

# Generating and revealing a quantum superposition of electromagnetic-field binomial states in a cavity

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We introduce the  $N$ -photon quantum superposition of two orthogonal generalized binomial states of an electromagnetic field. We then propose, using resonant atom-cavity interactions, nonconditional schemes to generate and reveal such a quantum superposition for the two-photon case in a single-mode high- $Q$  cavity. We finally discuss the implementation of the proposed schemes.

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Since the birth of the quantum macroscopic superposition (Schrödinger cat) phenomenon [1], the possibility of generating and detecting macroscopic quantum superpositions has held much interest in several frameworks [2–4]. A macroscopic quantum superposition of electromagnetic-field states is usually meant as a superposition of two coherent states with classically different phases [5,6]. In the context of cavity quantum electrodynamics (CQED), such a state has been generated by dispersive coupling between a circular Rydberg atom and a small coherent cavity field, the quantum decoherence of the superposition being there observed by probe atoms [7]. Other schemes have been proposed to generate or detect quantum superpositions of this kind, for example in a dispersive medium [8], in a nanomechanical resonator [9], and in a cavity [10–13], and a free-propagating light pulse was also recently prepared in such a state [14]. Nevertheless, two different coherent states can never be made exactly orthogonal, and therefore the coherent states of a quantum superposition are not completely distinguishable. In the CQED experiment of Ref. [7], for example, it is necessary to adjust the detuning between the atomic transition and the cavity frequency to partially distinguish the two components of the superposition. Thus, to propose schemes aimed at generating “optimized” quantum superpositions, defined as quantum superpositions of two orthogonal, distinguishable electromagnetic-field states with nonzero mean fields, appears to be an attractive challenge.

It is well known that the binomial states of an electromagnetic field are characterized by a finite maximum number of photons  $N$ , interpolate between coherent state and number state, and also exhibit nonzero mean fields [15–17]. In addition, it is always possible to find orthogonal couples among the  $N$ -photon generalized binomial states [18]. Such features make the generalized binomial states promising candidates to construct optimized quantum superpositions and to study the general problem of the classical-quantum border and quantum measurement. The point is then how to generate and reveal such states. This paper addresses this issue, by exploiting standard resonant atom-cavity interactions in the CQED framework.

The dynamics of the resonant interaction between a two-

level atom and a single-mode cavity field of frequency  $\omega$  is described by the Jaynes-Cummings Hamiltonian  $H_{JC} = \hbar\omega\sigma_z/2 + \hbar\omega a^\dagger a + i\hbar g(\sigma_+ a - \sigma_- a^\dagger)$ , where  $a$  and  $a^\dagger$  are the field annihilation and creation operators,  $\sigma_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ ,  $\sigma_+ = (\sigma_-)^\dagger = |\uparrow\rangle\langle\downarrow|$  are the pseudospin atomic operators,  $|\uparrow\rangle$  and  $|\downarrow\rangle$  being, respectively, the excited and ground states of the two-level atom, and  $g$  is the atom-field coupling constant. The  $H_{JC}$ -based time evolution of the states  $|\uparrow n\rangle \equiv |\uparrow\rangle|n\rangle$  and  $|\downarrow n\rangle \equiv |\downarrow\rangle|n\rangle$ , with  $a^\dagger a|n\rangle = n|n\rangle$ , is [19]

$$|\uparrow n\rangle \rightarrow \cos(g\sqrt{n+1}t)|\uparrow n\rangle - \sin(g\sqrt{n+1}t)|\downarrow n+1\rangle,$$

$$|\downarrow n\rangle \rightarrow \cos(g\sqrt{n}t)|\downarrow n\rangle + \sin(g\sqrt{n}t)|\uparrow n-1\rangle, \quad (1)$$

where  $t$  is the atom-cavity interaction time.

The normalized  $N$ -photon generalized binomial state is given by [15]

$$|N, p, \phi\rangle = \sum_{n=0}^N \left[ \binom{N}{n} p^n (1-p)^{N-n} \right]^{1/2} e^{in\phi} |n\rangle, \quad (2)$$

where  $0 \leq p \leq 1$  is the probability of single-photon occurrence and  $\phi$  is the mean phase [16]. The orthogonality property  $\langle N, p, \phi | N, 1-p, \pi+\phi \rangle = 0$  [18] allows us to define the  $N$ -photon quantum superposition of two orthogonal generalized binomial states (NQSBs) as

$$|\Psi_S^{(N)}\rangle \equiv \mathcal{N}[|N, p, \phi\rangle + \eta|N, 1-p, \pi+\phi\rangle], \quad (3)$$

where  $\eta$  is a complex number and  $\mathcal{N} = 1/\sqrt{1+|\eta|^2}$ . It should be noted that, for  $p=0, 1$ , the NQSB is reduced to a quantum superposition of the number states  $|0\rangle, |N\rangle$ . The NQSB effectively represents a macroscopic superposition of electromagnetic field states if  $N \gg 1$ . However, in order to remain in the grasp of the current experimental feasibility, we shall concentrate on both the generation and the revealing of the quantum superposition in the case  $N=2$ . We shall show that the 2QSB  $|\Psi_S^{(2)}\rangle$  may be generated in a cavity by the experimental scheme sketched in Fig. 1 and its components and coherence may be revealed by the schemes sketched in Figs. 2 and 3.

**Generating the quantum superposition.** In the generation scheme of Fig. 1, the cavity  $C$  is initially prepared in the vacuum state  $|0\rangle$ , and a pair of two-level atoms, namely, 1 and 2, is prepared in the state  $|\psi\rangle = \mathcal{N}(|\uparrow_1\downarrow_2\rangle + \eta_0|\downarrow_1\uparrow_2\rangle)$ , with  $\eta_0$  real. Entangled atomic states of this form have already

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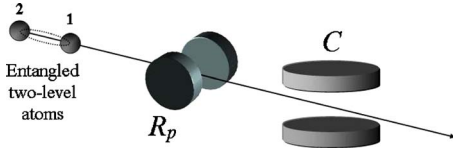


FIG. 1. (Color online) Experimental scheme for the generation of a 2QSB.  $R_p$  is the “preparing” Ramsey zone.

been obtained using a cavity as atomic entanglement catalyst [20,21], providing in addition that the two atoms enter the Ramsey zone, as well as the cavity, one at a time. Each atom first crosses a “preparing” Ramsey zone  $R_p$ . The Ramsey zone interaction makes the  $j$ th atom undergo the following transformations:

$$\begin{aligned} |\uparrow_j\rangle &\xrightarrow{R} \cos(\theta_j/2)|\uparrow_j\rangle - e^{i\varphi_j} \sin(\theta_j/2)|\downarrow_j\rangle, \\ |\downarrow_j\rangle &\xrightarrow{R} e^{-i\varphi_j} \sin(\theta_j/2)|\uparrow_j\rangle + \cos(\theta_j/2)|\downarrow_j\rangle, \end{aligned} \quad (4)$$

where the parameters  $\theta_j$  (“Ramsey pulse”) and  $\varphi_j$  are fixed by adjusting the classical field amplitude and the atom-field interaction time. The  $j$ th atom then resonantly interacts with  $C$  for a time  $T_j$  ( $j=1,2$ ). The atom-cavity interaction times  $T_j$  can be obtained by selecting either different velocities for each atom or the same velocity for the two atoms (“monokinetic atomic beam”) and applying a Stark shift inside the cavity for a time such as to have the desired resonant interaction time [21,22]. The appropriate atomic velocity may be selected by laser-induced atomic pumping [23]. We shall show that a 2QSB state can be efficiently generated by appropriately choosing the Ramsey zone settings and the atom-cavity interaction times.

In accordance with the scheme of Fig. 1, atom 1 crosses  $R_p$  set with a “pulse”  $\theta_1$  such that  $\cos(\theta_1/2) \equiv \sqrt{p}$ ,  $\sin(\theta_1/2) \equiv \sqrt{1-p}$ , and with  $\varphi_1$  to be related to the mean phase  $\phi$  appearing in Eq. (3). After a free-evolution time  $\tau_1$  between  $R_p$  and  $C$ , atom 1 interacts with the cavity  $C$  for a given time  $T_1$ . After it exits  $C$ , atom 2 crosses the Ramsey zone  $R_p$ , freely evolves for a time  $\tau_2$  from  $R_p$  to  $C$ , and then interacts with  $C$  for a time  $T_2$ . Let us indicate by  $T$  the time elapsed between the exit of the atom 1 from  $C$  and the entrance of the atom 2 in  $C$ . After the passage of atom 1, the  $R_p$  parameters must be reset at  $\theta_2 = \theta_1 + \pi$ , so that  $\cos(\theta_2/2) = -\sqrt{1-p}$  and  $\sin(\theta_2/2) = \sqrt{p}$  in Eq. (4), and  $\varphi_2 = \varphi_1 + \omega(\tau_1 + T - \tau_2)$ . Taking into account Eqs. (1) and (4), it is possible

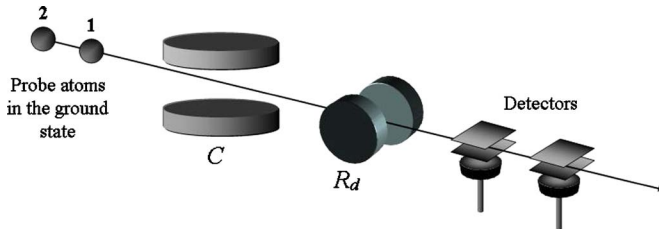


FIG. 2. (Color online) Experimental scheme for distinguishing the two components of the 2QSB.  $R_d$  is the “decoding” Ramsey zone.

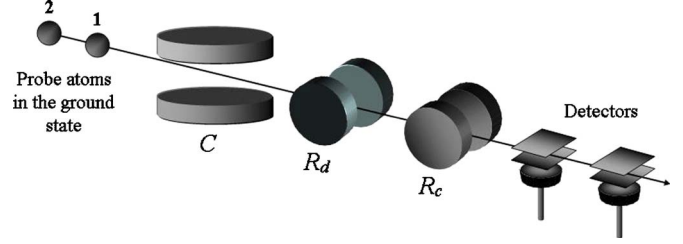


FIG. 3. (Color online) Experimental scheme for revealing the coherence of the 2QSB.  $R_c$  is the “coherence decoding” Ramsey zone. The cavity  $C$  is prepared in the 2QSB  $|\Psi_S^{(2)}\rangle_{\pm}$ .

to demonstrate that, if  $T_1 = (4m+1)\pi/2g$  ( $m$  is a non-negative integer) and  $T_2$  is such that the two equalities  $\sin(gT_2 + \pi/4) = 1$ ,  $\sin(g\sqrt{2}T_2) = 1$  are simultaneously satisfied, when the second atom leaves  $C$ , the state of the total system (atom 1 + atom 2 + cavity) turns out to be factorized as  $|\Psi_S^{(2)}\rangle_{\pm} |\downarrow_1 \downarrow_2\rangle$ . It is worth noting that, choosing  $T_2 = 41\pi/4g$ , both equalities above are satisfied within the error due to the typical experimental interaction time uncertainties [24]. Thus, the cavity field after the passage of the two atoms coincides with the quantum superposition of a pair of orthogonal two-photon generalized binomial states (2QSBs)

$$|\Psi_S^{(2)}\rangle = \mathcal{N} [ |2, p, \phi\rangle + \eta_0 e^{i\gamma} |2, 1-p, \pi + \phi\rangle ], \quad (5)$$

where  $\phi = -[\varphi_1 + \omega(\tau_1 + T)]$  and  $\gamma = \omega(t_{R_2} - t_{R_1} - T_1)$ ,  $t_{R_1}$  and  $t_{R_2}$  being, respectively, the interaction times of the atoms 1 and 2 with  $R_p$ . It is of relevance that our procedure to generate a 2QSB in a cavity does not require a final atomic measurement and then it is a nonconditional scheme.

We shall now analyze the possibility to probe the generated  $|\Psi_S^{(2)}\rangle$  state. Generally speaking, to probe a quantum superposition requires a measurement procedure permitting us both to resolve the two components and to reveal their relative quantum coherence. In the following, we present a procedure appropriate for the 2QSB  $|\Psi_S^{(2)}\rangle$  based on two-level probe atoms that “read” the cavity field. Our considerations will be developed for the maximal 2QSB of Eq. (5), corresponding to  $\eta_0 = \pm 1$ , that is,

$$|\Psi_S^{(2)}\rangle_{\pm} = [ |2, p, \phi\rangle \pm e^{i\gamma} |2, 1-p, \pi + \phi\rangle ] / \sqrt{2}. \quad (6)$$

**Revealing the two components.** The experimental scheme we propose is illustrated in Fig. 2. It exploits two consecutive probe atoms, both in their ground states, interacting one at a time with the apparatus. The atom 1 resonantly interacts with  $C$  for an appropriate time  $T_{P_1}$ , after a delay time  $t_1$  from  $C$  to  $R_d$  it crosses the “decoding” Ramsey zone  $R_d$  and it is finally measured by field ionization detectors. After this measurement, atom 2 enters the cavity  $C$ . Let us denote by  $T'$  the time interval between the exit of the atom 1 from  $C$  and the entrance of the atom 2 in  $C$ . Atom 2 resonantly interacts with  $C$  for a time  $T_{P_2}$ , takes a time  $t_2$  to go from  $C$  to  $R_d$ , crosses  $R_d$ , and its internal state is finally measured.

Let us suppose the cavity prepared in the state  $|2, p, \phi\rangle$  ( $|2, 1-p, \pi + \phi\rangle$ ) and perform the experiment previously described, fixing  $T_{P_1} = 41\pi/4g$ ,  $T_{P_2} = (4m+1)\pi/2g$ , the  $R_d$  parameters  $\theta_{d_1} = \theta_{d_2} = \theta_d$  such that  $\cos(\theta_d/2) = \sqrt{p}$ ,  $\sin(\theta_d/2)$

$=\sqrt{1-p}$ , and  $\varphi_{d_1}=-\phi+\omega t_1$ ,  $\varphi_{d_2}=-\phi+\omega(T'+t_2)$ . Under these conditions, the probability of finding the two atoms in the states  $|\uparrow_1\uparrow_2\rangle$  ( $|\downarrow_1\downarrow_2\rangle$ ) at the end of the experiment is equal to 1. This statement readily follows from the unitary evolutions [18,24]

$$\begin{aligned} |\downarrow_1\rangle|2,p,\phi\rangle &\xrightarrow{T_{P_1},R_d} e^{-i\varphi_{d_1}}|1,p,\phi'\rangle|\uparrow_1\rangle, \\ |\downarrow_2\rangle|1,p,\phi'\rangle &\xrightarrow{T_{P_2},R_d} e^{-i\varphi_{d_2}}|0\rangle|\uparrow_2\rangle, \end{aligned} \quad (7)$$

and

$$\begin{aligned} |\downarrow_1\rangle|2,1-p,\pi+\phi\rangle &\xrightarrow{T_{P_1},R_d} |1,1-p,\pi+\phi'\rangle|\downarrow_1\rangle, \\ |\downarrow_2\rangle|1,1-p,\pi+\phi'\rangle &\xrightarrow{T_{P_2},R_d} |0\rangle|\downarrow_2\rangle, \end{aligned} \quad (8)$$

where  $|1,p,\phi'\rangle$  is the one-photon generalized binomial state as given by Eq. (2) and  $\phi'=\phi-\omega T'$ . Thus, we claim that, if the only possible outcomes of our experiment are  $|\uparrow_1\uparrow_2\rangle$  or  $|\downarrow_1\downarrow_2\rangle$ , then the cavity is with certainty in a combination (quantum or statistical) of the two generalized binomial states  $|2,p,\phi\rangle$ ,  $|2,1-p,\pi+\phi\rangle$ . In a sense, the atoms act here as “quantum probes,” changing their state if they find the state  $|2,p,\phi\rangle$ , or maintaining the same state if they find the state  $|2,1-p,\pi+\phi\rangle$ .

*Revealing the quantum coherence.* Our goal of revealing with certainty the existence of the field state  $|\Psi_S^{(2)}\rangle_+$  or  $|\Psi_S^{(2)}\rangle_-$  of Eq. (6) inside the cavity requires a further step. The previous experimental scheme does not indeed enable us to discover the quantum nature of the superposition of the two components  $|2,p,\phi\rangle$ ,  $|2,1-p,\pi+\phi\rangle$ . To this end we propose a further experimental scheme after the previous one. The appropriate apparatus is sketched in Fig. 3 and differs from the one in Fig. 2 for the presence of a second Ramsey zone  $R_c$ , playing the peculiar role of decoding the information about the quantum coherence we are seeking. Once again, the two probe atoms are initially prepared in their ground states and interact one at a time with the apparatus. In this second scheme, the atom 1 (2) crosses the cavity  $C$ , the Ramsey zone  $R_d$ , and, after a free-evolution time  $t'_1$  ( $t'_2$ ), it enters the additional Ramsey zone  $R_c$ . The procedure ends in measuring the internal states of both probe atoms after atom 2 exits  $R_c$ . The atom-cavity interaction times  $T_{P_1}$ ,  $T_{P_2}$  and the  $R_d$  parameters  $\theta_d$ ,  $\varphi_{d_1}$ ,  $\varphi_{d_2}$  are the same as in the previous scheme, and we strategically set the  $R_c$  pulse  $\theta_c=\pi/2$ . In order to find a suitable value of the  $R_c$  parameter  $\varphi_c$ , it is worth noting that, with these experimental settings, after atom 2 leaves  $R_c$  the initial total state  $|\downarrow_1\downarrow_2\rangle|\Psi_S^{(2)}\rangle_{\pm}$  evolves into

$$|0\rangle\left\{\frac{1\pm e^{i(\gamma-\phi_r-2\varphi_c)}}{2\sqrt{2}}[|\uparrow_1\uparrow_2\rangle+e^{i\alpha}|\downarrow_1\downarrow_2\rangle]-e^{i\varphi_c}\frac{1\mp e^{i(\gamma-\phi_r-2\varphi_c)}}{2\sqrt{2}}[|\uparrow_1\downarrow_2\rangle+e^{i\beta}|\downarrow_1\uparrow_2\rangle]\right\}, \quad (9)$$

where  $|0\rangle$  is the cavity vacuum state,  $\phi_r=2\phi-\omega(T'+t_1+t'_1+t_2+t'_2)$ ,  $\alpha\equiv 2\varphi_c+\omega\bar{t}$ , and  $\beta\equiv\omega\bar{t}$ , with  $\bar{t}$  being the sum of some characteristic times of the procedure. From Eq. (9) it is readily seen that, setting  $\varphi_c=(\gamma-\phi_r)/2$ , the following unitary evolutions are obtained

$$\begin{aligned} |\downarrow_1\downarrow_2\rangle|\Psi_S^{(2)}\rangle_+ &\rightarrow |0\rangle[|\uparrow_1\uparrow_2\rangle+e^{i\alpha}|\downarrow_1\downarrow_2\rangle]/\sqrt{2}, \\ |\downarrow_1\downarrow_2\rangle|\Psi_S^{(2)}\rangle_- &\rightarrow |0\rangle[|\uparrow_1\downarrow_2\rangle+e^{i\beta}|\downarrow_1\uparrow_2\rangle]/\sqrt{2}. \end{aligned} \quad (10)$$

Note that all the free-evolution times can be determined from the atomic velocities, the delay time  $T_0$  between the two atoms and the geometrical parameters. Equation (10) says that the unitary evolution of the probe atoms and the cavity field  $|\Psi_S^{(2)}\rangle_+$  ( $|\Psi_S^{(2)}\rangle_-$ ) generates a vanishing probability amplitude for the atomic states  $|\uparrow_1\downarrow_2\rangle$  and  $|\downarrow_1\uparrow_2\rangle$  ( $|\uparrow_1\uparrow_2\rangle$  and  $|\downarrow_1\downarrow_2\rangle$ ), equally distributing the probability between the other two possible outcomes  $|\uparrow_1\uparrow_2\rangle$  and  $|\downarrow_1\downarrow_2\rangle$  ( $|\uparrow_1\downarrow_2\rangle$  and  $|\downarrow_1\uparrow_2\rangle$ ). Therefore, after repeating this experiment many times, including the preparation of the cavity field, we are able to confirm the quantum coherence (“sign” and relative phase  $\gamma$ ) of the initial cavity field state  $|\Psi_S^{(2)}\rangle_+$  or  $|\Psi_S^{(2)}\rangle_-$ . In fact, if the outcomes of the repeated measurements always give “parallel” atoms then the cavity field is with certainty in the quantum superposition  $|\Psi_S^{(2)}\rangle_+$ ; otherwise, if the outcomes always give “antiparallel” atoms, then the cavity field is with certainty in the quantum superposition  $|\Psi_S^{(2)}\rangle_-$ .

We now briefly analyze the experimental feasibility of the proposed schemes. They require precise atom-cavity interaction times. However, the experimental uncertainty of the selected velocity  $\Delta v$  induces an error  $\Delta T$  on the interaction time such that  $\Delta T/T\approx\Delta v/v$ . In current laboratory experiments, it is possible to select a given atomic velocity such that  $\Delta v/v\leq 10^{-2}$  [21,23]. This error does not appear to sensibly affect our schemes. We have also ignored the atomic or photon decay during the atom-cavity interactions. This assumption is valid if  $\tau_{\text{at}}, \tau_{\text{cav}}>T$ , where  $\tau_{\text{at}}$  and  $\tau_{\text{cav}}$  are the atomic and photon mean lifetimes, respectively, and  $T$  is the interaction time. For circular Rydberg atomic levels and microwave superconducting cavities with quality factors  $Q\sim 10^8-10^{10}$ , the required inequality on the mean lifetimes can indeed be satisfied, with  $\tau_{\text{at}}\sim 10^{-5}-10^{-2}$  s,  $\tau_{\text{cav}}\sim 10^{-4}-10^{-1}$  s and  $T\sim 10^{-5}-10^{-4}$  s [5,20]. Moreover, the typical mean lifetimes of circular Rydberg atomic levels  $\tau_{\text{at}}$  are such that the atoms do not decay during the entire sequence of the schemes [20,23]. The delay time  $T_0$  between the two atoms can be adjusted so that they cross the experimental apparatus one at a time, as required by our schemes. Recent laboratory developments open promising perspectives for a better and easy control of a well-defined atom number sequence [25] and for a high-efficiency atomic detection in microwave CQED experiments [26].

Finally, because the binomial states interpolate between number and coherent state, an estimate of the time scale of 2QSB decoherence can be provided by the corresponding experiment on coherent state superposition with small mean photon numbers [7]. In this experiment, the decoherence time comes out shorter than the photon decay time of the cavity; thus it may be taken as a good indication of the

mesoscopic nature of the superposition. We also wish to observe that our state check procedure gives an unambiguous signal when the state superposition is perfect. However, if, because of decoherence or state preparation, the final superposition is not perfect, our procedure is yet able to measure the degree of coherence (or the state preparation fidelity) of the 2QSB. Although a detailed *ab initio* analysis is required for the general case, we give here a quantitative simple example of this aspect. In fact, if the initial state of the system leads to a final state of the form given in the first line of Eq. (10) plus the term  $\delta|\uparrow_1\downarrow_2\rangle$  and with a new global normalization factor  $\mathcal{N}=1/\sqrt{2+|\delta|^2}$ , the probability of detecting the first atom in  $|\uparrow_1\rangle$  and the second atom in  $|\downarrow_2\rangle$  is  $\mathcal{P}(\uparrow_1\downarrow_2)=(\mathcal{N}|\delta|)^2$ . So, the state preparation fidelity is given by  $F=(1+|\delta|^2/2)^{-1}=1-\mathcal{P}(\uparrow_1\downarrow_2)$  and it is therefore determined by the detection outcomes.

In this paper, we have defined the  $N$ -photon quantum superposition of two orthogonal generalized binomial states of an electromagnetic field. Our main result is the proposal of nonconditional schemes to generate and reveal such an “op-

timized” quantum superposition in a single-mode high- $Q$  cavity in the case  $N=2$ . We wish to emphasize that the orthogonality of the two generalized binomial states forming this state plays a crucial role in revealing, using resonant probe atoms, the quantum nature of the superposition. The implementation of the proposed schemes has also been analyzed, showing how the unavoidable errors characterizing the current experiments do not seem to noticeably affect them. Because of the orthogonality property of generalized binomial states with any  $N$ , our procedure for generating and revealing their quantum superposition may be extended in principle to cases with  $N$  larger than 2. This would lead, for  $N\gg 1$ , to a regime of macroscopic quantum superpositions, the highest value of  $N$  being limited only by the experimental capabilities. The results of this paper can provide the basis for both new knowledge about the foundations of quantum theory (measurement processes, the quantum-classical border) and applications in quantum-information processing, in analogy with the superpositions of coherent states [27].

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