Observable effects of Kerr nonlinearity on slow light

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We show how the slow light propagation through a four-level *N*-type system can be significantly influenced by the application of the Kerr field under the condition of electromagnetically induced transparency. The change due to the Kerr field under fairly general conditions can be about 10–15 %.

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In a remarkable paper Schmidt and Imamoğlu [1] discovered that the Kerr nonlinearities could be enhanced by several orders of magnitude by taking advantage of electromagnetically induced transparency. Hau *et al.* reported one of the largest Kerr nonlinearity in a Bose condensate [2]. Kang and Zhu [3] reported a large enhancement of Kerr nonlinearity with vanishing linear susceptibilities in coherently prepared four-level Rb atoms. Using electromagnetically induced transparency (EIT) large cross phase modulation has also been reported [4]. Wang *et al.* proposed [5] the use of double EIT schemes [6–9] for optimal production of cross phase modulation. Kang and Zhu also reported the nature of probe absorption and dispersion under double EIT conditions [3]. However, it would be worthwhile to study the consequences of enhanced Kerr nonlinearities for slow light. This is a regime different from that of double EIT. We demonstrate in this report that under the condition of EIT (not double EIT) the Kerr effect can make a very significant contribution to the group velocity and one should be able to observe this rather easily.

The atomic medium under investigation is a N-shaped four-level system as depicted in Fig. 1. This level scheme has been extensively studied to exhibit large cross-phase modulation [1,3], enhanced nonlinear susceptibilities [10–13], as well as subluminal and superluminal propagation [14]. Here we define all fields as

$$\vec{E}_i(z,t) = \vec{\mathcal{E}}_i(z,t)e^{-i(\omega_i t - k_i z)} + \text{c.c.}, \quad (1)$$

where $\vec{\mathcal{E}}_i$ is the slowly varying envelope of the field and k_i is the wave vector; $i=1,2,3$ refers to the probe field, control field, and Kerr field, respectively. The level $|3\rangle$ is coupled to the lower levels $|1\rangle$ and $|2\rangle$ by the probe field E_1 at frequency ω_1 and the control field E_2 at frequency ω_2, respectively, as shown in Fig. 1. This results in a Λ-type EIT system provided the probe and control fields satisfy a two-photon resonance condition, i.e., $\omega_1-\omega_2=\omega_{12}$. Note that the states $|1\rangle$ and $|2\rangle$ are metastable states. We apply an additional field which acts on the transition $|2\rangle \leftrightarrow |4\rangle$ to demonstrate the effect of Kerr nonlinearity on slow light propagation. The density matrix equations of motion for the four level system under dipole and the rotating wave approximation can be written as follows:

$$\dot{\rho}_{22} = 2\gamma_2\rho_{33} + 2\gamma_3\rho_{44} + iG_1^*\rho_{32} - iG_1\rho_{23} + iG_2^*\rho_{42} - iG_2\rho_{24},$$

$$\dot{\rho}_{33} = -2(\gamma_1+\gamma_2)\rho_{33} + iG_1\rho_{23} + ig\rho_{13} - iG_1^*\rho_{32} - ig^*\rho_{31},$$

$$\dot{\rho}_{44} = -2\gamma_3\rho_{44} + iG_2\rho_{24} - iG_2^*\rho_{42},$$

$$\dot{\rho}_{31} = -[\gamma_1+\gamma_2 - i\Delta_1]\rho_{31} + iG_1\rho_{21} + ig(\rho_{11}-\rho_{33}),$$

$$\dot{\rho}_{32} = -[\gamma_1+\gamma_2 - i\Delta_2]\rho_{32} + ig\rho_{12} + iG_1(\rho_{22}-\rho_{33}) - iG_2\rho_{34},$$

$$\dot{\rho}_{21} = -[\Gamma - i(\Delta_1-\Delta_2)]\rho_{21} + iG_1^*\rho_{31} + iG_2^*\rho_{41} - ig\rho_{23},$$

$$\dot{\rho}_{43} = -[\gamma_1+\gamma_2+\gamma_3 - i(\Delta_3-\Delta_2)]\rho_{43} + iG_2\rho_{23} - ig^*\rho_{41} - iG_1^*\rho_{42},$$

$$\dot{\rho}_{42} = -[\gamma_3 - i\Delta_3]\rho_{42} + iG_2(\rho_{22}-\rho_{44}) - iG_1\rho_{43},$$

$$\dot{\rho}_{41} = -[\gamma_3 - i(\Delta_1-\Delta_2+\Delta_3)]\rho_{41} + iG_2\rho_{21} - ig\rho_{43}, \quad (2)$$

where Δ's and γ's represent the detuning and the rate of spontaneous emission, respectively. The decay rate of the off diagonal element ρ_{21} is given by Γ. These density matrix equations (2) are to be supplemented by the population conservation law

$$\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44}=1. \quad (3)$$

In the original frame of reference, the density matrix elements are given by $\rho_{31}e^{-i\omega_1 t}$, $\rho_{32}e^{-i\omega_2 t}$, $\rho_{21}e^{-i(\omega_1-\omega_2)t}$, $\rho_{42}e^{-i\omega_3 t}$, $\rho_{34}e^{-i(\omega_2-\omega_3)t}$, and $\rho_{41}e^{-i(\omega_1-\omega_2+\omega_3)t}$. The Rabi frequencies of

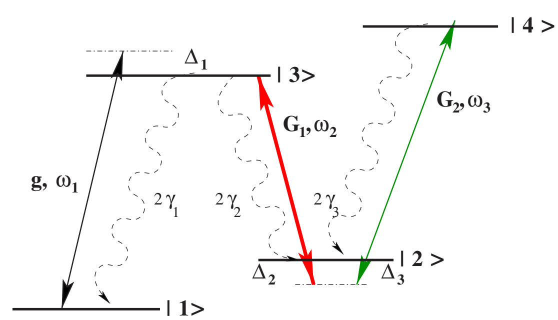

FIG. 1. (Color online) A schematic diagram of a four-level *N*-type atomic system.

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the probe, control and Kerr fields are related to the slowly varying amplitudes of $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$ according to the relation

$$2g = \frac{2\vec{d}_{31} \cdot \vec{\mathcal{E}}_1}{\hbar}, \quad 2G_1 = \frac{2\vec{d}_{32} \cdot \vec{\mathcal{E}}_2}{\hbar}, \quad 2G_2 = \frac{2\vec{d}_{42} \cdot \vec{\mathcal{E}}_3}{\hbar}, \quad (4)$$

where $\vec{d}_{ij}$ is the dipole matrix element corresponding to the atomic transitions. The susceptibility χ can be obtained by considering the steady-state solution of equations (2) to the first order in the probe field g and to all order in control field G_1 and Kerr field G_2. For this purpose we assume that $\gamma_1 = \gamma_2 = \gamma_3 = \gamma/2$ and write the solution as

$$\rho = \rho^0 + \frac{g}{\gamma}\rho^+ + \frac{g}{\gamma}^* \rho^- + \ \ldots . \quad (5)$$

The 13 element of ρ^+ will give the susceptibility at the angular frequency ω_1 which can be expressed as

$$\chi(\Delta_1) = \frac{in|d_{31}|^2}{\hbar} \frac{1}{(\gamma - i\Delta_1) + \cfrac{|G_1|^2}{[\Gamma - i(\Delta_1 - \Delta_2)] + \cfrac{|G_2|^2}{\frac{\gamma}{2} - i(\Delta_1 - \Delta_2 + \Delta_3)}}}, \quad \Delta_1 = \omega_1 - \omega_{31}, \quad (6)$$

where n is the density of the atoms. It is clear from the above expression that $\chi(\Delta_1)$ will depend strongly on the intensities and the detunings of the control and Kerr fields. The above equation (6) produces double EIT. In the limit $\gamma \to 0$, the denominator leads to a cubic equation for Δ_1 and the two real solutions (whenever they occur) of the cubic equation give positions of zero absorption. In this Brief Report, however, we are interested in demonstrating the effect of Kerr nonlinearity on slow light under the EIT condition. Therefore we do not work under the conditions of double EIT. In Figs. 2(a) and 2(b) we show the behavior of the susceptibility as a function of the detuning of the probe field in the presence of the control field as well as the Kerr field. The real part of susceptibility gives the normal dispersion. It is clear from Fig. 2(a) that the slope of normal dispersion decreases with an increase in the intensity of the Kerr field. The imaginary part of χ exhibits minimum absorption when the control field and the Kerr field satisfy equal detuning conditions, i.e., $\Delta_2 = \Delta_3 = \Delta$ as shown in Fig. 2(b). The next most relevant quantity is the group velocity v_g of the probe field which is related to the susceptibility $\chi(\Delta_1)$ as follows:

$$v_g = \frac{c}{n_g} = \frac{c}{1 + 2\pi\chi'(\Delta) + 2\pi\omega_1 \left.\dfrac{\partial \chi'}{\partial \Delta_1}\right|_{\Delta_1=\Delta}}, \quad (7)$$

where $\chi'(\Delta)$ is the real part of the susceptibility χ. Equation (7) shows that v_g depends on $\chi'(\Delta)$ and its slope. The group index n_g can be expressed as

$$n_g = 1 + 2\pi\chi'(\Delta) + 2\pi\omega_1 \left.\frac{\partial \chi'}{\partial \Delta_1}\right|_{\Delta_1=\Delta}$$
$$= 1 + \frac{2\pi\omega_1 n|d_{31}|^2}{\hbar}$$
$$\times \frac{G_1^2\{(\gamma - 2i\Delta)^2 - 4G_2^2\} - \{2G_2^2 + \Gamma(\gamma - 2i\Delta)\}^2}{[(\gamma - i\Delta)[2G_2^2 + \Gamma(\gamma - 2i\Delta)] + G_1^2(\gamma - 2i\Delta)]^2}. \quad (8)$$

In the above equation, probe, control, and Kerr fields satisfy equal detuning conditions i.e., $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$. To understand the effects of the Kerr field in the group index calculation, the susceptibility can be expanded in the following fashion:

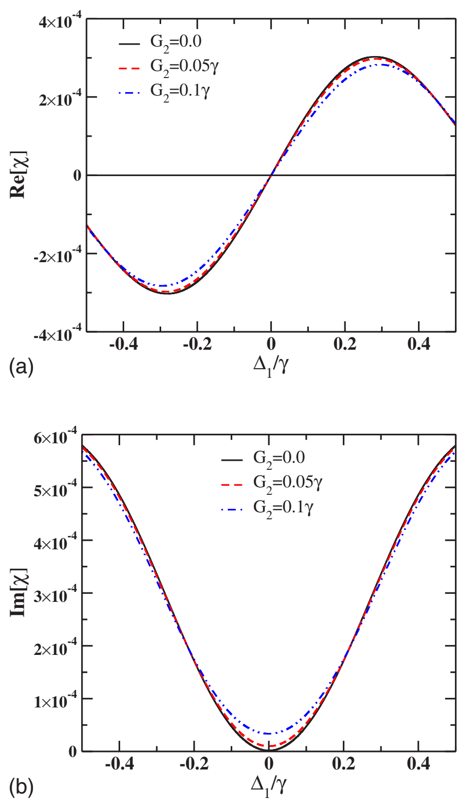

FIG. 2. (Color online) [(a),(b)] The real and imaginary parts, respectively, of the susceptibility χ at probe frequency ω_1 in the presence of the control field G_1 and the Kerr field G_2. Both control and Kerr field are satisfied equal detuning conditions, i.e., $\Delta_2 = \Delta_3 = 0$. The common parameters of the three plots for ^{87}Rb vapor are chosen as follows: density $n = 2 \times 10^{11}$ atoms cm^{-3}; $G_1 = 0.6\gamma$, $\Gamma = 0.001\gamma$, and $2\gamma = 2\pi \times 5.746 \times 10^6$ rad/s.

$$\chi^p = \chi(0) + I_2 \left.\frac{\partial\chi}{\partial I_2}\right|_{I_2=0} = \frac{n|d_{31}|^2}{\hbar}\left[\frac{i[\Gamma - i(\Delta_1-\Delta_2)]}{(\gamma - i\Delta_1)[\Gamma - i(\Delta_1-\Delta_2)] + |G_1|^2} + \frac{i|G_1|^2 I_2}{\left[\frac{\gamma}{2} - i(\Delta_1+\Delta_3-\Delta_2)\right][(\gamma-i\Delta_1)[\Gamma - i(\Delta_1-\Delta_2)]+|G_1|^2]^2}\right], \quad (9)$$

where $I_2=|G_2|^2$. The first term in the large square brackets corresponds to the standard EIT expression in absence of the Kerr field. The second term represents the properties of the EIT system modified by the application of the Kerr field. In the perturbation limit of $|G_2|^2$, the group index of the probe field can be derived as

$$n'_g = n'^{(0)}_g + n'^{(K)}_g = \mathrm{Re}\left[1 + \frac{2\pi\omega_1 n|d_{31}|^2}{\hbar}\left(\frac{G_1^2-\Gamma^2}{[G_1^2+\Gamma(\gamma-i\Delta)]^2} - \frac{4G_1^2G_2^2[G_1^2+\gamma(\gamma+2\Gamma)-3i\Delta(\gamma+\Gamma)-2\Delta^2]}{[G_1^2+\Gamma(\gamma-i\Delta)]^3(\gamma-2i\Delta)^2}\right)\right], \quad (10)$$

where

$$n'^{(0)}_g = \mathrm{Re}\left[1 + \frac{2\pi\omega_1 n|d_{31}|^2}{\hbar}\frac{G_1^2-\Gamma^2}{[G_1^2+\Gamma(\gamma-i\Delta)]^2}\right]$$

$$n'^{(K)}_g = -\mathrm{Re}\left[\frac{2\pi\omega_1 n|d_{31}|^2}{\hbar}\frac{4G_1^2G_2^2[G_1^2+\gamma(\gamma+2\Gamma)-3i\Delta(\gamma+\Gamma)-2\Delta^2]}{[G_1^2+\Gamma(\gamma-i\Delta)]^3(\gamma-2i\Delta)^2}\right]. \quad (11)$$

The expression $n'^{(0)}_g$ represents the effect of control field on the group velocity of the probe in the absence of the Kerr field and this was originally calculated by Harris *et al.* [15]. The effect of the Kerr field on the group index is denoted by $n'^{(K)}_g$. Here the prime of the group index indicates the approximate results obtained from the derivative of the susceptibility expression (9) with respect to the detuning of the probe field at equal detuning conditions. Figure 3 presents the fractional change in the group index $n^{(K)}_g/n^{(0)}_g$ due to the presence of the Kerr field as a function of the control field for different values of the common detuning parameter Δ. The solid black curve of Fig. 3 shows the results obtained from the approximate analysis of the group index as given by Eq. (11). The approximate results match well with the exact results when

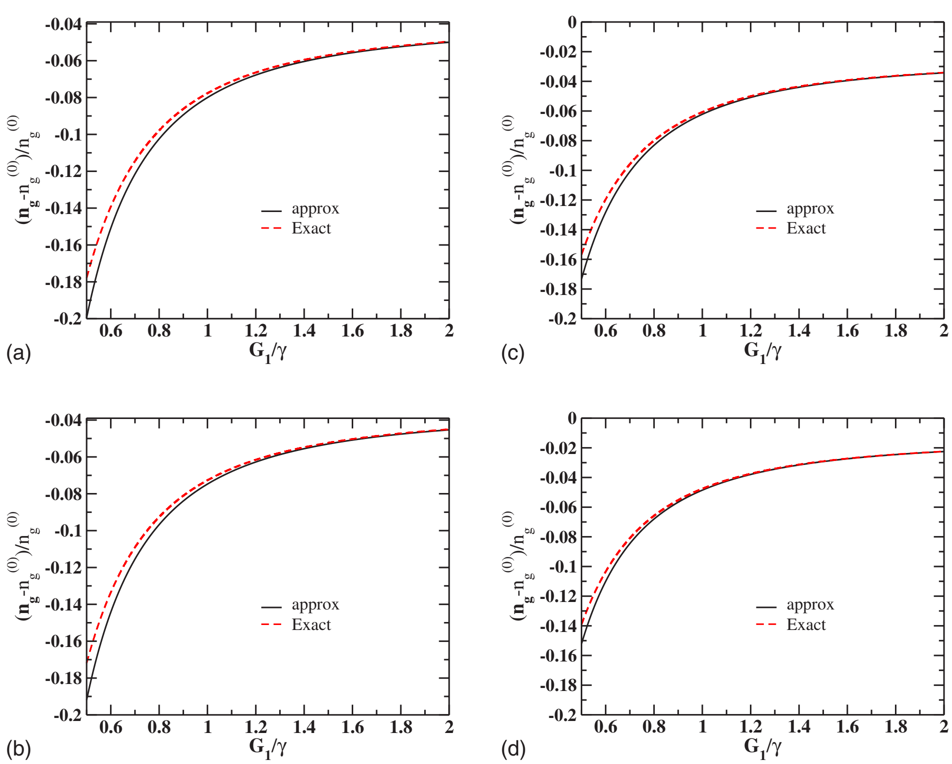

FIG. 3. (Color online) The fractional change in the group index of the probe as the function of the control field at different equal detuning conditions. The equal detuning Δ varies from 0 to 0.3γ in the step of 0.1γ from (a) to (d). The common parameters are chosen as $G_2=0.1\gamma$, $\Gamma=0.001\gamma$, $\Delta_1=\Delta_2=\Delta_3=\Delta$, and $2\gamma=2\pi\times5.746\times10^6$ rad/s.

the control field is much stronger than the Kerr field. From Fig. 3(b), we observe a 15% change in the group index of the probe for $\Delta=0.2\gamma$. Such a significant change is because of the enhanced nonlinearity due to EIT. It should be kept in mind that the width of the transparency window is like in any other EIT experiment using hot atomic vapors as, for example, in the experiment of Kash *et al.* [16]. It is of the order G_1^2/γ. The transparency window broadens with the field G_2 in the region of G_2 values considered here [17].

In conclusion, we have discussed the effect of Kerr nonlinearity on the slow light propagation through a four-level N system under an EIT condition. Our analytical and numerical results clearly show how the group index of the probe field [18] changes significantly due to the presence of Kerr nonlinearity.

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