal reservoir is weak and $\langle\hat{R}(t)\rangle_R=0$ is satisfied, we can obtain up to the second order with respect to the coupling constant λ,

$$\frac{\partial}{\partial t}\hat{\rho}'(t) = \phi_{-+}(t)[\hat{X}\hat{\rho}'(t),\hat{X}^\dagger] + \phi^*_{-+}(t)[\hat{X},\hat{\rho}'(t)\hat{X}^\dagger] + \phi_{+-}(t) \times[\hat{X}^\dagger\hat{\rho}'(t),\hat{X}] + \phi^*_{+-}(t)[\hat{X}^\dagger,\hat{\rho}'(t)\hat{X}], \tag{17}$$

with

$$\phi_{-+}(t) = \lambda^2\int_0^t dt'\langle\hat{R}(t)\hat{R}^\dagger(t')\rangle_R e^{i\omega(t-t')-i\mu(\theta_t-\theta_{t'})}, \tag{18}$$

$$\phi_{+-}(t) = \lambda^2\int_0^t dt'\langle\hat{R}^\dagger(t)\hat{R}(t')\rangle_R e^{-i\omega(t-t')+i\mu(\theta_t-\theta_{t'})}. \tag{19}$$

In the Schrödinger picture, the reduced quantum state $\hat{\rho}(t)$ of the relevant system is subject to

$$\frac{\partial}{\partial t}\hat{\rho}(t) = -i\left[\left(\omega-\mu\sum_k\delta(t-t_k)\right)\hat{Z},\hat{\rho}(t)\right] + \phi_{-+}(t)[\hat{X}\hat{\rho}(t),\hat{X}^\dagger] + \phi^*_{-+}(t)[\hat{X},\hat{\rho}(t)\hat{X}^\dagger] + \phi_{+-}(t)[\hat{X}^\dagger\hat{\rho}(t),\hat{X}] + \phi^*_{+-}(t) \times[\hat{X}^\dagger,\hat{\rho}(t)\hat{X}]. \tag{20}$$

In the derivation of this equation, we first apply the phase-modulation μ pulse to the relevant system and then eliminate the information of the thermal reservoir. If we first eliminate the information of the reservoir and then apply the phase-modulation μ pulse to the relevant system, we obtain the quantum master equation by replacing the phase factor

$e^{\pm i\mu(\theta_t-\theta_{t'})}$ with unity in Eqs. (18) and (19). In this treatment, the phase-modulation μ pulses do not affect the relaxation of the relevant system.

To see the effect of the phase-modulation μ pulses on the relaxation of the relevant system, we assume that the correlation functions of the thermal reservoir decay exponentially in time,

$$\lambda^2\langle\hat{R}(t)\hat{R}^\dagger(t')\rangle_R=(\kappa/\tau)(\bar{n}+1)e^{-i\omega(t-t')-(t-t')/\tau}, \tag{21}$$

$$\lambda^2\langle\hat{R}^\dagger(t)\hat{R}(t')\rangle_R=(\kappa/\tau)\bar{n}e^{i\omega(t-t')-(t-t')/\tau}, \tag{22}$$

where τ is the correlation time of the thermal reservoir, and the parameter κ reduces to the damping constant in the Markovian approximation, and $\bar{n}=(e^{\hbar\omega/k_{\rm B}T}-1)^{-1}$. In Eqs. (21) and (22), we have assumed that the frequencies of the reservoir modes are equal to ω (the resonance condition) [23–25]. Furthermore, we assume that the kth phase-modulation μ pulse is applied at time $t_k=k\Delta t$ $(k=1,2,\ldots)$. In this case, we have $\theta_t=\lfloor t/\Delta t\rfloor$, where $\lfloor x\rfloor$ is the largest integer not greater that x. Then substituting Eqs. (21) and (22) into Eqs. (18) and (19), we obtain

$$\phi_{-+}(t)=\kappa(\bar{n}+1)\phi_\mu(t), \tag{23}$$

$$\phi_{+-}(t)=\kappa\bar{n}\phi_\mu(t), \tag{24}$$

with

$$\phi_\mu(t)=1-e^{-(t-\lfloor t/\Delta t\rfloor\Delta t)/\tau}-\left(\frac{1-e^{-\Delta t/\tau}}{e^{i\mu}-e^{-\Delta t/\tau}}\right)\times(e^{-i\mu\lfloor t/\Delta t\rfloor-t/\tau}-e^{-(t-\lfloor t/\Delta t\rfloor\Delta t)/\tau}). \tag{25}$$

It is easy to see from this result that, if $\Delta t/\tau\gg1$, Eqs. (23) and (24) become $\phi_{-+}(t)\approx\kappa(\bar{n}+1)(1-e^{-t/\tau})$ and $\phi_{+-}(t)\approx\kappa\bar{n}(1-e^{-t/\tau})$. Hence when the reservoir correlation τ is sufficiently short in comparison with the pulse separation Δt, the application of the μ pulses does not affect the relaxation of the relevant system. For later convenience, we introduce real parameters $\Delta_\mu(t)$ and $\Lambda_\mu(t)$ by the relation

$$\begin{aligned}\int_0^t dt'\phi_\mu(t)=&\frac{t}{\tau}-\left\lfloor\frac{t}{\Delta t}\right\rfloor(1-e^{-\Delta t/\tau})-1+e^{-(t-\lfloor t/\Delta t\rfloor\Delta t)/\tau}\\&-\left(\frac{1-e^{-\Delta t/\tau}}{e^{i\mu}-e^{-\Delta t/\tau}}\right)\times\Bigg[\Bigg(e^{-(t-\lfloor t/\Delta t\rfloor\Delta t)/\tau}\\&+\frac{(1-e^{i\mu})e^{-\Delta t/\tau}}{e^{i\mu}-e^{-\Delta t/\tau}}\Bigg)(1-e^{-\lfloor t/\Delta t\rfloor(i\mu+\Delta t/\tau)})\\&-\left\lfloor\frac{t}{\Delta t}\right\rfloor(1-e^{-\Delta t/\tau})\Bigg]\equiv i\Delta_\mu(t)+\Lambda_\mu(t).\end{aligned} \tag{26}$$

Using the real parameters $\Delta_\mu(t)$ and $\Lambda_\mu(t)$, we can express the quantum master equation as

$$\begin{aligned}\frac{\partial}{\partial t}\hat{\rho}'(t)=&-i\kappa\tau(\bar{n}+1)\dot{\Delta}_\mu(t)[\hat{X}^\dagger\hat{X},\hat{\rho}'(t)]-i\kappa\tau\bar{n}\dot{\Delta}_\mu(t)\\&\times[\hat{X}\hat{X}^\dagger,\hat{\rho}'(t)]+\kappa\tau(\bar{n}+1)\dot{\Lambda}_\mu(t)\{[\hat{X}\hat{\rho}'(t),\hat{X}^\dagger]\\&+[\hat{X},\hat{\rho}'\hat{X}^\dagger]\}+\kappa\tau\bar{n}\dot{\Lambda}_\mu(t)\{[\hat{X}^\dagger\hat{\rho}'(t),\hat{X}]+[\hat{X}^\dagger,\hat{\rho}'(t)\hat{X}]\},\end{aligned} \tag{27}$$

where $\dot{\Delta}_\mu(t)=d\Delta_\mu(t)/dt$ and $\dot{\Lambda}_\mu(t)=d\Delta_\mu(t)/dt$.

III. PULSE-CONTROLLED IRREVERSIBLE TIME EVOLUTION OF QUBITS

This section considers a qubit (or a two-level system) as the relevant system. In this case, the operators $\hat{Z}$ and $\hat{X}$ are given by the Pauli matrices $(\hat{\sigma}^x,\hat{\sigma}^y,\hat{\sigma}^z)$, that is, $\hat{Z}=(1/2)\hat{\sigma}^z$ and $\hat{X}=\hat{\sigma}^-=(1/2)(\hat{\sigma}^x-i\hat{\sigma}^y)$. From Eq. (17) with Eqs. (23)–(25), we obtain the quantum master equation for the reduced density matrix $\hat{\rho}'(t)$ of the qubit in the interaction picture,

$$\begin{aligned}\frac{\partial}{\partial t}\hat{\rho}'(t)=&-\frac{1}{2}i\kappa\tau\dot{\Delta}_\mu(t)[\hat{\sigma}^z,\hat{\rho}'(t)]+\kappa\tau\bar{n}\dot{\Lambda}_\mu(t)\{[\hat{\sigma}^+\hat{\rho}'(t),\hat{\sigma}^-]\\&+[\hat{\sigma}^+,\hat{\rho}'(t)\hat{\sigma}^-]\}+\kappa\tau(\bar{n}+1)\dot{\Lambda}_\mu(t)\{[\hat{\sigma}^-\hat{\rho}'(t),\hat{\sigma}^+]\\&+[\hat{\sigma}^-,\hat{\rho}'(t)\hat{\sigma}^+]\}.\end{aligned} \tag{28}$$

The solution of this equation defines the quantum channel $\hat{\mathcal{L}}_t$ through the input-output relation $\hat{\rho}'(t)=\hat{\mathcal{L}}_t\hat{\rho}(0)$. The quantum channel $\hat{\mathcal{L}}_t$ is determined by the following relations:

$$\begin{aligned}\hat{\mathcal{L}}_t|0\rangle\langle0|=&\frac{1}{2}(1+e^{-2\Gamma_\mu(t)})|0\rangle\langle0|+\frac{1}{2}(1-e^{-2\Gamma_\mu(t)})|1\rangle\langle1|\\&+\frac{1}{2}(1-e^{-2\Gamma_\mu(t)})\sigma_{\rm eq}(|0\rangle\langle0|-|1\rangle\langle1|),\end{aligned} \tag{29}$$

$$\begin{aligned}\hat{\mathcal{L}}_t|1\rangle\langle1|=&\frac{1}{2}(1-e^{-2\Gamma_\mu(t)})|0\rangle\langle0|+\frac{1}{2}(1+e^{-2\Gamma_\mu(t)})|1\rangle\langle1|\\&+\frac{1}{2}(1-e^{-2\Gamma_\mu(t)})\sigma_{\rm eq}(|0\rangle\langle0|-|1\rangle\langle1|),\end{aligned} \tag{30}$$

$$\hat{\mathcal{L}}_t|0\rangle\langle1|=e^{-i\Omega_\mu(t)-\Gamma_\mu(t)}|0\rangle\langle1|, \tag{31}$$

$$\hat{\mathcal{L}}_t|1\rangle\langle0|=e^{i\Omega_\mu(t)-\Gamma_\mu(t)}|1\rangle\langle0|, \tag{32}$$

where $|0\rangle$ and $|1\rangle$ are the eigenstates of $\hat{\sigma}^z$ such that $\hat{\sigma}^z|0\rangle=|0\rangle$ and $\hat{\sigma}^z|1\rangle=-|1\rangle$, and the real parameters $\Omega_\mu(t)$, $\Gamma_\mu(t)$, and $\sigma_{\rm eq}$ are given by $\Omega_\mu(t)=\kappa\tau\Delta_\mu(t)$, $\Gamma_\mu(t)=\kappa\tau(2\bar{n}+1)\Lambda_\mu(t)$, and $\sigma_{\rm eq}=-(2\bar{n}+1)^{-1}=-\tanh(\hbar\omega/2k_BT)$. The quantum state $\hat{\rho}(t)$ of the qubit in the interaction picture is given by

$$\begin{aligned}\hat{\rho}'(t)=&\frac{1}{2}\{\hat{1}+e^{-i\Omega_\mu(t)-\Gamma_\mu(t)}a(0)\hat{\sigma}^++e^{i\Omega_\mu(t)-\Gamma_\mu(t)}a^*(0)\hat{\sigma}^-\\&+[e^{-2\Gamma_\mu(t)}a_z(0)+(1-e^{-2\Gamma_\mu(t)})\sigma_{\rm eq}]\hat{\sigma}^z\},\end{aligned} \tag{33}$$

where the complex parameter $a(0)$ and real parameter $a_z(0)$,

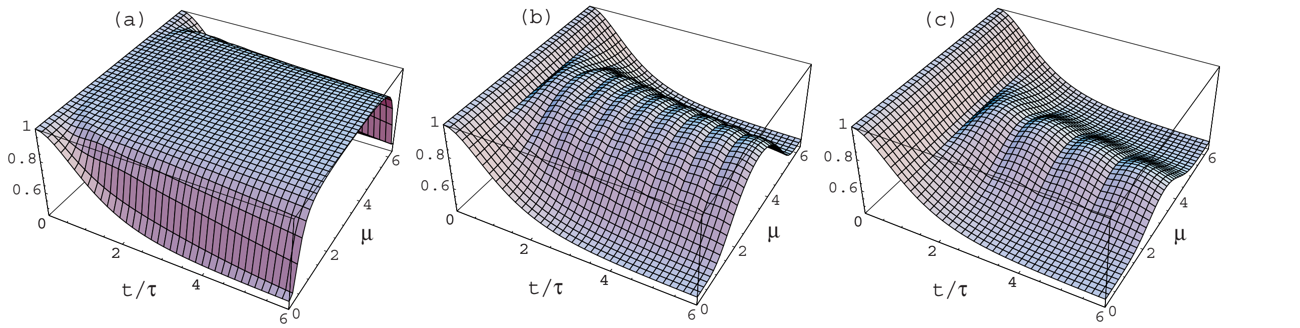

FIG. 1. (Color online) Average fidelity $\overline{F(t)}$ given by Eq. (37) as the function of time t and pulse area μ $(0 \leqslant \mu \leqslant 2\pi)$, where $\Delta t/\tau$ =(a) 0.1, (b) 0.6, and (c) 1.2. In the figure, we set $\kappa\tau=0.5$ and $\sigma_{\mathrm{eq}}=-0.5$.

satisfying the inequality $|a(0)|^2+a_z^2(0) \leqslant 1$, are determined by the initial qubit state $\hat{\rho}(0)$. The quantum state $\hat{\rho}(t)$ in the Schrödinger picture is obtained by replacing $\Omega_\mu(t)$ with $\omega t + \Omega_\mu(t) - \mu\lfloor t/\Delta t\rfloor$ in Eq. (33).

A. Decoherence of fidelity and purity

To investigate the phase-modulation pulse effect on the decoherence, we first obtain the fidelity of a qubit state. When the qubit is initially in the pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $(|\alpha|^2+|\beta|^2=1)$, the quantum state $\hat{\rho}(t)=\hat{\mathcal{L}}_t|\psi\rangle\langle\psi|$ in the Schrödinger picture is derived from Eqs. (29)–(32),

$$\hat{\rho}(t) = \frac{1}{2}[1 + e^{-2\Gamma_\mu(t)}(|\alpha|^2 - |\beta|^2) + (1 - e^{-2\Gamma_\mu(t)})\sigma_{\mathrm{eq}}]|0\rangle\langle 0| + e^{-i\omega t/2 + i\mu\lfloor t/\Delta t\rfloor - i\Omega_\mu(t) - \Gamma_\mu(t)}\alpha\beta^*|0\rangle\langle 1| + e^{i\omega t/2 - i\mu\lfloor t/\Delta t\rfloor i\Omega_\mu(t) - \Gamma_\mu(t)}\alpha^*\beta|1\rangle\langle 0| + \frac{1}{2}[1 - e^{-2\Gamma_\mu(t)}(|\alpha|^2 - |\beta|^2) - (1 - e^{-2\Gamma_\mu(t)})\sigma_{\mathrm{eq}}]|1\rangle\langle 1|. \tag{34}$$

If there is no thermal reservoir, we obtain the pure state

$$|\psi(t)\rangle = e^{-i\omega t/2 + i\mu\lfloor t/\Delta t\rfloor}\alpha|0\rangle + e^{i\omega t/2 - i\mu\lfloor t/\Delta t\rfloor}\beta|1\rangle. \tag{35}$$

Then the fidelity $F(t)=\langle\psi(t)|\hat{\rho}(t)|\psi(t)\rangle$ is calculated to be

$$F(t) = \frac{1}{2} + 2|\alpha|^2|\beta|^2 e^{-\Gamma_\mu(t)}\cos\Omega_\mu(t) + \frac{1}{2}(|\alpha|^2 - |\beta|^2)^2 e^{-2\Gamma_\mu(t)} + \frac{1}{2}(|\alpha|^2 - |\beta|^2)\sigma_{\mathrm{eq}}(1 - e^{-2\Gamma_\mu(t)}). \tag{36}$$

When we average the fidelity $F(t)$ over all possible pure qubit states with equal probabilities, we obtain the average fidelity

$$\overline{F(t)} = \frac{1}{2} + \frac{1}{3}e^{-\Gamma_\mu(t)}\cos\Omega_\mu(t) + \frac{1}{6}e^{-2\Gamma_\mu(t)}. \tag{37}$$

The average fidelity $\overline{F(t)}$ is plotted in Fig. 1. We find from this figure that the phase-modulation μ pulses can suppress the decay of the average fidelity if μ is around neither 0 nor 2π. In particular, when the pulse separation Δt is sufficiently small in comparison with the reservoir correlation time τ, the decay of the average fidelity becomes negligible. Although the π pulses are most effective, the non-π pulses still work well for suppressing the decoherence.

We next investigate the decay of the purity of the quantum state $\hat{\rho}(t)$, where the purity is quantified by the linear entropy $S_L(t)=1-\mathrm{Tr}\hat{\rho}^2(t)$. When the qubit is initially in the pure state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$, we obtain the linear entropy from Eq. (34),

$$S_L(t) = \frac{1}{2} - 2|\alpha|^2|\beta|^2 e^{-2\Gamma_\mu(t)} - \frac{1}{2}[(|\alpha|^2 - |\beta|^2)e^{-2\Gamma_\mu(t)} + (1 - e^{-2\Gamma_\mu(t)})\sigma_{\mathrm{eq}}]^2. \tag{38}$$

The average of the linear entropy over all possible pure qubit states becomes

$$\overline{S_L(t)} = \frac{1}{2}(1 - e^{-2\Gamma_\mu(t)}) \times \left[1 + \frac{1}{3}e^{-2\Gamma_\mu(t)} - (1 - e^{-2\Gamma_\mu(t)})\sigma_{\mathrm{eq}}^2\right]. \tag{39}$$

The average value of the linear entropy is plotted in Fig. 2. The figure shows that dynamical decoupling takes place if $\Delta t/\tau \ll 1$ and $|\mu - \pi| \ll 1$ are satisfied.

B. Decoherence of entanglement

To investigate the decoherence of entanglement, we suppose that one of two qubits prepared in the Bell state $|\Phi_+\rangle$ is put into the quantum channel $\hat{\mathcal{L}}_t$ and the other remains unchanged. Then the output state $\hat{\rho}(t)$ is given by

$$\hat{\rho}(t) = (\hat{\mathcal{L}}_t \otimes \hat{\mathcal{I}})|\Phi_+\rangle\langle\Phi_+|, \tag{40}$$

where $\hat{\mathcal{I}}$ is an identity map. When the output state $\hat{\rho}(t)$ becomes separable, the quantum channel $\hat{\mathcal{L}}_t$ is called an entanglement-breaking channel [26,27]. Using Eqs. (29)–(32), we obtain the two-qubit state $\hat{\rho}(t)$,

$$\hat{\rho}(t) = \frac{1}{4}[1 + e^{-2\Gamma_\mu(t)} + 2e^{-\Gamma_\mu(t)}\cos\Omega_\mu(t)]|\Phi_+\rangle\langle\Phi_+| + \frac{1}{4}[1 + e^{-2\Gamma_\mu(t)} - 2e^{-\Gamma_\mu(t)}\cos\Omega_\mu(t)]|\Phi_-\rangle\langle\Phi_-| + \frac{1}{4}(1 - e^{-2\Gamma_\mu(t)})(|\Psi_+\rangle\langle\Psi_+| + |\Psi_-\rangle\langle\Psi_-|)$$

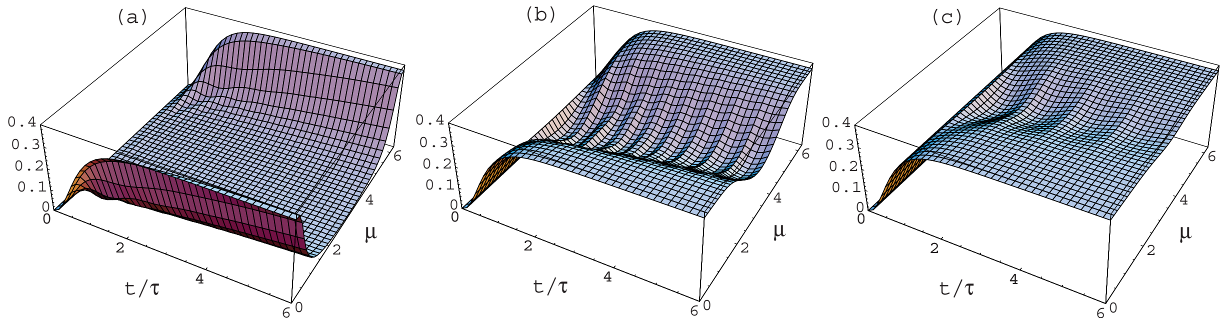

FIG. 2. (Color online) Average value $\overline{S_L(t)}$ of the linear entropy given by Eq. (39) as the function of time t and pulse area μ $(0\leqslant\mu\leqslant 2\pi)$, where $\Delta t/\tau=$(a) 0.1, (b) 0.6, and (c) 1.2. In the figure, we set $\kappa\tau=0.5$ and $\sigma_{\rm eq}=-0.5$.

$$+\frac{1}{4}[(1-e^{-2\Gamma_\mu(t)})\sigma_{\rm eq}+2ie^{-\Gamma_\mu(t)}\sin\Omega_\mu(t)]|\Phi_+\rangle\langle\Phi_-|$$
$$+\frac{1}{4}[(1-e^{-2\Gamma_\mu(t)})\sigma_{\rm eq}-2ie^{-\Gamma_\mu(t)}\sin\Omega_\mu(t)]|\Phi_-\rangle\langle\Phi_+|$$
$$+\frac{1}{4}(1-e^{-2\Gamma_\mu(t)})\sigma_{\rm eq}(|\Psi_+\rangle\langle\Psi_-|+|\Psi_-\rangle\langle\Psi_+|), \qquad (41)$$

with $|\Phi_\pm\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$ and $|\Psi_\pm\rangle=(|01\rangle\pm|10\rangle)/\sqrt{2}$. The concurrence C_t [28] of the output state $\hat{\rho}(t)$ is calculated to be

$$C_t=\max\left[0,e^{-\Gamma_\mu(t)}-\frac{1}{2}\sqrt{1-\sigma_{\rm eq}^2}(1-e^{-2\Gamma_\mu(t)})\right], \qquad (42)$$

in terms of which the entanglement of formation [28,29] is given by

$$E_t=H\left(\frac{1+\sqrt{1-C_t^2}}{2}\right), \qquad (43)$$

with the binary entropic function $H(x)=-x\ln x-(1-x)\ln(1-x)$. The necessary and sufficient condition for the output state $\hat{\rho}(t)$ to be entangled or inseparable is that the concurrence is positive. The concurrence C_t is plotted in Fig. 3. The figure shows that the application of the phase-modulation μ pulses can suppress the decoherence of qubit entanglement. When we apply around ten pulses with $|\mu-\pi|\ll 1$ within the reservoir correlation time τ, the decay of the entanglement becomes negligible [see Fig. 3(a)].

IV. PULSE-CONTROLLED IRREVERSIBLE TIME EVOLUTION OF PHOTONS

In this section, we consider a photon system as the relevant system. In this case, we set $\hat{Z}=\hat{a}^\dagger\hat{a}$ and $\hat{X}=\hat{a}$ with $\hat{a}$ being the bosonic annihilation operator. Then the quantum master equation (27) for the reduced quantum state in the interaction picture becomes

$$\frac{\partial}{\partial t}\hat{\rho}'(t)=-i\kappa\tau\dot{\Delta}_\mu(t)[\hat{a}^\dagger\hat{a},\hat{\rho}'(t)]+\kappa\tau\bar{n}\dot{\Lambda}_\mu(t)\{[\hat{a}^\dagger\hat{\rho}'(t),\hat{a}]+[\hat{a}^\dagger,\hat{\rho}'(t)\hat{a}]\}+\tau\kappa(\bar{n}+1)\{[\hat{a}\hat{\rho}'(t),\hat{a}^\dagger]+[\hat{a},\hat{\rho}'(t)\hat{a}^\dagger]\}. \qquad (44)$$

Using the solution of this equation, we can calculate the average value by

$$\langle\hat{a}^{\dagger m}(t)\hat{a}^n(t)\rangle=e^{i(m-n)(\omega t-\mu\lfloor t/\Delta t\rfloor)}{\rm Tr}[\hat{a}^{\dagger m}\hat{a}^n\hat{\rho}'(t)]. \qquad (45)$$

Since the nonclassicality and entanglement of quantum states remain unchanged under a local unitary transformation $e^{if(t)\hat{a}^\dagger\hat{a}}$ with real function $f(t)$, in the rest of this section we ignore the free motion and the frequency shift $\kappa\tau\Delta_\mu(t)$. We can solve the quantum master equation (44) by means of the phase-space method.

To use the phase-space method [30,31], we introduce the s-ordered phase-space function $F_s(t;z)$ of the quantum state $\hat{\rho}(t)$,

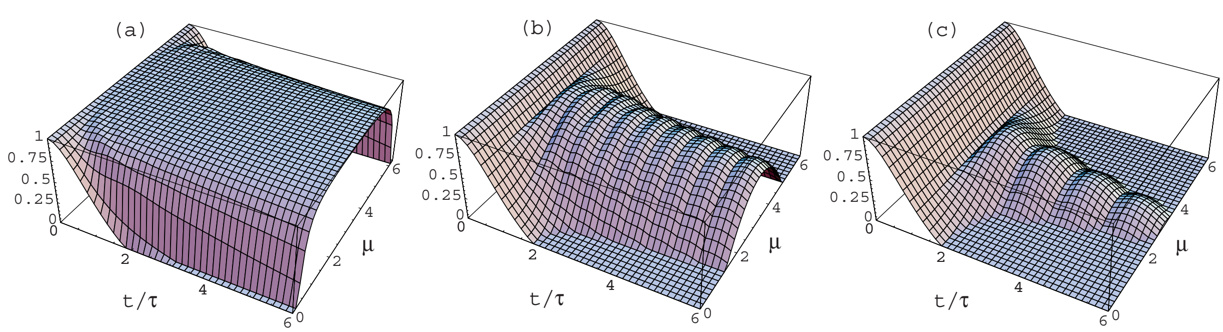

FIG. 3. (Color online) Concurrence C_t as a function of time t and pulse area μ $(0\leqslant\mu\leqslant 2\pi)$, where $\Delta t/\tau=$(a) 0.1, (b) 0.6, and (c) 1.2. In the figure, we set $\kappa\tau=0.5$ and $\sigma_{\rm eq}=-0.5$.

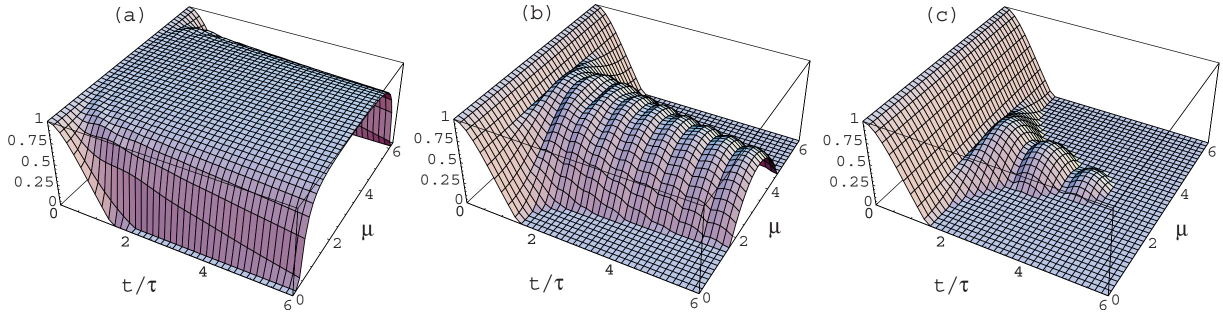

FIG. 4. (Color online) Nonclassical depth $\tau_c(t)$ given by Eq. (57) as a function of time t and pulse area μ $(0 \leq \mu \leq 2\pi)$, where $\Delta t/\tau$ =(a) 0.1, (b) 0.6, and (c) 1.2. In the figure, we set $\tau_c(0)=0.0$, $\kappa\tau=0.5$, and $\bar{n}=1.0$.

$$F_s(t;z) = \frac{1}{\pi}\mathrm{Tr}[\hat{\Delta}_{-s}(z)\hat{\rho}(t)], \tag{46}$$

where the Hermitian Δ operator is defined by

$$\hat{\Delta}_s(z) = \int \frac{d^2\alpha}{\pi} D(\alpha) e^{(1/2)s|\alpha|^2 - z^*\alpha + z\alpha^*}, \tag{47}$$

with $\hat{D}(\alpha)$ being the usual displacement operator. The quantum state $\hat{\rho}(t)$ is expressed as

$$\hat{\rho}(t) = \int \frac{d^2 z}{\pi} F_s(t;z)\hat{\Delta}_s(z). \tag{48}$$

In particular, when $s=-1$, 0, and 1, the phase-space function $F_s(t;z)$ becomes the Glauber-Sudarshan P function $P(t;z)$, the Wigner function $W(t;z)$, and the Husimi-Kano Q function $Q(t;z)$. When the quantum state $\hat{\rho}(t)$ is the solution of the non-Markovian master equation (44), the corresponding phase-space function is given by [32,33]

$$F_s(t;z) = \frac{1}{D_s(t,0)} \int \frac{d^2 z'}{\pi} e^{-|z - z' e^{-\Gamma(t,0)}|^2/D_s(t)} F_s(0;z'), \tag{49}$$

with

$$\Gamma(t,t') = \kappa\tau[\Lambda_\mu(t) - \Lambda_\mu(t')] \equiv \Gamma_\mu(t) - \Gamma_\mu(t'), \tag{50}$$

$$D_s(t,t') = \frac{1}{2}[(1+s)(\bar{n}+1) + (1-s)\bar{n}] \times (1 - e^{-2\Gamma_\mu(t) + 2\Gamma_\mu(t')}). \tag{51}$$

The average value of the annihilation operator and the average photon number are given by

$$\langle \hat{a}(t)\rangle = \sqrt{G_\mu(t)}\langle \hat{a}(0)\rangle, \tag{52}$$

$$\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle = G_\mu(t)[\langle \hat{a}^\dagger(0)\hat{a}(0)\rangle + A_\mu(t)], \tag{53}$$

with

$$G_\mu(t) = e^{-2\Gamma_\mu(t)}, \tag{54}$$

$$A_\mu(t) = \bar{n}(e^{2\Gamma_\mu(t)} - 1). \tag{55}$$

The parameter $G_\mu(t)$ represents the gain in the process and the parameter $A_\mu(t)$ is the average number of the noise per unit gain. The input-output relation $\hat{\rho}(t)=\hat{\mathcal{L}}_t\hat{\rho}(0)$ derived from Eq. (49) defines the quantum channel $\hat{\mathcal{L}}_t$. If the inequality $A_\mu(t) \geq 1$ is satisfied, the quantum channel $\hat{\mathcal{L}}_t$ becomes an entanglement-breaking channel [33].

A. Decoherence of nonclassicality

When a quantum state does not have the Glauber-Sudarshan P function, which is neither a non-negative analytic function nor a δ function, the quantum state is called nonclassical. In this case, the nonclassicality of a quantum state can be quantified by means of the nonclassical depth [34]. To define the nonclassical depth of the quantum state $\hat{\rho}(t)$, we introduce

$$R(t;z) = \frac{1}{\tau}\int \frac{d^2 z'}{\pi} e^{-|z - z'|^2/\tau} P(t;z), \tag{56}$$

where $P(t;z)$ is the Glauber-Sudarshan P function of the quantum state $\hat{\rho}(t)$. The nonclassical depth $\tau_c(t)$ is defined as the minimum value of τ so that $R(t;z)$ becomes a non-negative analytic function or a δ function. It satisfies the inequality $0 \leq \tau_c(t) \leq 1$, where $\tau_c(t)=0$ for classical states and $\tau_c(t)=1$ for maximally nonclassical states. Substituting Eq. (49) with $s=-1$ into Eq. (56), we can obtain the nonclassical depth $\tau_c(t)$ of the quantum state $\hat{\rho}(t)$ [33],

$$\tau_c(t) = \max[G_\mu(t)[\tau_c(0) - A_\mu(t)], 0]. \tag{57}$$

This result shows that the nonclassicality of the quantum state $\hat{\rho}(t)$ disappears at the minimum time t_c so that the inequality $A_\mu(t_c) \geq \tau_c(0)$ is established. The nonclassical depth $\tau_c(t)$ is plotted in Fig. 4. The figure clearly shows that when the pulse area μ is not so far from π, the phase modulation μ pulses work well for the dynamical suppression of the decoherence.

B. Decoherence of entanglement

We finally investigate the phase-modulation μ-pulse effect on the decoherence of entanglement of the bipartite

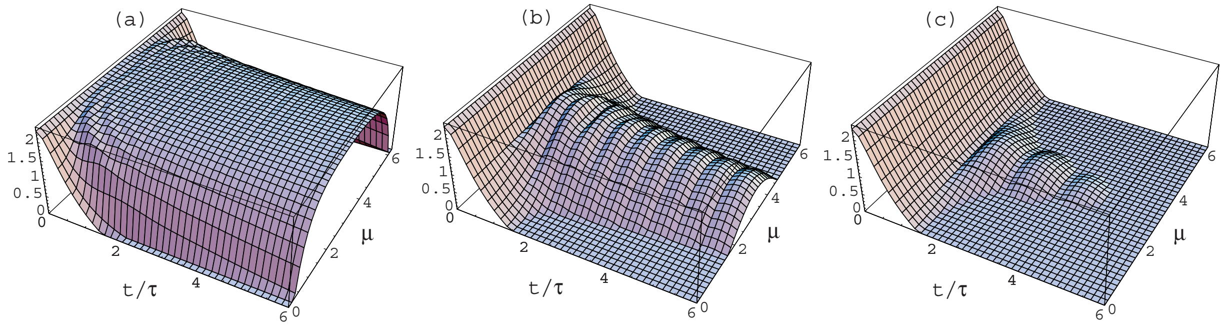

FIG. 5. (Color online) Logarithmic negativity $E_{\mathcal{N}}(t)$ of the quantum state $\hat{\rho}(t)$ as a function of time t and pulse area μ $(0 \leq \mu \leq 2\pi)$, where $\Delta t/\tau =$(a) 0.1, (b) 0.6, and (c) 1.2. In the figure, we set $\kappa\tau = 0.5$, $\bar{n} = 1.0$, and $\bar{n}(0) = 2.0$ with $\bar{m}(0) = 2.45$.

Gaussian state, the characteristic function of which is given by

$$C(0) = \mathrm{Tr}[(e^{z_1\hat{a}_1^\dagger - z_1^*\hat{a}_1}e^{z_2\hat{a}_2^\dagger - z_2^*\hat{a}_2})\hat{\rho}(0)] = e^{-(1/2)\mathsf{z}^\dagger \mathsf{C}(0)\mathsf{z}}, \quad (58)$$

where $\hat{a}_k$ is the bosonic annihilation operator of each mode and $\mathsf{z}^\dagger = (z_1^*, z_1, z_2^*, z_2)$. The 4×4 Hermitian matrix $\mathsf{C}(0)$ is given by

$$\mathsf{C}(0) = \begin{pmatrix} \bar{n}(0) + \frac{1}{2} & 0 & 0 & \bar{m}(0) \\ 0 & \bar{n}(0) + \frac{1}{2} & \bar{m}^*(0) & 0 \\ 0 & \bar{m}(0) & \bar{n}(0) + \frac{1}{2} & 0 \\ \bar{m}^*(0) & 0 & 0 & \bar{n}(0) + \frac{1}{2} \end{pmatrix}, \quad (59)$$

with $\bar{n}(0) = \langle \hat{a}_k^\dagger(0)\hat{a}_k(0)\rangle$ and $\bar{m}(0) = -\langle \hat{a}_1(0)\hat{a}_2(0)\rangle$. The inequality $\bar{n}(0)[\bar{n}(0)+1] \geq |\bar{m}(0)|^2$ holds due to the uncertainty relation. In Eq. (58), we set $\langle \hat{a}_k(0)\rangle = 0$ since the average value of the annihilation operator does not affect the entanglement of the quantum state. We use the logarithmic negativity as a computable measure of entanglement [35]. For the Gaussian state whose characteristic function is given by Eq. (58), the logarithmic negativity is calculated to be

$$E_{\mathcal{N}}(0) = \max[-\ln(2\bar{n}(0) - 2|\bar{m}(0)| + 1), 0]. \quad (60)$$

A quantum state is inseparable or entangled if and only if the logarithmic negativity is positive. When one of the two modes in the quantum state $\hat{\rho}(0)$ evolves by the quantum channel $\hat{\mathcal{L}}_t$ and the other remains unchanged, the two-mode state becomes $\hat{\rho}(t) = (\hat{\mathcal{L}}_t \otimes \hat{\mathcal{I}})\hat{\rho}(0)$, which is also Gaussian. The characteristic function $C(t)$ of the quantum state $\hat{\rho}(t)$ is obtained by replacing the matrix $\mathsf{C}(0)$ in Eq. (58) with

$$\mathsf{C}(t) = \begin{pmatrix} \bar{n}(t) + \frac{1}{2} & 0 & 0 & \bar{m}(t) \\ 0 & \bar{n}(t) + \frac{1}{2} & \bar{m}^*(t) & 0 \\ 0 & \bar{m}(t) & \bar{n}(0) + \frac{1}{2} & 0 \\ \bar{m}^*(t) & 0 & 0 & \bar{n}(0) + \frac{1}{2} \end{pmatrix}, \quad (61)$$

with

$$\bar{n}(t) = G_\mu(t)[\bar{n}(0) + A_\mu(t)], \quad (62)$$

$$\bar{m}(t) = \sqrt{G_\mu(t)}\bar{m}(0). \quad (63)$$

Then the logarithmic negativity $E_{\mathcal{N}}(t)$ of the quantum state $\hat{\rho}(t)$ is given by

$$E_{\mathcal{N}}(t) = \max\left[-\frac{1}{2}\ln[2\Lambda(t)], 0\right], \quad (64)$$

with

$$\Lambda(t) = \left(\bar{n}(t) + \frac{1}{2}\right)^2 + \left(\bar{n}(0) + \frac{1}{2}\right)^2 + 2|\bar{m}(t)|^2 - (\bar{n}(t) + \bar{n}(0) + 1)\sqrt{(\bar{n}(t) - \bar{n}(0))^2 + 4|\bar{m}(t)|^2}. \quad (65)$$

The logarithmic negativity $E_{\mathcal{N}}(t)$ of the quantum state $\hat{\rho}(t)$ is plotted in Fig. 5. We find from this figure that the phase-modulation μ pulses with $|\mu - \pi| \ll 1$ can suppress the decoherence of the entanglement of the Gaussian state.

V. CONCLUDING REMARKS

In this paper, using the non-Markovian quantum master equation derived by means of the projection operator method, we have shown that the decoherence of the relevant system interacting with the thermal reservoir via linear dissipative coupling can be suppressed by means of a sequence of phase-modulation μ pulses. When the pulse separation is sufficiently small in comparison with the reservoir correlation time and the pulse area μ is not so far from π, dynamical decoupling of the decoherence can take place. To see the effect of the phase-modulation μ pulses, we have investigated the time evolutions of the fidelity, purity, and entanglement of qubits and nonclassicality and entanglement of photon systems.

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