Unified theory for Goos-Hänchen and Imbert-Fedorov effects

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A unified theory is advanced to describe both the lateral Goos-Hänchen (GH) effect and the transverse Imbert-Fedorov (IF) effect, through representing the vector angular spectrum of a three-dimensional light beam in terms of a two-form angular spectrum consisting of its two orthogonal polarized components. From this theory, the quantization characteristics of the GH and IF displacements are obtained, and the Artmann formula for the GH displacement is derived. It is found that the eigenstates of the GH displacement are the two orthogonal linear polarizations in this two-form representation, and the eigenstates of the IF displacement are the two orthogonal circular polarizations. The theoretical predictions are found to be in agreement with recent experimental results.

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I. INTRODUCTION

In 1947, Goos and Hänchen [1] experimentally demonstrated that a totally reflected light beam at a plane dielectric interface is laterally displaced in the incidence plane from the position predicted by geometrical reflection. Artmann [2] in the next year advanced a formula for this displacement on the basis of a stationary-phase argument. This phenomenon is now referred to as the Goos-Hänchen (GH) effect. In 1955, Fedorov [3] expected a transverse displacement of a totally reflected beam from the fact that an elliptical polarization of the incident beam entails a nonvanishing transverse energy flux inside the evanescent wave. Imbert [4] calculated this displacement using an energy flux argument developed by Renard [5] for the GH effect and experimentally measured it. This phenomenon is usually called the Imbert-Fedorov (IF) effect. The investigation of the GH effect has been extended to the cases of partial reflection and transmission in transmitting configurations [6,7] and to other areas of physics, such as acoustics [8], nonlinear optics [9], plasma physics [10], and quantum mechanics [5,11], and the IF effect has been connected with the angular momentum conservation and the Hall effect of light [12,13]. But the comment of Beauregard and Imbert [14] is still valid up to now that there are, strictly speaking, no completely rigorous calculations of the GH or IF displacement. Though the argument of the stationary phase was used to explain [2] the GH displacement and to calculate the IF displacement [15], it was on the basis of the formal properties of the Poynting vector inside the evanescent wave [14] that the quantization characteristics were acquired for both the GH and IF displacements in total reflection. On the other hand, it has been found that the GH displacement in transmitting configurations has nothing to do with the evanescent wave [7].

The purpose of this paper is to advance a unified theory for the GH and IF effects through representing the vector angular spectrum of a three-dimensional (3D) light beam in terms of a two-form angular spectrum, consisting of its two orthogonal polarizations. From this theory, the quantization characteristics of the GH and IF displacements are obtained, and the Artmann formula [2] for the GH displacement is derived. The amplitude of the two-form angular spectrum describes the polarization state of a beam in such a way that the eigenstates of the GH displacement are the two orthogonal linear polarizations and the eigenstates of the IF displacement are the two orthogonal circular polarizations.

II. GENERAL THEORY

Consider a monochromatic 3D light beam in a homogenous and isotropic medium of refractive index n that intersects the plane $x=0$. In order to have a beam representation that can describe the propagation parallel to the x axis, the vector electric field of the beam is expressed in terms of its vector angular spectrum as follows [16]:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{A}(k_y,k_z)\exp(i\mathbf{k}\cdot\mathbf{r})dk_y dk_z, \qquad (1)$$

where time dependence $\exp(-i\omega t)$ is assumed and suppressed, $\mathbf{A}=(A_x \quad A_y \quad A_z)^T$ is the vector amplitude of the angular spectrum, $\mathbf{k}=(k_x \quad k_y \quad k_z)^T$ is the wave vector satisfying $k_x^2+k_y^2+k_z^2=k^2$, $k=2n\pi/\lambda_0$, λ_0 is the vacuum wavelength, the superscript T means transpose, and the beam is supposed to be well collimated so that its angular distribution function is sharply peaked around the principal axis (k_{y0},k_{z0}) and that the integration limits have been extended to $\pm\infty$ for both variables k_y and k_z [17]. When this beam intersects the plane $x=0$, the electric-field distribution on this plane is thus

$$\mathbf{\Psi}(y,z) \equiv \mathbf{E}(\mathbf{r})|_{x=0} = \frac{1}{2\pi}\int\int \mathbf{A}e^{i(k_y y+k_z z)}dk_y dk_z,$$

hereafter the integration limits will be omitted as such. The position coordinates of the centroid of the beam (1) on the plane $x=0$ are defined by

$$\langle y\rangle = \frac{\int\int \mathbf{\Psi}^\dagger y\mathbf{\Psi}dydz}{\int\int \mathbf{\Psi}^\dagger\mathbf{\Psi}dydz} = \frac{\int\int \mathbf{A}^\dagger i\frac{\partial\mathbf{A}}{\partial k_y}dk_y dk_z}{\int\int \mathbf{A}^\dagger\mathbf{A}dk_y dk_z} \qquad (2)$$

and

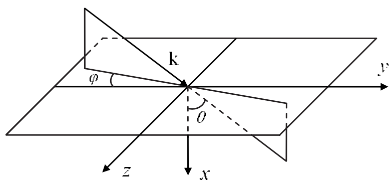

FIG. 1. Schematic diagram for the rotation of the vector amplitude **A** of an arbitrarily polarized light beam.

$$\langle z\rangle = \frac{\int\int \mathbf{\Psi}^\dagger z \mathbf{\Psi} dydz}{\int\int \mathbf{\Psi}^\dagger \mathbf{\Psi} dydz} = \frac{\int\int \mathbf{A}^\dagger i \frac{\partial \mathbf{A}}{\partial k_z} dk_y dk_z}{\int\int \mathbf{A}^\dagger \mathbf{A} dk_y dk_z}, \quad (3)$$

where $\frac{\partial}{\partial k_y}$ means partial derivative with respect to k_y with k_z fixed, $\frac{\partial}{\partial k_z}$ means partial derivative with respect to k_z with k_y fixed, and superscript † stands for transpose conjugate.

Since the Fresnel formula for the amplitude reflection coefficient at a dielectric interface depends on whether the incident plane wave is in s or p polarization, it is profitable to represent the vector amplitude of the angular spectrum in terms of its s and p polarized components. To this end, let us first consider one plane-wave element of the angular spectrum whose wave vector is $\mathbf{k}^0=(k\cos\theta \quad k\sin\theta \quad 0)^T$, where θ is its incidence angle. Its vector amplitude is given by $\mathbf{A}^0=\mathbf{A}_s^0+\mathbf{A}_p^0 \equiv A_s\mathbf{s}^0+A_p\mathbf{p}^0$, where A_s and A_p are the complex amplitudes of $\mathbf{A}_s^0$ and $\mathbf{A}_p^0$, respectively, $\mathbf{s}^0=(0 \quad 0 \quad 1)^T$ is the unit vector of $\mathbf{A}_s^0$ and is perpendicular to the plane xoy, and $\mathbf{p}^0=(-\sin\theta \quad \cos\theta \quad 0)^T$ is the unit vector of $\mathbf{A}_p^0$ and is parallel to the plane xoy. This means that $\mathbf{A}^0$ can be represented as

$$\mathbf{A}^0 = \begin{pmatrix} 0 & -\sin\theta \\ 0 & \cos\theta \\ 1 & 0 \end{pmatrix} \tilde{A},$$

where

$$\tilde{A} = \begin{pmatrix} A_s \\ A_p \end{pmatrix} \equiv A_s\tilde{s} + A_p\tilde{p} \quad (4)$$

is what we introduce in this paper and is referred to as the two-form amplitude of the angular spectrum,

$$\tilde{s} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

represents the normalized state of s polarization, and

$$\tilde{p} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

represents the normalized state of p polarization. $\tilde{s}$ and $\tilde{p}$ form the orthogonal complete set of linear polarizations.

After this element is rotated by angle φ around the x axis as is displayed in Fig. 1, its wave vector becomes

$$\mathbf{k} = M(\varphi)\mathbf{k}^0 = \begin{pmatrix} k\cos\theta \\ k\sin\theta\cos\varphi \\ k\sin\theta\sin\varphi \end{pmatrix},$$

and its vector amplitude becomes

$$\mathbf{A} = M(\varphi)\mathbf{A}^0 = A_s\mathbf{s} + A_p\mathbf{p}, \quad (5)$$

where

$$M(\varphi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

is the rotation matrix,

$$\mathbf{s} = M(\varphi)\mathbf{s}^0 = (0 \quad -\sin\varphi \quad \cos\varphi)^T$$

is the unit vector of $\mathbf{A}_s = M(\varphi)\mathbf{A}_s^0$, and

$$\mathbf{p} = M(\varphi)\mathbf{p}^0 = \begin{pmatrix} -\sin\theta \\ \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \end{pmatrix}$$

is the unit vector of $\mathbf{A}_p = M(\varphi)\mathbf{A}_p^0$. This shows that the vector amplitude (5) can be represented as

$$\mathbf{A} = \begin{pmatrix} 0 & p_x \\ s_y & p_y \\ s_z & p_z \end{pmatrix} \tilde{A} \equiv P\tilde{A}, \quad (6)$$

where the matrix P represents the projection of two-form amplitude $\tilde{A}$ onto vector amplitude **A** and is thus referred to as the projection matrix, and

$$s_y = -\sin\varphi = -\frac{k_z}{(k_y^2+k_z^2)^{1/2}},$$

$$s_z = \cos\varphi = \frac{k_y}{(k_y^2+k_z^2)^{1/2}},$$

$$p_x = -\sin\theta = -\frac{(k_y^2+k_z^2)^{1/2}}{k},$$

$$p_y = \cos\theta\cos\varphi = \frac{k_x k_y}{k(k_y^2+k_z^2)^{1/2}},$$

$$p_z = \cos\theta\sin\varphi = \frac{k_x k_z}{k(k_y^2+k_z^2)^{1/2}}.$$

Now we have successfully represented, through the projection matrix, the vector amplitude **A** in terms of the two-form amplitude $\tilde{A}$, that is to say, in terms of the two orthogonal linear polarizations $\mathbf{A}_s$ and $\mathbf{A}_p$. It should be pointed out that in this representation, **s** is not necessarily perpendicular to the plane xoy, and **p** is not necessarily parallel to this plane. Denoting $\mathbf{k}_r=k_r\mathbf{e}_r=k_y\mathbf{e}_y+k_z\mathbf{e}_z$, where $k_y=k_r\cos\varphi$, $k_z=k_r\sin\varphi$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are the unit vectors in the y and z directions, respectively, and $\mathbf{e}_r$ is the unit vector in the radial

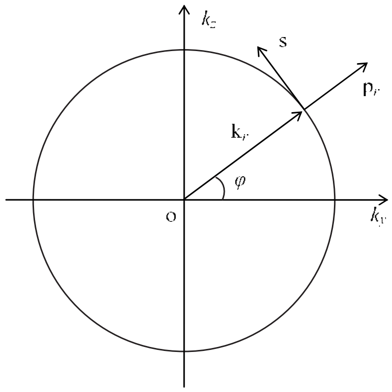

FIG. 2. $\mathbf{s}$ and $\mathbf{p}_r$ are in the azimuthal and radial directions, respectively.

direction, we find that $\mathbf{s}$ is in fact the unit vector in the azimuthal direction, $\mathbf{s}=\mathbf{e}_\varphi$. Furthermore, letting $\mathbf{p}_r=p_y\mathbf{e}_y+p_z\mathbf{e}_z$, it is apparent that $\mathbf{p}_r=\frac{k_x}{k}\mathbf{e}_r$. In other words, $\mathbf{p}_r$ is in the radial direction. The directions of $\mathbf{s}$ and $\mathbf{p}_r$ are schematically shown in Fig. 2.

Unit vectors $\mathbf{s}$ and $\mathbf{p}$ and the wave vector $\mathbf{k}$ are orthogonal to each other and thus satisfy the following relations:

$$s_y^2+s_z^2=1,$$

$$p_x^2+p_y^2+p_z^2=1,$$

$$s_yp_y+s_zp_z=0,$$

$$k_ys_y+k_zs_z=0,$$

$$k_xp_x+k_yp_y+k_zp_z=0. \tag{7}$$

The first three equations guarantee

$$\mathbf{A}^\dagger\mathbf{A}=\tilde{A}^\dagger\tilde{A}. \tag{8}$$

From expression (6) for the vector amplitude and with the help of Eq. (7), we obtain

$$\mathbf{A}^\dagger\frac{\partial\mathbf{A}}{\partial k_y}=\tilde{A}^\dagger\frac{\partial\tilde{A}}{\partial k_y}-\frac{k_xk_z}{k(k_y^2+k_z^2)}(A_s^*A_p-A_p^*A_s), \tag{9}$$

$$\mathbf{A}^\dagger\frac{\partial\mathbf{A}}{\partial k_z}=\tilde{A}^\dagger\frac{\partial\tilde{A}}{\partial k_z}+\frac{k_xk_y}{k(k_y^2+k_z^2)}(A_s^*A_p-A_p^*A_s). \tag{10}$$

Equations (2), (3), and (8)–(10) are the central results of this paper, from which the GH and IF displacements are derived below.

III. DESCRIPTION OF INCIDENT AND REFLECTED BEAMS

Without loss of generality, we consider an arbitrarily polarized *incident* beam of the following two-form amplitude:

$$\tilde{A}_i=(l_{i1}\tilde{s}+l_{i2}\tilde{p})A\equiv\tilde{L}_iA, \tag{11}$$

where $\tilde{L}_i=l_{i1}\tilde{s}+l_{i2}\tilde{p}$ describes the polarization state of the beam and is assumed to satisfy the normalization condition

$$\tilde{L}_i^\dagger\tilde{L}_i=1, \tag{12}$$

the angular distribution function $A(k_y,k_z)$ is assumed to be a positively definite sharply peaked symmetric function around the principal axis $(k_{y0},k_{z0})=(k\sin\theta_0,0)$ and satisfy the normalization condition

$$\int\int A^2(k_y,k_z)dk_ydk_z=1, \tag{13}$$

and θ_0 stands for the incidence angle of the beam. Equations (12) and (13) guarantee the following normalization condition for the two-form amplitude (11):

$$\int\int\tilde{A}_i^\dagger\tilde{A}_idk_ydk_z=1. \tag{14}$$

One example of such a distribution function that satisfies normalization condition (13) is the following Gaussian function [16,18]:

$$A_G=\left(\frac{w_yw_z}{\pi}\right)^{1/2}\exp\left[-\frac{w_y^2}{2}(k_y-k_{y0})^2\right]\exp\left(-\frac{w_z^2}{2}k_z^2\right), \tag{15}$$

where $w_y=w_0/\cos\theta_0$, $w_z=w_0$, w_0 is half the width of the beam at waist. $\Delta\theta=\frac{1}{kw_0}$ is half the divergence angle of the beam.

According to Eq. (6), the vector amplitude of the incident beam is given by $\mathbf{A}_i=P\tilde{A}_i$. For a uniformly polarized beam that was obtained from a linearly polarized beam in experiments [19–21], the s components of all its plane-wave elements are in the same direction, and the same to the p components. But in our representation advanced here, the s polarizations of different plane-wave elements are generally in different directions; so are the p polarizations. Considering Eqs. (6), (11), and (15) together, one concludes that in order to describe a uniformly polarized beam mentioned above, it is essential that the incidence angle θ_0 be much larger than $\Delta\theta$. So we will only consider the case of large θ_0 below. Fortunately, this is just what we have in the case of total reflection.

It will be convenient to express $\tilde{L}_i$ on the orthogonal complete set of circular polarizations as follows:

$$\tilde{L}_i=c_{i1}\tilde{r}+c_{i2}\tilde{l}=U\tilde{C}_i, \tag{16}$$

where c_{i1} represents the complex amplitude of right circular polarization, c_{i2} represents the complex amplitude of left circular polarization,

$$\tilde{r}=U\tilde{s}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

is the normalized state of right circular polarization,

$$\tilde{l} = U\tilde{p} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

is the normalized state of left circular polarization, U is the unitary transformation matrix

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix},$$

and

$$\tilde{C}_i = \begin{pmatrix} c_{i1} \\ c_{i2} \end{pmatrix}.$$

$\tilde{r}$ and $\tilde{l}$ form the orthogonal complete set of circular polarizations. Unitary transformation guarantees $\tilde{L}_i^\dagger \tilde{L}_i = \tilde{C}_i^\dagger \tilde{C}_i$.

When the beam is reflected at plane $x=0$, the reflected beam has the following two-form amplitude:

$$\tilde{A}_r = R_l \tilde{A}_i = \tilde{L}_r A, \tag{17}$$

where

$$R_l = \begin{pmatrix} R_s & 0 \\ 0 & R_p \end{pmatrix}$$

is the reflection coefficient matrix,

$$\tilde{L}_r = R_l \tilde{L}_i \equiv \begin{pmatrix} l_{r1} \\ l_{r2} \end{pmatrix}$$

describes the polarization state of the reflected beam, and $R_s \equiv |R_s|\exp(i\Phi_s)$ and $R_p \equiv |R_p|\exp(i\Phi_p)$ are the reflection coefficients for s and p polarizations, respectively. It will be convenient to express $\tilde{L}_r$ on the orthogonal complete set of circular polarizations as follows:

$$\tilde{L}_r = c_{r1}\tilde{r} + c_{r2}\tilde{l} = U\tilde{C}_r, \tag{18}$$

where c_{r1} represents the complex amplitude of right circular polarization for reflected beam, c_{r2} represents the complex amplitude of left circular polarization,

$$\tilde{C}_r = \begin{pmatrix} c_{r1} \\ c_{r2} \end{pmatrix} = R_c \tilde{C}_i,$$

and $R_c = U^\dagger R_l U$. Unitary transformation guarantees $\tilde{L}_r^\dagger \tilde{L}_r = \tilde{C}_r^\dagger \tilde{C}_r$.

IV. GH EFFECT AND ITS QUANTIZATION

Applying Eqs. (2), (8), and (9) to $\tilde{A}_i$ produces the y coordinate of the centroid of the incident beam on the plane $x=0$,

$$\langle y \rangle_i = 0.$$

Since R_s and R_p are all even functions of k_z, we have for the y coordinate of the centroid of the reflected beam on the plane $x=0$, on applying Eqs. (2), (8), and (9) to $\tilde{A}_r$,

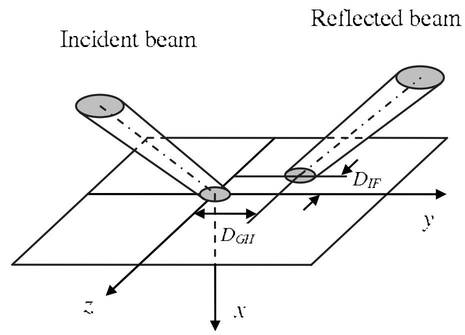

FIG. 3. Schematic diagram for the GH and IF effects, where plane $x=0$ represents the interface between two different dielectric media, and plane $z=0$ represents the incidence plane.

$$\langle y \rangle_r = -\frac{1}{\mathfrak{R}} \int\int \left(|l_{r1}|^2 \frac{\partial \Phi_s}{\partial k_y} + |l_{r2}|^2 \frac{\partial \Phi_p}{\partial k_y} \right) A^2 dk_y dk_z, \tag{19}$$

where

$$\mathfrak{R} = \int\int (|l_{r1}|^2 + |l_{r2}|^2) A^2 dk_y dk_z \tag{20}$$

describes the reflectivity of a 3D beam. The above equation can also be written as

$$\mathfrak{R} = |l_{i1}|^2 \mathfrak{R}_s + |l_{i2}|^2 \mathfrak{R}_p,$$

with

$$\mathfrak{R}_s = \int\int |R_s|^2 A^2 dk_y dk_z$$

and

$$\mathfrak{R}_p = \int\int |R_p|^2 A^2 dk_y dk_z.$$

The displacement of $\langle y \rangle_r$ from $\langle y \rangle_i$ is the GH effect as is schematically shown in Fig. 3 and is thus given by

$$D_{GH} = -\frac{|l_{i1}|^2}{\mathfrak{R}} \int\int |R_s|^2 A^2 \frac{\partial \Phi_s}{\partial k_y} dk_y dk_z - \frac{|l_{i2}|^2}{\mathfrak{R}} \int\int |R_p|^2 A^2 \frac{\partial \Phi_p}{\partial k_y} dk_y dk_z. \tag{21}$$

It is obviously quantized with eigenstates being the s and p polarization states. The eigenvalues are

$$D_{GHj} = -\frac{1}{\mathfrak{R}_j} \int\int |R_j|^2 A^2 \frac{\partial \Phi_j}{\partial k_y} dk_y dk_z,$$

with $j=s,p$. When the angular distribution function $A(k_y,k_z)$ is so sharp that $\frac{\partial \Phi_s}{\partial k_y}$ and $\frac{\partial \Phi_p}{\partial k_y}$ are approximately constant in the area in which A is appreciable, we arrive at the Artmann formula [2],

$$D_{GHj} = -\frac{\partial \Phi_j}{\partial k_y}. \tag{22}$$

It is now clear that the quantization description of GH displacement depends closely on the two-form representation of the angular spectrum.

A. Total reflection

When the beam is *totally* reflected, the reflection coefficients take the form of

$$R_s = \exp(i\Phi_s), \quad R_p = \exp(i\Phi_p), \tag{23}$$

and $\mathfrak{R}=1$. Substituting Eq. (23) into Eq. (21), we obtain

$$D_{GH} = -\int\int\left(|l_{i1}|^2\frac{\partial \Phi_s}{\partial k_y} + |l_{i2}|^2\frac{\partial \Phi_p}{\partial k_y}\right)A^2 dk_y dk_z.$$

If $\frac{\partial \Phi_s}{\partial k_y}$ and $\frac{\partial \Phi_p}{\partial k_y}$ are approximately constant in the area in which A is appreciable, the reflected beam maintains the shape of the incident beam [22] and the GH displacement takes the form of

$$D_{GH} = -|l_{i1}|^2\frac{\partial \Phi_s}{\partial k_y} - |l_{i2}|^2\frac{\partial \Phi_p}{\partial k_y},$$

which leads naturally to the Artmann formula (22) for s or p polarization and agrees well with the recent experimental results [19,20].

B. Partial reflection and generalized GH displacement

When the beam is partially reflected, the reflected beam is also displaced from $\langle y\rangle_i$ to $\langle y\rangle_r$ in the y direction. This is the so-called generalized GH displacement [7] and is given by Eq. (21). Such generalized GH displacements may also occur in attenuated total reflection [10], amplified total reflection [23], and in reflections from absorptive [24] and active [25] media. If $\frac{\partial \Phi_s}{\partial k_y}$ and $\frac{\partial \Phi_p}{\partial k_y}$ are approximately constant in the area in which $A(k_y,k_z)$ is appreciable, Eq. (21) reduces to

$$D_{GH} = -\frac{|l_{i1}|^2\mathfrak{R}_s}{\mathfrak{R}}\frac{\partial \Phi_s}{\partial k_y} - \frac{|l_{i2}|^2\mathfrak{R}_p}{\mathfrak{R}}\frac{\partial \Phi_p}{\partial k_y},$$

which also leads to the Artmann formula (22) for s or p polarized beams.

V. IF EFFECT AND ITS QUANTIZATION

Now let us pay our attention to the problem of the IF effect. As before, we first want to find out the z coordinate of the centroid of the incident beam on the plane $x=0$. On applying Eqs. (3), (8), and (10) to $\tilde{A}_i$ and with the help of Eq. (16), we have

$$\langle z\rangle_i = (|c_{i1}|^2 - |c_{i2}|^2)\langle z\rangle_i^c, \tag{24}$$

where

$$\langle z\rangle_i^c = \frac{1}{k}\int\int\frac{k_x k_y}{k_y^2 + k_z^2}A^2 dk_y dk_z.$$

Equation (24) shows that $\langle z\rangle_i$ does not vanish and is quantized with eigenstates being the two circular polarizations.

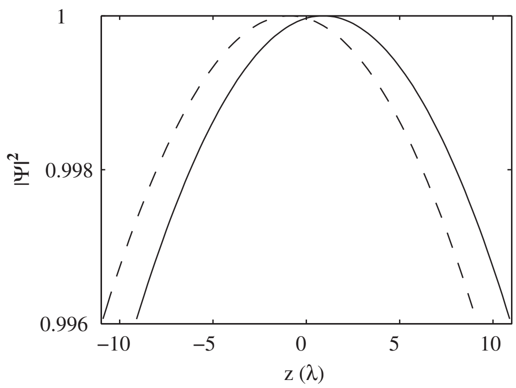

FIG. 4. Normalized intensity distributions of right circularly polarized beam (solid curve) and left circularly polarized beam (dashed curve) on the z axis, where $\theta_0=10°$, $\Delta\theta=10^{-3}$ rad, and z is in units of λ.

The eigenvalues are the same in magnitude and opposite in direction. For the Gaussian distribution function (15), we have [26]

$$\langle z\rangle_i^c \approx \frac{1}{k\tan\theta_0}$$

at large incidence angle, $\theta_0 \gg \Delta\theta$.

The nonvanishing transverse displacement of the incident beam from the plane $z=0$ is in fact an evidence of the so-called translational inertial spin effect of light that was once discussed by Beauregard [27]. Beauregard found that although the transverse wave vector of a two-dimensional beam is identically zero, the two circular polarizations have nonvanishing transverse Poynting vector, and called this phenomenon the translational inertial spin effect. The problem is that the electromagnetic field of so defined two-dimensional beam is uniform in the transverse direction, $\frac{\partial}{\partial z}=0$. In order to observe this effect, it is necessary to have a bound beam that is not transversely uniform, provided that the expectation of transverse wave vector is zero. The 3D beam that we consider here is such a beam satisfying

$$\langle k_z\rangle = \int\int \mathbf{A}^\dagger k_z \mathbf{A} dk_y dk_z = \int\int \tilde{A}^\dagger k_z \tilde{A} dk_y dk_z = 0.$$

For example, when $\theta_0=10°$, we have $\langle z\rangle_i^c \approx 0.9\lambda$. This displacement has been confirmed by the numerical calculation of the field intensity distribution, $|\mathbf{\Psi}(0,z)|^2$, on the z axis as is shown in Fig. 4, where the Gaussian distribution function (15) is considered with $\Delta\theta=10^{-3}$ rad $\ll \theta_0$, and $|\mathbf{\Psi}|^2$ is normalized to unity. The right circularly polarized beam (solid curve) is displaced 0.9λ to the positive direction, and the left circularly polarized beam (dashed curve) is displaced 0.9λ to the negative direction.

When the beam is totally reflected, the two-form amplitude of the reflected beam is represented by Eq. (17), with R_s and R_p given by Eq. (23). Applying Eqs. (3), (8), and (10) to this amplitude and with the help of Eq. (18) gives the transverse displacement of the reflected beam from the plane $z=0$. So defined displacement is the IF effect [4,14,19,20] and is given by

$$D_{IF} \equiv \langle z \rangle_r = \frac{1}{k} \int \int (|c_{r1}|^2 - |c_{r2}|^2) \frac{k_x k_y}{k_y^2 + k_z^2} A^2 dk_y dk_z. \tag{25}$$

This shows that the IF displacement of the reflected beam is quantized with eigenstates being the two circular polarizations. The eigenvalues are the same in magnitude and are opposite in direction. Equations (24) and (25) indicate that the quantization description of IF displacement depends closely on the two-form representation of the angular spectrum.

In order to compare with the recent experimental results [19–21], we consider such an incident beam that has the following elliptical polarization and Gaussian distribution function:

$$\tilde{A}_i = \begin{pmatrix} \cos \psi \\ e^{-i\phi} \sin \psi \end{pmatrix} A_G, \tag{26}$$

where $-\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2}$. In this case, the IF displacement of totally reflected beam is

$$D_{IF} = \frac{w_y w_z \sin(2\psi)}{k\pi} \int \int \frac{k_x k_y}{k_y^2 + k_z^2} e^{-w_y^2 (k_y - k_{y0})^2} e^{-w_z^2 k_z^2} \times \sin(\phi + \Phi_s - \Phi_p) dk_y dk_z. \tag{27}$$

Since Eq. (27) holds whether the beam is totally reflected by a single dielectric interface [19] or by a thin dielectric film in a resonance configuration [20], it is no wonder that the observed IF displacement in the resonance configuration [20] is not enhanced in the way that the lateral GH displacement is enhanced.

If the total reflection takes place at a single dielectric interface and the incidence angle is far away from the critical angle for total reflection and the angle of grazing incidence in comparison with $\Delta\theta$, the first and the last factors of the integrand in Eq. (27) can be regarded as constants for a well-collimated beam [22] and thus can be taken out of the integral with k_y, k_z, Φ_s, and Φ_p evaluated at $k_y = k_{y0}$ and $k_z = k_{z0}$, producing

$$D_{IF} = \frac{\sin(2\psi)\sin(\phi + \Phi_{s0} - \Phi_{p0})}{k \tan \theta_0}. \tag{28}$$

This shows that for given θ_0, the magnitude of D_{IF} is maximum for circularly polarized *reflected* beams [$\psi = \pm\pi/4$ and $\phi + \Phi_{s0} - \Phi_{p0} = (m+1/2)\pi$]. It also shows that the nonvanishing IF displacement for the case of oblique linear polarization of the incident beam ($\phi = m\pi$) [4] results from the different phase shifts between s and p polarizations in total reflection.

With the angular spectrum (26) and the above-mentioned incidence condition, one easily finds in the same way for the transverse displacement of the incident beam,

$$\langle z \rangle_i = \frac{\sin(2\psi)\sin \phi}{k \tan \theta_0}. \tag{29}$$

Comparison of Eq. (28) with Eq. (29) clearly shows that D_{IF} appears as $\langle z \rangle_i$ modified by total reflection. The modification is represented by the replacement of factor $\sin \phi$ in Eq. (29) with factor $\sin(\phi + \Phi_{s0} - \Phi_{p0})$ in Eq. (28). In the experiment performed by Girard [21], a quarter wave plate is used to produce $\phi = \pi/2$. When ψ is changed continuously from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$ by rotating the quarter wave plate, the incident beam is changed from left circular polarization to right circular polarization, and D_{IF} is changed from $-\frac{\cos(\Phi_{s0}-\Phi_{p0})}{k \tan \theta_0}$ to $\frac{\cos(\Phi_{s0}-\Phi_{p0})}{k \tan \theta_0}$. Thus what the experiment measures is nothing but $D_{IF} = \langle z \rangle_r$. The incidence angle dependence $\sim 1/\tan \theta_0$ of D_{IF} in Eq. (28) is in consistency with the experimental result. Since θ_0 is larger than the critical angle for total reflection, it is no wonder that the IF displacement is of the order of $\lambda_0 / 2\pi$ [4,19,20].

VI. CONCLUDING REMARKS

We have advanced a unified theory for the GH and IF effects by representing the vector angular spectrum of a 3D light beam in terms of a two-form angular spectrum consisting of the s and p polarized components. The two-form amplitude of the angular spectrum describes the polarization state of a beam in such a way that the GH displacement is quantized with eigenstates being the orthogonal linear polarizations and the IF displacement is quantized with eigenstates being the two orthogonal circular polarizations. We have also derived the Artmann formula for the GH displacement and found an observable evidence of the so-called translational inertial spin effect that was discussed more than 40 years ago [27]. It was shown that the IF displacement is in fact the translational inertial spin effect happening to the totally reflected beam.

In the two-form representation of a bound beam presented here, only large incidence angle θ_0 in the angular distribution function $A(k_y, k_z)$ corresponds to the uniformly polarized beams [19–21]. When θ_0 is very small, especially when $\theta_0 = 0$, this representation gives quite different beams with peculiar polarization distributions, which needs further investigations.

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[1] F. Goos and H. Hänchen, Ann. Phys. **1**, 333 (1947).
[2] K. Artmann, Ann. Phys. **2**, 87 (1948).
[3] F. I. Fedorov, Dokl. Akad. Nauk SSSR **105**, 465 (1955).
[4] C. Imbert, Phys. Rev. D **5**, 787 (1972).
[5] R. H. Renard, J. Opt. Soc. Am. **54**, 1190 (1964).
[6] C. W. Hsue and T. Tamir, J. Opt. Soc. Am. A **2**, 978 (1985).
[7] C.-F. Li and Q. Wang, Phys. Rev. E **69**, 055601(R) (2004).
[8] R. Briers, O. Leroy, and G. Shkerdin, J. Acoust. Soc. Am. **108**, 1622 (2000).
[9] B. M. Jost, Abdul-Azeez R. Al-Rashed, and B. E. A. Saleh, Phys. Rev. Lett. **81**, 2233 (1998).
[10] X. Yin, L. Hesselink, Z. Liu, N. Fang, and X. Zhang, Appl. Phys. Lett. **85**, 372 (2004).
[11] V. K. Ignatovich, Phys. Lett. A **322**, 36 (2004).
[12] M. Onoda, S. Murakami, and N. Nagaosa, Phys. Rev. Lett. **93**, 083901 (2004).
[13] K. Yu. Bliokh and Y. P. Bliokh, Phys. Rev. Lett. **96**, 073903 (2006).
[14] O. Costa de Beauregard and C. Imbert, Phys. Rev. D **7**, 3555 (1973).
[15] H. Schilling, Ann. Phys. **16**, 122 (1965).
[16] A. K. Ghatak and K. Thyagarajan, *Contemporary Optics* (Plenum Press, New York, 1978), p. 128.
[17] D. Marcuse, *Light Transmission Optics*, 2nd ed. (Van Nostrand Reinhold, New York, 1982), 107.
[18] Y. Zhang and C.-F. Li, Eur. J. Phys. **27**, 779 (2006).
[19] F. Pillon, H. Gilles, and S. Girard, Appl. Opt. **43**, 1863 (2004).
[20] F. Pillon, H. Gilles, S. Girard, M. Laroche, R. Kaiser, and A. Gazibegovic, J. Opt. Soc. Am. B **22**, 1290 (2005).
[21] F. Pillon, H. Gilles, S. Girard, M. Laroche, and O. Emile, Appl. Phys. B: Lasers Opt. **80**, 355 (2005).
[22] J.-L. Shi, C.-F. Li, and Q. Wang, Int. J. Mod. Phys. B (to be published).
[23] J. Fan, A. Dogariu, and L. J. Wang, Opt. Express **11**, 299 (2003).
[24] H. M. Lai and S. W. Chan, Opt. Lett. **27**, 680 (2002).
[25] Y. Yan, X. Chen, and C.-F. Li, Phys. Lett. A **361**, 178 (2007).
[26] Schilling once obtained this result on the basis of stationary phase argument [15].
[27] O. Costa de Beauregard, Phys. Rev. **139**, B1443 (1965).