Improving coherent atomic vapor optical buffers

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We report on a theoretical study of the influence of incoherent optical pumping and beam profiling on slow light propagation in three-level atomic vapor. The pumping reduces the residual unwanted absorption of the signal while destroying the atomic coherence and increasing the group velocity of light. We show that the pumping enhances the resulting group delay for a certain range of parameters. We also examine changing the cross section of the beam of light along the propagation direction by, e.g., focusing the beam, for the purpose of maintaining the drive intensity to effectively counter its absorption.

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I. INTRODUCTION

In practical optical buffers an optical pulse is delayed for a period of time that exceeds the pulse duration (large fractional delay), while the absorption and distortion of the delayed pulse is kept reasonably small [1,2]. Such delays were demonstrated in cold coherent atomic vapors [3]. This, generally, is not the case in slow light experiments with hot coherent atomic vapors [4,5], where the measured time delays are smaller than the pulse duration. The incomplete compensation of the probe absorption by the atomic coherence as well as the high order dispersion of the electromagnetically induced transparency (EIT) resonance is the reason in some cases [1,6]. In other cases, the performance of the optical buffer becomes limited by the high nonlinearity of the atomic media [7] and the wave mixing processes [6]. Nonetheless, it was shown theoretically that through optimization it should be possible to delay a pulse by very many pulse lengths in a slow light coherent atomic medium [8] that is naturally broadened.

Large fractional delays were realized in hot atomic vapors without involvement of low frequency atomic coherence. For instance, delays exceeding seven pulse durations with 3 dB absorption were realized in hot ^{85}Rb atomic vapors [9] when carrier frequency of the pulses was tuned to be halfway between the D_2 line hyperfine resonances. The pulses were delayed because their frequency was tuned to the transparency window limited by two strongly absorbing resonances [10]. An optical buffer based on spectral hole-burning and characterized with fractional delay on the order of three at 3 dB absorption was realized in hot rubidium vapor [11]. Again, no EIT-like effects were involved in the experiment. The disadvantage of those delays compared with EIT-based delays is in the slow tunability of the buffers.

The possibility of achieving large tunable delays arises also in experiments with stopped light [12,13]. It was shown, however, that the light storage experiments have several limitations similar to those of slow light experiments [6]. To the best of our knowledge, there have been no realizations of storage and release of an entire arbitrary optical pulse in a hot atomic vapor cell; only a part of a pulse is generally stored and retrieved. Moreover, atomic cells with a buffer gas are frequently used in the experiments. Diffusion processes in such cells [14,15] alter the shape and the phase of the released pulse, making the operation of the system different from the original proposal. Very recently a dynamic control of the driving light power was proposed to realize an efficient storage of the probe pulses [16]. However, this technique is suitable only for particular wave forms of the probe. A similar optimal control strategy for storage and retrieval of a photon wave packet of any given shape by dynamic control of the drive field was proposed in [17]. Finally, an optimum quantum optical memory based on the off-resonant Raman interaction of a single photon, with the optimization based on the same principle as mentioned above, was studied in [18].

The goal of the present contribution is to study possible ways of improving delay lines based on a hot coherent atomic medium to approach the ideal performance predicted in [8]. Residual absorption of light in the medium is one of the main reasons for the small relative group delays observed experimentally. To suppress the residual absorption of the probe light we propose using incoherent pumping (Fig. 1). It is worth noting that the idea of improving the performance of slow light optical buffers with incoherent pumping has a certain overlap with Raman-based slow light systems [19–21]. However, studies considering those systems did not quantitatively answer the question if the electromagnetically induced transparency (EIT) medium with Raman gain is superior when compared with a passive EIT medium.

It is known that incoherent pumping results in suppression of absorption or even amplification [22] of the probe on one hand, and in the destruction of the atomic coherence and broadening of the EIT resonance on the other. The suppression of absorption improves the delay line, while the destruction of the coherence degrades its performance. We calculate the susceptibility of a Λ level configuration (Fig. 1), find the group delay in the system for the case of inhomogeneous broadening, and determine the region of parameters where the overall result of the incoherent pumping is beneficial.

Let us find the maximum value of the product $N = \tau_g(L)\delta_{EIT}(L)$ for a case of a finite absorption of the drive field. We assume that the Rabi frequency of the drive exceeds the minimum required for the EIT value $|\Omega(z)|^2 \geq \gamma\gamma_0$. Here L is the length of the interaction region, $\tau_g(L)$ is the group delay, $\delta_{EIT}(L)$ is the width of the EIT resonance, $\Omega(z)$ is the Rabi frequency of the driving field, 2γ is the radiative decay rate, and γ_0 is the coherence decay rate. N determines the maximum fractional delay of a pulse by the medium.

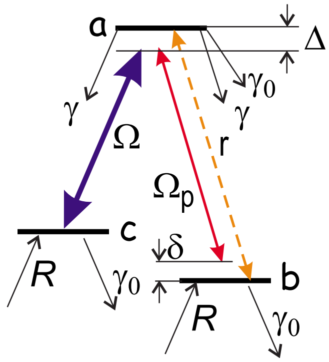

FIG. 1. (Color online) Three-level Λ scheme. Solid blue arrow stands for the drive field, solid red arrow stands for the probe field, and dashed orange arrow stands for the incoherent optical pump. The selection of the external pump ($R=\gamma_0/2$) and decay (γ_0) rates for the atomic levels is required to maintain the normalization condition for the level populations in the particular open level system (in the steady state we have $\rho_{aa}+\rho_{bb}+\rho_{cc}=1$). Physically, the equality of the external decay rates for all the levels (γ_0) means that an atom in any state has equal probability to exit the interaction region. Equal external pumping rates for the ground states $R=\gamma_0/2$ mean that an atom has equal probabilities to enter the interaction region either in state $|b\rangle$ or $|c\rangle$. In addition to the external decay there is the radiative decay rate, γ, which has to be selected independently. The details of the model are discussed in [23].

II. HOMOGENEOUSLY BROADENED THREE LEVEL SYSTEM

A. No incoherent pumping

For the case of a resonant homogeneously broadened Λ system and no incoherent pump the absolute value of the Rabi frequency of the driving field obeys the equation (see Appendix B)

$$|\Omega(z)|^2 = |\Omega(L)|^2\left[1+\kappa\frac{\gamma\gamma_0}{|\Omega(L)|^2}(L-z)\right], \qquad (1)$$

where $\kappa=3\mathcal{N}\lambda^2/(8\pi)$. Group velocity is given by expression [8]

$$V_g(z) \approx \frac{|\Omega(z)|^2}{\kappa\gamma}. \qquad (2)$$

We find from Eqs. (1) and (2),

$$\tau_g(L) = \int_0^L \frac{dz}{V_g(z)} = \ln\left[\frac{P_p(0)}{P_p(L)}\right]\frac{1}{\gamma_0}, \qquad (3)$$

$$\delta_{EIT}(L) = \left(2\ln 2\frac{|\Omega(L)|^2}{\gamma\gamma_0}\frac{P_p(0)/P_p(L)}{P_p(0)/P_p(L)-1}\right)^{1/2}\gamma_0, \qquad (4)$$

$$\frac{P_p(0)}{P_p(L)} \simeq 1+\kappa L\frac{\gamma\gamma_0}{|\Omega(L)|^2}; \qquad (5)$$

where $P_p(L)/P_p(0)=P(L)/P(0)$ is the relative probe (drive) power transmission through the cell [$P(z)$ is the power of the drive and $P_p(z)$ is the power of the probe]. If we restrict the relative probe absorption by e times [$P_p(L)/P_p(0)=e^{-1}$], then $N_{max}\simeq|\Omega(L)|/\sqrt{\gamma\gamma_0}$. The width of the EIT resonance has been found from the usual expression for the full width at half maximum (δ_w) of the Gaussian distribution $\exp(-4\ln 2\delta^2/\delta_w^2)$.

An important result following from Eq. (4) is that *the width of the EIT resonance cannot be less than the coherence decay rate* γ_0 *even in the optically thick media*. We should emphasize that the width can easily be subnatural (less than γ) [24]. It is important to note also that an active system, e.g., a maser based on the low frequency transition $|c\rangle\to|b\rangle$, could have a narrower linewidth than γ_0 [25].

B. Incoherent pumping is present

To obtain useful results we have to restrict the space of the free parameters of the system. We assume that (i) the drive field is strong enough, $|\Omega|^2\gg(\gamma_0+r)\gamma$ and $|\Omega|^2\gg(\gamma_0+r)\Delta$, and (ii) the value of the incoherent pump rate r is chosen in such a way that $\alpha_p(\delta=0)=0$, i.e., that the system is transparent at zero two-photon detuning. We find (see Appendix B)

$$r \simeq \gamma_0\left(\frac{\gamma^2}{\Delta^2}+\sqrt{1+\frac{2\gamma|\Omega|^2}{\gamma_0\Delta^2}}-1\right). \qquad (6)$$

Let us consider two opposite cases with respect to the ratio of the incoherent pump r and coherence decay rate γ_0. As follows from Eq. (6) $\gamma_0\gg r$ is possible only if $\Delta^2\gamma_0\gg\gamma|\Omega|^2$ and $\Delta^2\gg\gamma^2$:

$$r \simeq \gamma_0\left(\frac{\gamma^2}{\Delta^2}+\frac{\gamma|\Omega|^2}{\gamma_0\Delta^2}\right). \qquad (7)$$

The group velocity does not depend on the power of the drive in this case,

$$V_g \approx \frac{\gamma_0}{\kappa}\frac{\Delta^2}{\gamma^2}, \qquad (8)$$

and drive is absorbed exponentially

$$|\Omega(z)|^2 = |\Omega(0)|^2\exp\left(-\kappa z\frac{\gamma^2}{\Delta^2}\right), \qquad (9)$$

i.e., the EIT effect is absent. The group delay is not tunable and this is an apparent disadvantage of the selected regime. Interestingly, using Eqs. (8) and (9) we arrive at an expression for the group delay that is identical to the expression (3) obtained for the case of EIT and no Raman gain:

$$\tau_g(L) = \ln\left[\frac{P(0)}{P(L)}\right]\frac{1}{\gamma_0}, \qquad (10)$$

with the difference that the probe light is not absorbed [$P_p(0)=P_p(L)$]. Increasing the relative absorption of the drive field results in the increase of the maximal group delay beyond the previously obtained limit.

The width of the transparency resonance in this case is

$$\delta_{TR}(L) = \frac{2|\Omega(0)|^2}{|\Delta|}\left[2\ln 2\left(\exp\frac{2\kappa L\gamma^2}{\Delta^2}-1\right)^{-1}\right]^{1/2}. \quad (11)$$

The fractional delay $N \simeq 2.4|\Omega(L)|^2/|\Delta|\gamma_0$ is maximal for small absorption of the drive and approaches unity for strong absorption of the drive (large group delay).

In the opposite case when $r \gg \gamma_0$ and $\gamma|\Omega|^2 \gg \Delta^2\gamma_0$ we have $r \simeq (2\gamma_0\gamma)^{1/2}|\Omega|/\Delta$,

$$V_g \approx \frac{|\Omega|^2}{\kappa\gamma}, \quad (12)$$

and

$$\tau_g(L) = \ln\left[\frac{P(0)}{P(L)}\right]\frac{1}{\gamma_0 + r}. \quad (13)$$

The absorption of the drive is linear, hence the EIT effect is present. The width of the EIT resonance is

$$\delta_{EIT}(L) = \left(2\ln 2\frac{|\Omega(L)|^2}{\gamma(\gamma_0+r)}\frac{P(0)/P(L)}{P(0)/P(L)-1}\right)^{1/2}(\gamma_0 + r). \quad (14)$$

The only difference with the case of zero incoherent pump [Eqs. (3)–(5)] is that we have substituted γ_0 with $\gamma_0 + r \gg \gamma_0$. Such a modification reduces the value of the group delay if the absorption of the drive is small. However, the fractional group delay increases because the probe is not absorbed and at the same time drive can be absorbed much stronger compared with the case of no incoherent pump.

An advantage of the considered scheme is that it allows cascading the delay lines to increase the probe delay. Moreover, if a cloud of cold atoms has a prolonged shape the drive light could be sent perpendicularly to the probe light propagating along the longest direction of the cloud. It reduces drive absorption but keeps group velocity at the smallest possible level through entire cloud.

III. INHOMOGENEOUSLY BROADENED THREE LEVEL SYSTEM

Let us now turn to the inhomogeneously broadened atomic vapor. We consider a quasi-Doppler-free configuration which is characterized by a copropagating drive and probe. The configuration can be considered as Doppler-free if $\omega_{ab}\gamma_{bc}/\Delta_D \gg \omega_{bc}$, where Δ_D the single-photon frequency Doppler width. We approximate the Doppler distribution $f(\Delta_k)$ by a Lorentzian function, following [23,27],

$$f(\Delta_k) = \frac{\Delta_D}{\pi}\frac{1}{\Delta_D^2 + \Delta_k^2}, \quad (15)$$

where Δ_k is the atomic velocity dependent single-photon detuning.

A. No incoherent pumping

The resulting EIT resonance shape is not Lorentzian. Assuming, for the sake of simplicity, that the probe light and the drive light are resonant with motionless atoms, we approximate the dispersion of the medium in the vicinity of zero two-photon detuning:

$$V_g = \frac{|\Omega|^2}{\kappa\gamma}\left(1 + \frac{\Delta_D\sqrt{\gamma_0}}{|\Omega|\sqrt{\gamma}}\right). \quad (16)$$

While Eq. (16) is valid for any intensity of the drive light that obeys to $|\Omega|^2 \gg \gamma\gamma_0$, the expression for the absorption profile of the EIT resonance is rather involved [27]. We present this expression in the approximation $\Delta_D\sqrt{2\gamma_0} \gg |\Omega|\sqrt{2\gamma}$:

$$\alpha_{pD} \simeq \kappa\frac{\gamma\gamma_0}{2|\Omega|^2}\frac{|\Omega|\sqrt{\gamma}}{\Delta_D\sqrt{\gamma_0}} + \kappa\frac{\gamma^2\delta^2}{2\gamma_0\Delta_D|\Omega|^2}. \quad (17)$$

The dispersion β and absorption α coefficients at the drive transition are

$$\beta = 0, \quad (18)$$

$$\alpha \simeq \kappa\frac{\gamma\gamma_0}{2|\Omega|^2}\left(1 + \frac{\Delta_D\sqrt{\gamma_0}}{|\Omega|\sqrt{\gamma}}\right)^{-1}. \quad (19)$$

Using the expressions presented above we derive

$$\tau_g(L) \simeq \frac{1}{\gamma_0}\ln\left[\frac{P(0)}{P(L)}\right], \quad (20)$$

$$\delta_{EIT}(L) \simeq \gamma_0\left\{2\ln 2\frac{|\Omega(L)|}{\sqrt{\gamma\gamma_0}}\frac{[P(0)/P(L)]^{1/2}}{[P(0)/P(L)]^{1/2}-1}\right\}^{1/2}, \quad (21)$$

$$\frac{P(0)}{P(L)} \simeq \left[1 + \frac{\kappa\gamma L}{2\Delta_D}\sqrt{\frac{\gamma\gamma_0}{|\Omega(L)|}}\right]^2. \quad (22)$$

Interestingly, the width of the EIT resonance (21) depends on the drive intensity very weakly. The expression for the group delay (20) is the same as Eq. (3) derived for the case of a naturally broadened system.

B. Incoherent pumping is present

Let us now consider the case of an inhomogeneously broadened system with the incoherent pumping field turned on, $r \neq 0$. We obtain an equation for the absolute value of the Rabi frequency of the driving field assuming that $\Delta_D\sqrt{\gamma_0 + r/2} \gg |\Omega(L)|\sqrt{\gamma}$ and $|\Omega(L)|^2 > \gamma(\gamma_0 + r)$ and solve it (see Appendixes for details):

$$|\Omega(z)| = |\Omega(L)| + \sqrt{\frac{2\gamma}{2\gamma_0 + r}\frac{\gamma(\gamma_0+r)}{2\Delta_D}\kappa(L - z)}, \quad (23)$$

where Δ_D is the width of the Doppler distribution. Using Eq. (23) we derive the equation for the absolute value of the Rabi frequency of the probe light. We solve this equation and select the incoherent pump term r such that the absorption of the probe light is compensated by the Raman gain at the exit of the atomic cell for zero two-photon detuning (δ=0),

$$\frac{(\gamma_0+r)r}{\gamma_0^2} = \frac{|\Omega(L)|^2}{\kappa L \gamma \gamma_0}\sqrt{\frac{P(0)}{P(L)}}\ln\frac{P(0)}{P(L)}, \tag{24}$$

and obtain

$$\tau_g(L) \simeq \frac{1}{\gamma_0+r}\ln\left[\frac{P(0)}{P(L)}\right], \tag{25}$$

$$\delta_{EIT}(L) \simeq (\gamma_0+r)\left\{2\ln 2\frac{|\Omega(L)|}{\sqrt{\gamma(\gamma_0+r)}} \times \times\sqrt{\frac{2\gamma_0+r}{2(\gamma_0+r)}\frac{[P(0)/P(L)]^{1/2}}{[P(0)/P(L)]^{1/2}-1}}\right\}^{1/2}, \tag{26}$$

$$\frac{P(0)}{P(L)} \simeq \left[1+\frac{\kappa\gamma L}{2\Delta_D}\frac{\sqrt{\gamma}(\gamma_0+r)}{\sqrt{\gamma_0+r/2}|\Omega(L)|}\right]^2. \tag{27}$$

To derive Eq. (25) we have used the expression for the group velocity of the probe light propagating in the Doppler broadened medium at the condition of zero two-photon detuning.

In the case of no incoherent pumping ($r=0$) we always have $P_p(0)/P_p(L)=P(0)/P(L)$, and it is reasonable to assume that $P(0)/P(L)=e$, which limits the value of the group delay $\tau_g \leq \gamma_0^{-1}$. In the case of the proper Raman gain we can allow drive absorption $P(0)/P(L)=100$ with no probe absorption. This allows us to achieve a fivefold increase in the group delay.

Moreover, Eqs. (25) and (26) result in $N_{max} \simeq 5.7[|\Omega(L)|/(\gamma\gamma_0)^{1/2}]^{1/2}$ if $\gamma_0 \geq r$ and, e.g., $P(0)/P(L)=100$. On the other hand, we obtain $N_{max} \simeq 2[|\Omega(L)|/(\gamma\gamma_0)^{1/2}]^{1/2}$ if $r=0$ and $P(0)/P(L)=e$. Hence the incoherent pumping also results in an increase of the fractional group delay achieved in the inhomogeneously broadened atomic system.

IV. FOCUSING OF DRIVE AND PROBE BEAMS TO COMPENSATE FOR THEIR ABSORPTION

The basic limit of the atomic group delay with Raman amplification of the probe is set by the absorption of the drive that results in a gradual decreasing of the width of the EIT resonance along the propagation direction. One possible approach to compensation of the absorption is to focus the light in such a way that its intensity does not change inside the atomic cell. Such a compensation leads to longer fractional group delays as compared with the atomic group delay lines utilizing collimated light. However, as we will see below, a special focusing technique is required.

A. Ideal case

Let us study propagation of the focused drive light in the atomic medium (see, e.g., Fig. 2). The equation for the absolute value of the drive Rabi frequency for the collimated light beam propagating in an inhomogeneously broadened atomic vapor is

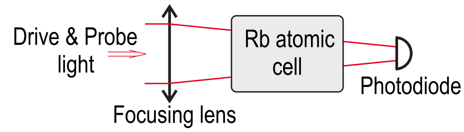

FIG. 2. (Color online) A slow light scheme with changing beam profile. A long-focus thin lens is inserted on the front of the atomic cell.

$$\frac{\partial|\Omega(z)|}{\partial z} = -\kappa\frac{\gamma(\gamma_0+r)}{2\Delta_D}\sqrt{\frac{2\gamma}{2\gamma_0+r}}. \tag{28}$$

The power of the drive changes in accordance with equation

$$\frac{\partial P(z)}{\partial z} = -\frac{\kappa}{|\Omega(z)|}\frac{\gamma(\gamma_0+r)}{\Delta_D}\sqrt{\frac{2\gamma}{2\gamma_0+r}}P(z), \tag{29}$$

where $|\Omega(z)|$ is proportional to $P^{1/2}(z)$. Let us assume now that both the drive and probe are focused such that $|\Omega(z)| = |\Omega(0)|$, then

$$P(z) = P(0)\exp\left(-\frac{\kappa z}{|\Omega(0)|}\frac{\gamma(\gamma_0+r)}{\Delta_D}\sqrt{\frac{2\gamma}{2\gamma_0+r}}\right). \tag{30}$$

The condition $|\Omega(z)|=|\Omega(0)|$ is fulfilled only if the area of the drive beam changes as

$$S(z) = S(0)\exp\left(-\frac{\kappa z}{|\Omega(0)|}\frac{\gamma(\gamma_0+r)}{\Delta_D}\sqrt{\frac{2\gamma}{2\gamma_0+r}}\right). \tag{31}$$

The width of EIT resonance does not change while the group delay is built up with the distance if such a focusing is achieved. This would allow one to increase the fractional delay and make it independent on the drive power $N \simeq 2(\kappa L\gamma/\Delta_D)^{1/2}$.

If both the drive and the probe beams are focused and incoherent pump r is not equal to zero, than the Rabi frequency of the probe increases, while the Rabi frequency of the drive stays intact. It is worth noting that in the experiment with a changing beam profile, parameters γ_0 and r depend on the coordinate z. However, this dependence is slow, especially if the atomic cell contains some buffer gas, so we neglected it in our calculations.

B. Thin spherical lens

A realization of the condition (31) is challenging. Let us find the improvement that can be achieved with the usual thin lens. Thin spherical lens with focal distance f changes the beam profile as

$$\frac{S(z)}{S(0)} = \left(1-\frac{z}{f}\right)^2, \tag{32}$$

where condition $z<f$ is assumed to be valid. Hence selecting the focal distance as

$$f = \frac{2\,\Delta_D|\Omega(0)|}{\kappa\,\gamma(\gamma_0+r)}\sqrt{\frac{2\gamma_0+r}{2\gamma}} \tag{33}$$

we can suppress the linear decrease of the intensity of the drive field. However, such focusing does not result in a significant improvement of the group delay as well as fractional group delay. The focusing is too fast to exactly compensate the linear absorption arising in Eq. (23). We have performed numerical simulations and found that the focusing does not provide any advantage in the case of no Raman amplification. In the case where Raman amplification is present, the additional focusing does not change the group delay [Eq. (25) is valid], but increases the EIT width by approximately a factor of 2. Taking all of the above into account we conclude that focusing of light by thin lenses in the geometrical optics limit is useless for improving the delay time-bandwidth product of the EIT optical buffer. Applications of more complicated optical systems are required. The usefulness of this approach in the wave optics limit, or with thick caustic lenses that possibly provide for a slower changing beam profile, remains to be analyzed.

V. CONCLUSION

In conclusion, we have shown that the application of incoherent optical pumping in a slow light experiment with inhomogeneously broadened Λ-type atomic vapor results in an increase of the achievable optical delay due to the compensation of the absorption of the probe light at the expense of the drive light. We also show that a proper profiling of the probe and drive beams would be beneficial for increasing the slow light delay.

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APPENDIX A: BASIC EQUATIONS

Let us consider the scheme shown in Fig. 1. The drive and probe light waves interact with $|c\rangle \rightarrow |a\rangle$ and $|b\rangle \rightarrow |a\rangle$ transitions, respectively. The incoherent pump with the rate r is applied to the probe transition. We neglect the four wave mixing process and any possible new field generation in the atomic vapor.

We introduce the drive Ω and the probe Ω_p Rabi frequencies as $\Omega = E\wp/\hbar$ and $\Omega_p = E_p\wp_p/\hbar$, where E and E_p are the slow amplitudes of the electric fields, assuming the probe and drive atomic transitions have the same matrix elements $\wp_p = \wp$. We introduce the radiative decay 2γ for level $|a\rangle$. The matrix elements are connected with the radiative decay of the corresponding transition as $\gamma = 4\omega_i^3\wp^2/(3\hbar c^3)$, where ω_i is the circular frequency of the transition. The Rabi frequency of the probe is much less than the Rabi frequency of the drive, $|\Omega| \gg |\Omega_p|$. The probe and drive fields have (one-photon) detunings Δ and $\Delta + \delta$ from the corresponding single-photon resonances. The coherence has a decay rate γ_0 ($\gamma_0 \ll \gamma$) resulting from the finite time of interaction of an atom with the light.

Propagation of the drive and probe is described by equations

$$\frac{1}{c}\frac{\partial E}{\partial t} + \frac{\partial E}{\partial z} = i\frac{2\pi\nu}{c}\mathcal{N}\wp\,\rho_{ac}, \tag{A1}$$

$$\frac{1}{c}\frac{\partial E_p}{\partial t} + \frac{\partial E_p}{\partial z} = i\frac{2\pi\nu_p}{c}\mathcal{N}\wp\,\rho_{ab}, \tag{A2}$$

where $\mathcal{N}$ is the atomic density, ν and ν_p are the carrier frequencies of the drive and probe light, respectively, ρ_{ac} and ρ_{ab} are the density matrix elements describing drive and probe atomic transitions, and c is the speed of light in the vacuum. The density matrix $\hat{\rho} = \Sigma\rho_{ij}|i\rangle\langle j|$ is calculated from [26]

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{1}{2}\{\hat{\Gamma},\hat{\rho}\}, \tag{A3}$$

where $\hat{H}$ is the Hamiltonian of the atomic system; matrix $\hat{\Gamma}$ stands for the relaxation mechanism of the atomic populations and polarizations. We solve Eqs. (A3) in steady state and substitute the solutions for polarizations into a steady state version of Eqs. (A2) and (A1).

APPENDIX B: HOMOGENEOUS BROADENING

Set (A3) can be presented in a more explicit form if the atomic system is homogeneously broadened:

$$\dot{\rho}_{ab} = -\Gamma_{ab}\rho_{ab} - i\Omega_p(\rho_{aa} - \rho_{bb}) + i\Omega\rho_{cb}, \tag{B1}$$

$$\dot{\rho}_{ac} = -\Gamma_{ac}\rho_{ac} - i\Omega(\rho_{aa} - \rho_{cc}) + i\Omega_p\rho_{bc}, \tag{B2}$$

$$\dot{\rho}_{cb} = -\Gamma_{cb}\rho_{cb} + i\Omega^*\rho_{ab} - i\Omega_p\rho_{ca}, \tag{B3}$$

$$\dot{\rho}_{bb} = \gamma_0(1/2 - \rho_{bb}) + r(\rho_{aa} - \rho_{bb}) + \gamma\rho_{aa} - i(\Omega_p\rho_{ba} - \Omega_p^*\rho_{ab}), \tag{B4}$$

$$\dot{\rho}_{cc} = \gamma_0(1/2 - \rho_{cc}) + \gamma\rho_{aa} - i(\Omega\rho_{ca} - \Omega^*\rho_{ac}), \tag{B5}$$

$$\begin{aligned}\rho_{aa} = &-(\gamma_0 + 2\gamma)\rho_{aa} - r(\rho_{aa} - \rho_{bb}) + i(\Omega_p\rho_{ba} - \Omega_p^*\rho_{ab}) \\ &+ i(\Omega\rho_{ca} - \Omega^*\rho_{ac});\end{aligned} \tag{B6}$$

where

$$\Gamma_{ab} = \gamma + \gamma_0 + r + i(\omega_{ab} - \nu_p) = \gamma + \gamma_0 + r + i(\Delta + \delta),$$

$$\Gamma_{ac} = \gamma + \gamma_0 + r/2 + i(\omega_{ac} - \nu) = \gamma + \gamma_0 + r/2 + i\Delta,$$

$$\Gamma_{cb} = \gamma_0 + r/2 + i(\omega_{ab} - \nu_p) - i(\omega_{ac} - \nu) = \gamma_0 + r/2 + i\delta.$$

We solve Eqs. (B1)–(B6) in the steady state and substitute the solutions for polarizations into the steady state version of Eqs. (A1) and (A2),

$$\frac{\partial \Omega}{\partial z} = i\kappa\gamma\rho_{ac} \equiv -(i\beta+\alpha)\Omega, \tag{B7}$$

$$\frac{\partial \Omega_p}{\partial z} = i\kappa\gamma\rho_{ab} \equiv -(i\beta_p+\alpha_p)\Omega_p, \tag{B8}$$

where $\kappa = 3N\lambda^2/(8\pi)$.

The dispersion and absorption of light at the probe transition is described by

$$\beta_p \simeq \kappa \frac{|\Omega|^2\gamma[2\delta\gamma-(r+\gamma_0)\Delta][|\Omega|^2-\delta(\Delta+\delta)+\gamma(\gamma_0+r/2)]}{2[\gamma|\Omega|^2+(\gamma_0+r/2)(\Delta^2+\gamma^2)]\{[|\Omega|^2-\delta(\Delta+\delta)+\gamma(\gamma_0+r/2)]^2+[\delta\gamma+\Delta(\gamma_0+r/2)]^2\}}, \tag{B9}$$

$$\alpha_p \simeq \kappa \frac{|\Omega|^2\gamma\{2\gamma\gamma_0[|\Omega|^2-\delta(\Delta+\delta)+\gamma(\gamma_0+r/2)]+4\delta^2\gamma^2-\Delta^2r^2\}}{4[\gamma|\Omega|^2+(\gamma_0+r/2)(\Delta^2+\gamma^2)]\{[|\Omega|^2-\delta(\Delta+\delta)+\gamma(\gamma_0+r/2)]^2+[\delta\gamma+\Delta(\gamma_0+r/2)]^2\}}, \tag{B10}$$

where we have assumed that $\gamma > r, \gamma_0$.

The dispersion and absorption of the medium at the drive frequency are given by

$$\beta \simeq -\frac{\kappa}{2}\frac{\Delta\gamma(\gamma_0+r)}{\Delta^2(\gamma_0+r/2)+\gamma[|\Omega|^2+\gamma(\gamma_0+r/2)]}, \tag{B11}$$

$$\alpha \simeq \frac{\kappa}{2}\frac{\gamma^2(\gamma_0+r)}{\Delta^2(\gamma_0+r/2)+\gamma[|\Omega|^2+\gamma(\gamma_0+r/2)]}. \tag{B12}$$

The EIT condition (linear attenuation of the drive power with the propagation distance) is fulfilled when $\gamma|\Omega|^2 \gg \Delta^2(\gamma_0+r/2)$ and $|\Omega|^2 \gg \gamma(\gamma_0+r/2)$, and the absolute value of the Rabi frequency of the drive obeys the equation

$$\frac{\partial|\Omega(z)|}{\partial z} = -\kappa\frac{\gamma(\gamma_0+r)}{2|\Omega(z)|} \tag{B13}$$

that can be easily solved,

$$|\Omega(z)|^2 = |\Omega(L)|^2\left[1+\kappa\frac{\gamma(\gamma_0+r)}{|\Omega(L)|^2}(L-z)\right]. \tag{B14}$$

Similarly, we derive the equation for the absolute value of the probe

$$\frac{\partial|\Omega_p(z)|}{\partial z} = -\frac{\kappa}{2}\left(\frac{\gamma\gamma_0}{|\Omega(z)|^2}+\frac{2\delta^2\gamma^2}{|\Omega(z)|^4}\right)|\Omega_p(z)|, \tag{B15}$$

assuming that $\Delta=0$ and $r=0$, for the sake of simplicity. The solution of the equation can be presented as

$$\frac{|\Omega_p(L)|^2}{|\Omega_p(0)|^2} = \frac{|\Omega(L)|^2}{|\Omega(0)|^2}\exp\left[-\frac{2\delta^2\gamma^2\kappa L}{|\Omega(L)|^2|\Omega(0)|^2}\right], \tag{B16}$$

from where we immediately get Eqs. (4) and (5).

Let us now consider propagation of the probe if the incoherent pump is present. Solving equation $\alpha_p=0$ with respect to r, where α_p is given by Eq. (B10), we get Eq. (6). Finally, we use the formula

$$V_g \simeq \left(\frac{\partial\beta_p}{\partial\delta}\right)^{-1}\Bigg|_{\delta=0} \tag{B17}$$

to find the group velocity.

APPENDIX C: INHOMOGENEOUS BROADENING AND INCOHERENT PUMP

The absolute value of the Rabi frequency of the drive field can be presented in the form

$$\frac{\partial|\Omega(z)|}{\partial z} = -\kappa\frac{\gamma(\gamma_0+r)}{2\Delta_D}\sqrt{\frac{2\gamma}{2\gamma_0+r}}, \tag{C1}$$

for the case of a nonzero incoherent pump and inhomogeneously broadened system. The solution of Eq. (C1) is

$$|\Omega(z)| = |\Omega(L)| + \sqrt{\frac{2\gamma}{2\gamma_0+r}}\frac{\gamma(\gamma_0+r)}{2\Delta_D}\kappa(L-z), \tag{C2}$$

where we assumed that $\Delta_D\sqrt{\gamma_0} \gg |\Omega|\gamma$ and $|\Omega(L)| > \sqrt{\gamma\gamma_0}$.

We next derive the equation for the probe absorption

$$\frac{\partial|\Omega_p(z)|}{\partial z} = -\left[\kappa\frac{\gamma\gamma_0}{2|\Omega(z)|^2}\left(\frac{|\Omega(z)|\sqrt{2\gamma}}{\Delta_D\sqrt{2\gamma_0+r}}-\frac{r}{\gamma_0}\right) + \kappa\frac{\gamma^2\delta^2}{(2\gamma_0+r)\Delta_D|\Omega(z)|^2}\right]|\Omega_p(z)|.$$

Selecting gain term r such that $\Omega_p(L)=\Omega_p(0)$ we derive Eqs. (6)–(27).

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