# Properties of the localized field emitted from degenerate Λ-type atoms in photonic crystals

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The spontaneous emission from a degenerate Λ-type three-level atom, embedded in a photonic crystal, is studied. The emitted field, as a function of time and position, is calculated by solving the three coupled differential equations governing the amplitudes. We show that the spontaneously emitted field is characterized by three components (as in the case of two-level and *V*-type atoms): a localized part, a traveling part, and a $t^{-3/2}$ decaying part. Our calculations indicate that under specific conditions the atoms do not emit propagating fields, while the localized field, having shorter localization length and time, is intensified. As a consequence, the population of the upper level, after a short period of oscillations, approaches a constant value. It is also shown that this steady value, under the same conditions, is much larger than its counterpart in *V*-type atoms.

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## I. INTRODUCTION

A major interest of modern quantum optics is to devise ways to modify and control the properties of spontaneous emission. One possible way for this is to use periodic dielectric structures (photonic crystals) having a photonic band gap. It is well known that, because of the complicated dispersions inside the photonic crystals [1] and the consequent interferences [2], the structure does not allow the electromagnetic field to propagate in any direction [3,4]. The field energy may therefore be mostly trapped inside the crystals. Recently, the entrapment of the fields, emitted by atoms, inside photonic crystals has formed the subject of numerous reports [3–8]. However, in none but a few of such reports is that part of the field, trapped in the vicinity of the atom, the localized field, discussed [6–8]. In the present work, we extend the report of Ref. [8] to investigate the properties of spontaneous emission of a Λ atom in an isotropic photonic crystal. By calculating the time dependent probability amplitudes, we show that, as in the case of two-level [9] and *V*-type atoms [10,11], the emitted field consists of localized, propagating, and diffusing components. Our results, however, indicate that when the upper level lies below the gap, the propagating field is absent, while a strong localized one exists. The properties of this local field, such as its strength and localization length, as compared to that of the *V* type, are also discussed. In particular, we show that the exchange of energy between the localized field (with a shorter localization length and higher intensity) and the atoms causes the excited state population to reach a steady value which is much higher than that in the *V*-type models [12]. This result is drastically altered when absorbing agents, as in Ref. [7], are present. We further show that when the upper level energy exceeds the photonic band gap, the localized field diminishes rather rapidly, contrary to the two-level models in which the localized field may exist even when the upper level lies above the gap [10]. This paper is outlined as follows. In the next section we present the model and the solutions to the atomic dynamical equations for the amplitudes. In Sec. III, the properties of the emitted fields, with due attention to the localized one, are given in detail. Consequently, and for comparison with previous reports [8], the population of the excited state is also discussed in Sec. IV. Finally, we summarize our findings in Sec. V.

## II. MODEL

We consider a degenerate Λ-type atom with one upper level $|a\rangle$ and two lower degenerate levels $|b\rangle$ and $|c\rangle$ (as shown in Fig. 1), embedded in an isotropic photonic crystal. The resonant transition frequencies between levels $|a\rangle$, $|b\rangle$, and $|c\rangle$ are $\omega_{ab}$ and $\omega_{ac}$, respectively, which are assumed to be near the band edge. In the figure the relative position of the upper level with respect to the band gap frequency is also indicated. Close to the band edge, the dispersion relation can be approximated by the effective mass dispersion relation (isotropic crystal) [13]

$$\omega_k \simeq \omega_e + A(k-k_0)^2 \qquad (1)$$

where $\omega_e$ is the cutoff frequency of the corresponding band edge, $k_0$ is the wave vector corresponding to the band edge frequency, and $A$ is a model-dependent constant. The model Hamiltonian under the electric dipole and rotating wave approximations can be written as follows:

$$H = \sum_{\vec{k},\alpha} \hbar\omega_{\vec{k}} a^{\dagger}_{\vec{k},\alpha} a_{\vec{k},\alpha} + \sum_i E_i \sigma_{ii} + \hbar \sum_{\vec{k},\alpha} g^{ab}_{\vec{k},\alpha} (\sigma_{ab} a_{\vec{k},\alpha} - \sigma_{ba} a^{\dagger}_{\vec{k},\alpha}) + \hbar \sum_{\vec{k},\alpha} g^{ac}_{\vec{k},\alpha} (\sigma_{ac} a_{\vec{k},\alpha} - \sigma_{ca} a^{\dagger}_{\vec{k},\alpha}), \qquad (2)$$

where $a_{\vec{k},\alpha}$ ($a^{\dagger}_{\vec{k},\alpha}$) is the radiation field annihilation (creation) operator for the $k'$th electromagnetic mode with frequency $\omega_{\vec{k},\alpha}$. The coupling constants between the $k'$th electromagnetic mode and the atomic transitions $|a\rangle \to |j\rangle$ ($j=b,c$), are $g_{\vec{k},\alpha} = (\omega_a d/\hbar)(\hbar/2\epsilon_0 \omega_{k,\alpha} V)^{1/2} \hat{\varepsilon}_{\vec{k},\alpha} \cdot \hat{u}$, which are assumed to

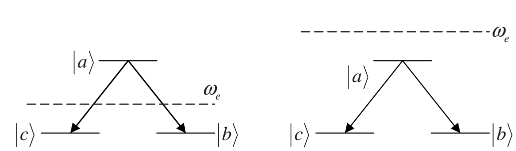


FIG. 1. Schematic representation of the model.

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be real. Here, $(\vec{k},\alpha)$ represent the momentum and polarization (along $\hat{\varepsilon}_{\vec{k},\alpha}$) of the radiation modes, $d$ and $\hat{u}$ are the magnitude and unit vector of the atomic transition dipole moments. The state of the system at any time $t$ can be written as

$$|\psi(t)\rangle = A(t)|a\rangle|0\rangle_f + \sum_{\vec{k},\alpha} B_{\vec{k},\alpha}(t)|b\rangle|1_{\vec{k},\alpha}\rangle_f + \sum_{\vec{k},\alpha} C_{\vec{k},\alpha}(t)|c\rangle|1_{\vec{k},\alpha}\rangle_f, \tag{3}$$

where the state vector $|a\rangle|0\rangle_f$ describes the atom in its excited state $|a\rangle$ and no photons present in the reservoir modes, and the state vectors $|j\rangle|1_{\vec{k},\alpha}\rangle_f$ $(j=b,c)$ describe the atom in its ground states with a single photon in the $k'$th mode with frequency $\omega_{\vec{k},\alpha}$. We assume the atom is initially excited, i.e., $|A_1(0)|^2=1$, $B_{\vec{k},\alpha}(0)=C_{\vec{k},\alpha}(0)=0$, and the energy of the two degenerate lower levels are set to zero $(\omega_b=\omega_c=0)$. Substituting the latter equations into the Schrödinger equation, we obtain

$$\frac{\partial A(t)}{\partial t} = -\sum_{\vec{k},\alpha} g_{\vec{k},\alpha} B_{\vec{k},\alpha}(t) e^{i(\omega_{ab}-\omega_{\vec{k},\alpha})t} - \sum_{\vec{k},\alpha} g_{\vec{k},\alpha} C_{\vec{k},\alpha}(t) e^{-i(\omega_{ac}-\omega_{\vec{k},\alpha})t}, \tag{4}$$

$$\partial B_{\vec{k},\alpha}(t)/\partial t = g_{\vec{k},\alpha} A(t) e^{-i(\omega_{ab}-\omega_{\vec{k},\alpha})t}, \tag{5}$$

$$\partial C_{\vec{k},\alpha}(t)/\partial t = g_{\vec{k},\alpha} A(t) e^{i(\omega_{ac}-\omega_{\vec{k},\alpha})t}. \tag{6}$$

Using the Laplace transforms of Eqs. (4)–(6) and solving for $A(s)[=\int e^{-st}A(t)dt]$, we find

$$A(s) = -\frac{1}{s+2\Gamma}, \tag{7}$$

where $\Gamma = -\frac{i\beta^{3/2}}{\sqrt{\omega_e}+\sqrt{-\omega_{ae}-is}}$, $\omega_{ae}=\omega_a-\omega_e$, and $\beta^{3/2} = \frac{(\omega_a d)^2}{8\epsilon_0 \hbar \pi A^{3/2}} \sin^2\theta$, $\theta$ being the angle between the atomic transition dipole moments and $\vec{k}_0$. The amplitude $A(t)$ can be calculated by means of the inverse Laplace transform,

$$A(t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} A(s) e^{st} ds, \tag{8}$$

where the real constant $\sigma$ is chosen such that the line $s=\sigma$ lies to the right of all singularities (poles and branch points) of $A(s)$. With proper selections of branch cuts and contours, the inverse Laplace transform is performed, leading to

$$A(t) = \sum_j \frac{e^{x_j^{(1)}t}}{F'(x_j^{(1)})} + \sum_j \frac{e^{x_j^{(2)}t}}{E'(x_j^{(2)})} - \frac{e^{i\omega_{ae}t}}{2\pi i}\int_0^\infty \frac{I_1(x)}{I_1(x)L(x)+2i\beta^{3/2}} e^{-xt}dx - \frac{e^{i\omega_{ae}t}}{2\pi i}\int_0^\infty \frac{I_2(x)}{I_2(x)L(x)+2i\beta^{3/2}} e^{-xt}dx, \tag{9}$$

where $F(x)=\frac{2i\beta^{3/2}}{\sqrt{\omega_e}+\sqrt{-\omega_{ae}-ix}}-x$, $E(x)\frac{2i\beta^{3/2}}{\sqrt{\omega_e}-\sqrt{-\omega_{ae}-ix}}-x$, $I_1(x)=\sqrt{\omega_e}+\sqrt{ix}$, $I_2(x)=\sqrt{\omega_e}-\sqrt{ix}$, $L(x)=x-i\omega_{ae}$, $x_j^{(1)}$ are the roots of the equation $F(x)=0$, in the regions $[\mathrm{Im}(x_j^{(1)})>\omega_{ae}$ or $\mathrm{Re}(x_j^{(1)})>0]$ and $x_j^{(2)}$ are the roots of the equation $E(x)=0$, in the regions $[\mathrm{Im}(x_j^{(2)})<\omega_{ae}$ and $\mathrm{Re}(x_j^{(2)})<0]$. The structure of Eq. (9) along with the above mentioned properties are drastically different from those encountered in the case of the $V$-type atoms [14]. These differences, as we shall see, lead to a much stronger localized field which, in turn, affects the upper level population. From the differential equation for $B_{\vec{k},\alpha}(t)$ we have

$$\begin{aligned}\vec{B}_{\vec{k},\alpha}(t) &= g_{\vec{k},\alpha}\sum_j\left(\frac{1}{F'(x_j^{(1)})}\frac{e^{-i(\omega_a-\omega_k)t+x_j^{(1)}}t-1}{-i(\omega_a-\omega_k)+x_j^{(1)}}\right) + g_{\vec{k},\alpha}\sum_j\left(\frac{1}{E'(x_j^{(2)})}\frac{e^{-i(\omega_a-\omega_k)t+x_j^{(2)}}t-1}{-i(\omega_a-\omega_k)+x_j^{(2)}}\right)\\ &- \frac{g_{\vec{k},\alpha}}{2\pi i}\int_0^\infty \frac{I_1(x)}{I_1(x)L(x)+2i\beta^{3/2}}\frac{e^{-i(\omega_a-\omega_k)t-xt}-1}{-i(\omega_a-\omega_k)-x}dx - \frac{g_{\vec{k},\alpha}}{2\pi i}\int_0^\infty \frac{I_2(x)}{I_2(x)L(x)+2i\beta^{3/2}}\frac{e^{-i(\omega_a-\omega_k)t-xt}-1}{-i(\omega_a-\omega_k)-x}dx.\end{aligned} \tag{10}$$

In a similar manner, one could calculate $C_{\vec{k},\alpha}(t)$, but because of the assumption that $(\omega_b=\omega_c=0)$, $B_{\vec{k},\alpha}(t)=C_{\vec{k},\alpha}(t)$.

## III. THE EMITTED FIELDS

The emitted field at a fixed space point $\vec{r}$ away from the source can be calculated from $B_{\vec{k},\alpha}(t)=[C_{\vec{k},\alpha}(t)]$ as [15]

$$\vec{E}(\vec{r},t) = 2\sum_{\vec{k},\alpha}\sqrt{\frac{\hbar\omega_{\vec{k},\alpha}}{2\epsilon_0 V}} e^{-i(\omega_{\vec{k},\alpha}t-\vec{k}\cdot\vec{r})} B_{\vec{k},\alpha}(t)\hat{\varepsilon}_{\vec{k},\alpha}. \tag{11}$$

Using Eq. (10) in Eq. (11), the radiation field $\vec{E}(\vec{r},t)$ is expressed as the sum of three parts:

$$\vec{E}(\vec{r},t) = \vec{E}_l(\vec{r},t) + \vec{E}_p(\vec{r},t) + \vec{E}_d(\vec{r},t). \tag{12}$$

The first part, the localized field, $\vec{E}_l(\vec{r},t)=\Sigma_j E_l^j(\vec{r},t)$, coming from the first term in Eq. (10), is

$$\vec{E}_l(\vec{r},t) = \sum_j E_l^j(0)\frac{1}{r} e^{-i(\omega_a-b_j^{(1)})t-r/l_j}\Theta\left(t-\frac{l_j r}{2A}\right), \tag{13}$$

where

$$E_l^j(0) = \frac{\omega_a d_a}{4A\pi\epsilon_0 i}\frac{1}{F'(x_j^{(1)})} e^{i\vec{k}_0\cdot\vec{r}}\left[\hat{u} - \frac{\vec{k}_0(\vec{k}_0\cdot\hat{u})}{(\vec{k}_0)^2}\right],$$

with $x_j^{(1)}=ib_j^{(1)}$ denoting the pure imaginary roots of $F(x)=0$ and $\theta(x)$ is the Heaviside step function. The frequency of $E_l^j(\vec{r},t)$ is $\omega_a-b_j^{(1)}$, which is smaller than $\omega_e$ and within the forbidden band. Obviously, $E_l^j(\vec{r},t)$ represents a localized mode in the localized field. The amplitude of the localized mode drops exponentially against the distance from the atom as $e^{-r/l_j}$. The size of the localized field is determined by the localization length, $l_j=[(-ix_j^{(1)}-\omega_{ae})/A]^{-1/2}$. The presence of

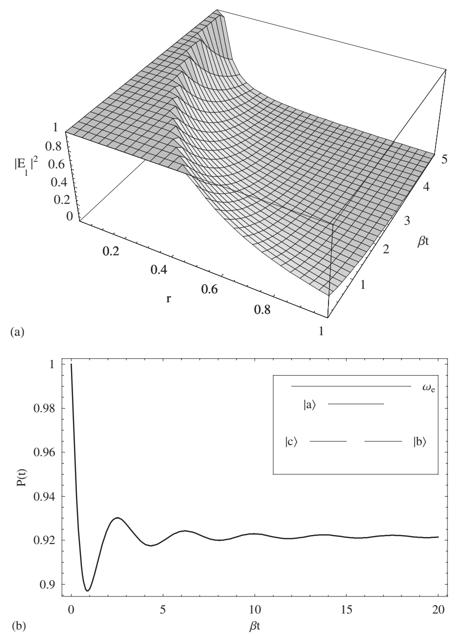


FIG. 2. (a) The normalized localized field as a function of scaled time $\beta t$ and distance $r$. Here $\omega_e = 3\beta$, $\omega_{ae} = -\beta$. (b) Atomic population on the upper level.

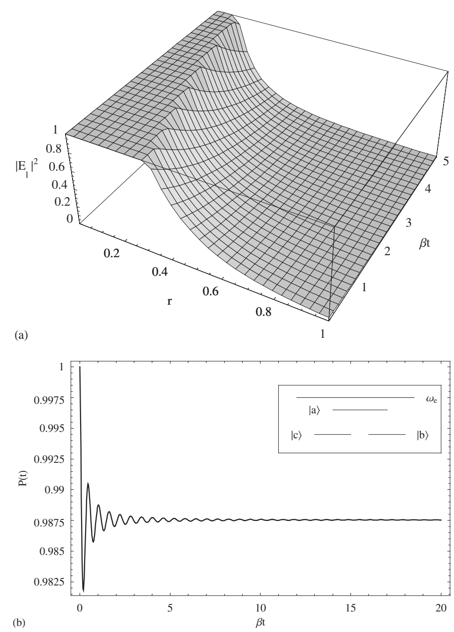


FIG. 3. (a) The normalized localized field as a function of scaled time $\beta t$ and distance $r$. Here $\omega_e = 3\beta$, $\omega_{ae} = -10\beta$. (b) Atomic population on the upper level.

this local field causes the population in the lower levels to jump back to the upper level by absorbing photons from the localized field [10].

The second part, $\vec{E}_p(\vec{r},t) = \Sigma_j E_l^j(\vec{r},t)$, coming from the second term in Eq. (10), forms the propagating field. We emphasize that in calculating the propagating field only the poles of the complex roots $x_j^{(2)}$ should be included. We then find that

$$\vec{E}_p(\vec{r},t) = \sum_j E_p^j(0) \frac{1}{r} e^{(x_j^{(2)} - i\omega_a)t + iq_j r} \times \Theta\{t - r/2A[\mathrm{Re}(q_j) + \mathrm{Im}(q_j)]\}, \tag{14}$$

where $q_j = [(ix_j^{(2)} + \omega_{ae})/A]^{1/2}$ and

$$E_p^j(0) = \frac{\omega_a d_a}{4A\pi\epsilon_0 i} \frac{1}{E'(x_j^{(2)})} e^{i\vec{k}_0 \cdot \vec{r}} \left[ \hat{u} - \frac{\vec{k}_0(\vec{k}_0 \cdot \hat{u})}{(\vec{k}_0)^2} \right].$$

$x_j^{(2)} = a_j^{(2)} + ib_j^{(2)}$ with $a_j^{(2)} < 0$ and $b^{(2)} < \omega_{ae}$, the frequency of the propagating field, $(\omega_a - b^{(2)})$, is larger than $\omega_e$, and within the transmitting band: $E_p^j(\vec{r},t)$ is a propagating mode, which travels away coherently from the atom in the form of a traveling pulse.

The third part $\vec{E}_d(\vec{r},t)$ is a diffusing field and comes from the integration along the branch cuts in Eq. (10). It can be written as

$$\vec{E}_d(\vec{r},t) = E_d(0) \frac{1}{r} e^{-i\omega_e t + ir^2/4At + 3i\pi/4} \times \int_0^\infty \left( \frac{I_1(x)}{I_1(x)L_1(x) + 2i\beta^{3/2}} - \frac{I_2(x)}{I_2(x)L_1(x) + 2i\beta^{3/2}} \right) \times dx \int_{-\infty}^{\infty} \frac{[ye^{3\pi i/4} + r/(2\sqrt{At})]e^{-y^2}}{-xt + i[ye^{3\pi i/4} + r/(2\sqrt{At})]^2} dy, \tag{15}$$

where

$$4E_d^j(0) = \frac{\omega_a d_a}{4A\pi^3 \epsilon_0} e^{i\vec{k}_0 \cdot \vec{r}} \left[ \hat{u} - \frac{\vec{k}_0(\vec{k}_0 \cdot \hat{u})}{(\vec{k}_0)^2} \right].$$

In the long time limit, $\vec{E}_d(\vec{r},t)$ can be approximated as

$$\vec{E}_d(\vec{r},t) \sim t^{-3/2} e^{-i\omega_e t + ir^2/(4At)}. \tag{16}$$

The above equation indicates that the energy of this field diffuses out incoherently. Contrary to the localized and propagating fields, there is no well-defined frequency to be assigned to the diffusing field [14]. The number of the localized and propagating modes strongly depends upon the relative position of the upper level and the band edge. We consider the case of $\omega_e > \omega_a$ and we investigate the properties of the emitted field for this special case. In this case there is no propagating mode in the emitted spectrum because there is no complex root $x_j^{(2)}$ for the equation $E(x) = 0$. In the next section we calculated the localized field for this case and consequently for comparison with a previous report [8], the population of the excited state is also discussed. The population in the upper level of the atom can be obtained from Eq. (9), $P(t) = |A(t)|^2$.

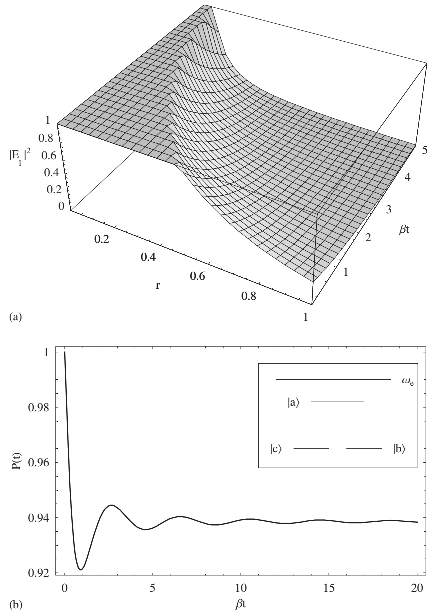


FIG. 4. (a) The normalized localized field as a function of scaled time $\beta t$ and distance $r$. Here $\omega_e=5\beta$, $\omega_{ae}=-\beta$. (b) Atomic population on the upper level.

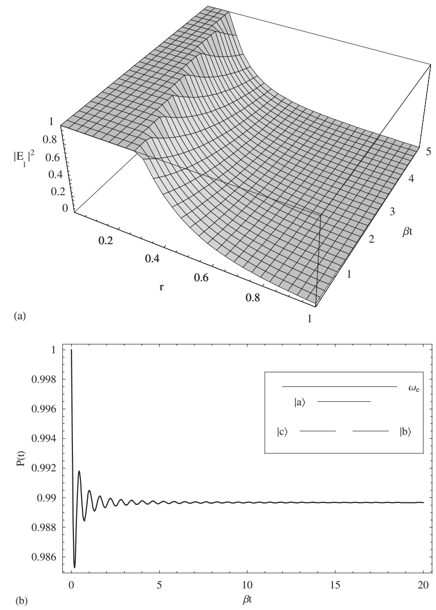


FIG. 5. (a) The normalized localized field as a function of scaled time $\beta t$ and distance $r$. Here $\omega_e=5\beta$, $\omega_{ae}=-10\beta$. (b) Atomic population on the upper level.

## IV. THE PROPERTIES OF EMITTED FIELDS AND UPPER LEVEL POPULATION

As we described in the previous section, if the band gap frequencies lie above the upper level, there is no complex root $x_j^{(2)}$ for the equation $E(x)=0$ and consequently in the emitted field spectrum the propagating field is absent. We calculate the propagating field for the case $\omega_e > \omega_a$. The exchange of energy between the localized field (with a shorter localization length and higher intensity) and the atoms causes the excited state population to reach a steady value which is much higher than that in the *V*-type models. In Figs. 2(a), 2(b), 3(a), 3(b), 4(a), 4(b), 5(a), and 5(b) the localized field, as a function of position (relative to the atoms) and time, for the cases in which the band gap frequencies lie above the upper level, for different relative position and different band gap frequencies, is presented. For this case the atomic population on the lower levels can jump back to the upper level by absorbing energy from the localized field and after a short period of oscillations, approaches a large constant value [see Fig. 2(b)]. In Fig. 3(a) we show that by increasing the difference between the energy of the excited state and band gap, the localization length of the localized field decreases while its intensity increases. Consequently the population in this case approaches a larger constant value [see Fig. 3(b)]. Furthermore, a comparison of Figs. 2(a) and 4(a), or Figs. 3(a) and 5(a), in which the band edge frequency is increased, while the detuning, $\omega_{ae}=\omega_a-\omega_e$, is fixed, shows that the intensity of the localized field also increases. This point, in turn, gives a larger constant value for the population of the upper level [see Figs. 2(b) and 4(b) or Figs. 3(b) and 5(b)].

## V. CONCLUSION

In this Brief Report we have presented the properties of the localized field emitted by three-level $\Lambda$-type atoms inside a photonic crystal. We have shown that, in contrast to the *V*-type models, the localized field only exists when the energy of the upper atomic level lies below the photonic gap. In that case, it is further shown that the localization length and time are reduced (leading to higher intensify of this field) in comparison with that of the localized fields, emitted by *V*-type atoms. Consequently, the atomic upper level population, due to the higher intensity of the localized field, approaches a higher steady value than its counterpart in *V*-type models.

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