

# Feasibility of studying vortex noise in two-dimensional superconductors with cold atoms

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We investigate the feasibility of using ultracold neutral atoms trapped near a thin superconductor to study vortex noise close to the Kosterlitz-Thouless-Berezinskii transition temperature. Alkali atoms such as rubidium probe the magnetic field produced by the vortices. We show that the relaxation time  $T_1$  of the Zeeman sublevel populations can be conveniently adjusted to provide long observation times. We also show that the transverse relaxation times  $T_2$  for Zeeman coherences are ideal for studying the vortex noise. We briefly consider the motion of atom clouds held close to the surface as a method for monitoring the vortex motion.

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Ultracold neutral atoms trapped and manipulated on atom chips [1] can be used as sensitive probes for a wide range of phenomena. Examples include accurate measurements of gravity [2], imaging the magnetic or electric landscape near wires and magnetic films [3–5], measuring the field due to Johnson noise near metallic and dielectric surfaces [6–9], and probing the Casimir-Polder force as a function of atom-surface distance [10]. Until recently, the atom chips used in these kinds of measurements have been at room temperature and the corresponding thermal fluctuations of the magnetic field have caused mixing of the Zeeman sublevels. Depending on the material of the surface and its distance from the atoms, this spin relaxation time is typically of order 1–100 s.

The use of superconducting films at cryogenic temperatures has been proposed as a way to reduce thermal noise [11] and cold atoms have now been trapped near a superconducting surface [12]. In theoretical studies of atom-superconductor interactions, the arguments employed so far have been based on a simple two-fluid model [13] or on data extracted from surface impedance measurements on bulk superconductors [11]. These approaches apply to three-dimensional superconducting materials, but the thin films normally used on atom chips are typically closer to two-dimensional objects for which three-dimensional (3D) theory is not strictly valid. However, fluctuations resulting from bulk losses seem to be completely negligible [13] so that two-dimensional (2D) effects become dominant. A superconducting film can be approximated by a two-dimensional object if its thickness is smaller than the penetration depth which is what we assume in the following.

Near thin film superconductors, the magnetic noise is generated primarily by vortex motion, which is absent in 3D superconductors. This raises the possibility that atoms trapped near the surface of a superconducting atom chip might be able to probe the physics of vortices. In this Brief Report, we start to explore the feasibility of using cold atoms to investigate vortex noise in a 2D superconductor.

Rather than going into the theory of 2D superconductors and their vortex dynamics, it is enough for our present purpose to be guided by existing experimental data. In the ex-

periment reported in [14], a superconducting quantum interference device (SQUID) loop roughly 1 mm in diameter measured the flux noise spectrum produced by vortices in a 2D superconducting Josephson junction array some 100  $\mu\text{m}$  away. The detected flux provided statistical information on the dynamics of the vortices in the Josephson array. Here we explore how the observed vortex behavior would influence the magnetic sublevel populations and spin coherence of ultracold atoms trapped above such a surface.

Our first goal is to relate the flux noise power measured in [14] to the expected spin flip rate for a magnetic atom trapped near the surface. To this end, we write the spectral density of the flux noise (per Hz of bandwidth) at angular frequency  $\omega$  as

$$S_\phi(f) = 2\pi \int_A d^2x d^2y \langle \hat{B}_z(\mathbf{x}, z; \omega) \hat{B}_z^\dagger(\mathbf{y}, z; \omega) \rangle, \quad (1)$$

where  $A$  is the area of the pick-up loop placed parallel to the surface at height  $z$ . The points  $(\mathbf{x}, z)$  and  $(\mathbf{y}, z)$  are any two points in the plane of the loop and  $\hat{B}_z$  is the operator for the magnetic field component normal to the loop. Both coordinates are integrated over the surface of the loop to obtain the flux noise. In thermal equilibrium, the integrand can be written in the standard way in terms of the dyadic Green function and the mean thermal photon number  $\bar{n}_{\text{th}}$ :

$$\begin{aligned} & \langle \hat{B}_z(\mathbf{x}, z; \omega) \hat{B}_z^\dagger(\mathbf{y}, z; \omega') \rangle \\ &= \delta(\omega - \omega') \frac{\hbar \mu_0}{\pi} \\ & \times \text{Im}[\vec{\nabla} \times \mathbf{G}(\mathbf{x}, z; \mathbf{y}, z; \omega) \times \vec{\nabla}]_{zz} (\bar{n}_{\text{th}} + 1). \end{aligned} \quad (2)$$

If atoms are trapped near the superconductor with their spins parallel to the surface, this same noise in the magnetic field component  $B_z$  can drive spin flip transitions. For an atom at position  $\mathbf{r}_A$ , the rate is given by

$$\Gamma_z = \frac{2\mu_0\mu_{12}^2}{\hbar} \text{Im}[\vec{\nabla} \times \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A; \omega) \times \vec{\nabla}]_{zz} (\bar{n}_{\text{th}} + 1), \quad (3)$$

where  $\mu_{12}$  is the magnetic dipole transition matrix element between the initial and final Zeeman sublevels. Here  $\omega$  is the resonant transition frequency, corresponding to the atomic

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level splitting. The total spin-flip rate  $\Gamma$  is related to  $\Gamma_z$  by  $\Gamma=3/2\Gamma_z$ .

Neglecting the vacuum contribution to the spin flip rate and using the Weyl expansion for the scattering part of the Green function [15,16], we find that

$$[\vec{\nabla} \times \mathbf{G}(\mathbf{x}, z; \mathbf{y}, z; \omega) \times \vec{\nabla}]_{zz} = \int \frac{d^2 k_{\parallel}}{(2\pi)^2} e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{x}-\mathbf{y})} r_s \frac{ik_{\parallel}^2}{k_z} e^{2ik_z z}, \quad (4)$$

where the perpendicular and parallel wave-vector components are related by  $k_z = \sqrt{\omega^2/c^2 - k_{\parallel}^2}$  and  $r_s$  is the Fresnel reflection coefficient for  $s$ -polarized (TE) waves, whose electric vector is perpendicular to the plane of incidence.

In order to integrate over the circular pick-up loop of radius  $R$ , we use the identity

$$\int_0^{2\pi} d\varphi e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{x}-\mathbf{y})} = 2\pi J_0(k_{\parallel} l), \quad (5)$$

where  $\mathbf{k}_{\parallel} \cdot (\mathbf{x}-\mathbf{y}) = k_{\parallel} l \cos \varphi$  with  $l = |\mathbf{x}-\mathbf{y}|$  and  $J_0(k_{\parallel} l)$  is the zeroth-order Bessel function. For the radial integration we have

$$\int_0^R dl l J_0(k_{\parallel} l) = \frac{R}{k_{\parallel}} J_1(k_{\parallel} R), \quad (6)$$

giving the result

$$\int_A d^2 x d^2 y e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{x}-\mathbf{y})} = A^2 \frac{4}{(k_{\parallel} R)^2} J_1^2(k_{\parallel} R) \quad (7)$$

for double integration over the pickup loop. Here we explicitly pull out the factor  $A^2 = (\pi R^2)^2$ , which is the squared area of the loop.

The Weyl expansion (4) of the Green function also requires us to integrate over transverse wave vectors. Since this cannot be done in closed form, we express the right-hand side of Eq. (7) as a power series in  $k_{\parallel}$  [17],

$$\frac{4J_1^2(k_{\parallel} R)}{(k_{\parallel} R)^2} = \sum_{s=0}^{\infty} \frac{4(-1)^s R^{2s} \Gamma\left(s + \frac{3}{2}\right)}{\sqrt{\pi} \Gamma(s+1) \Gamma(s+2) \Gamma(s+3)} k_{\parallel}^{2s}. \quad (8)$$

In cases of practical interest, the distance  $z$  between the trapped atoms and the surface (typically 1–100  $\mu\text{m}$ ) is very small in comparison with the free-space wavelength of the spin-flip transition (typically 3 cm–300 m). As a result, the integral over  $k_{\parallel}$  is entirely dominated by the region in which  $k_{\parallel}^2 \gg \omega^2/c^2$ , where  $k_z \approx ik_{\parallel}$ . When Eq. (4) is integrated to obtain the flux, in accordance with Eq. (1), the powers  $k_{\parallel}^{2s}$  arising from the expansion (8) of the Bessel function can be obtained by differentiating with respect to the atom-surface distance  $z$ , that is,  $k_{\parallel} \equiv -\frac{1}{2} \frac{\partial}{\partial z}$  [15]. Thus inserting Eq. (7), together with the power series expansion (8), into Eq. (1), we obtain

$$S_{\phi}(f) = \frac{\hbar^2 (\pi R^2)^2}{\mu_{12}^2} \sum_{s=0}^{\infty} \frac{4(-1)^s \Gamma\left(s + \frac{3}{2}\right)}{\sqrt{\pi} \Gamma(s+1) \Gamma(s+2) \Gamma(s+3)} \times \left(\frac{R}{2}\right)^{2s} \frac{\partial^{2s}}{\partial z^{2s}} \Gamma_z. \quad (9)$$

The final approximation consists of assuming that the spin flip rate  $\Gamma_z$  follows a strict power law with respect to the atom-surface distance:  $\Gamma_z \propto z^{-n}$ . This is certainly the case in various limiting regimes [11,18–20] when the length scales relevant to the problem (transition wavelength, atom-surface distance, skin depth of the substrate material, etc.) can be well-separated. The derivatives in Eq. (9) then become

$$\frac{\partial^{2s}}{\partial z^{2s}} \Gamma_z = \frac{(n+2s-1)!}{(n-1)!} z^{-n-2s} \Gamma_z. \quad (10)$$

In this way, we perform the summation over  $s$  in Eq. (9) to obtain

$$S_{\phi}(f) = \frac{\hbar^2 A^2}{\mu_{12}^2} \Gamma_z {}_3F_2 \left[ \left\{ \frac{3}{2}, \frac{n+1}{2}, \frac{n}{2} \right\}, \{2, 3\}, -\frac{R^2}{z^2} \right], \quad (11)$$

where  ${}_3F_2$  denotes a hypergeometric function. Equation (11) is the result we were seeking, connecting the measured flux noise spectrum  $S_{\phi}(f)$  to the anticipated atomic spin flip rate  $\frac{3}{2}\Gamma_z$ . The dependence of this connection on distance is controlled by the argument  $(R/z)^2$  of the hypergeometric function and by the power law associated with the spin flip rate.

In the limit of small  $R/z$ , when the size of the pick-up loop is small compared to its distance from the surface, the hypergeometric function in Eq. (11) can be approximated [15] by

$${}_3F_2 \left[ \left\{ \frac{3}{2}, \frac{n+1}{2}, \frac{n}{2} \right\}, \{2, 3\}, -\frac{R^2}{z^2} \right] \approx 1 - \frac{n(n+1)R^2}{16z^2} + O\left(\frac{R^4}{z^4}\right). \quad (12)$$

Hence the flux noise spectrum is essentially proportional to the spin flip rate times the squared area of the pick-up loop. This reflects the fact that the loop is small compared with the transverse correlation length (of order  $z$ ) and therefore the flux directly samples the local magnetic field.

The flux measurements reported in [14] were made in the opposite limit,  $R \gg z$ , with a large pick-up loop located very close to the superconducting surface. Assuming a power law  $\Gamma_z \propto 1/z^4$  (corresponding to the limit  $\delta \ll z$  with  $\delta$  being the penetration depth of the substrate material [11,18–20]), we obtain the limiting behaviour

$${}_3F_2 \left[ \left\{ \frac{3}{2}, \frac{5}{2}, 2 \right\}, \{2, 3\}, -\frac{R^2}{z^2} \right] \equiv {}_2F_1 \left[ \frac{3}{2}, \frac{5}{2}, 3, -\frac{R^2}{z^2} \right] \approx \frac{16z^3}{3\pi R^3} + \frac{4z^5}{3\pi R^5} \left[ 5 + 6 \log \frac{z}{4R} \right] + O\left(\frac{z^7}{R^7}\right). \quad (13)$$

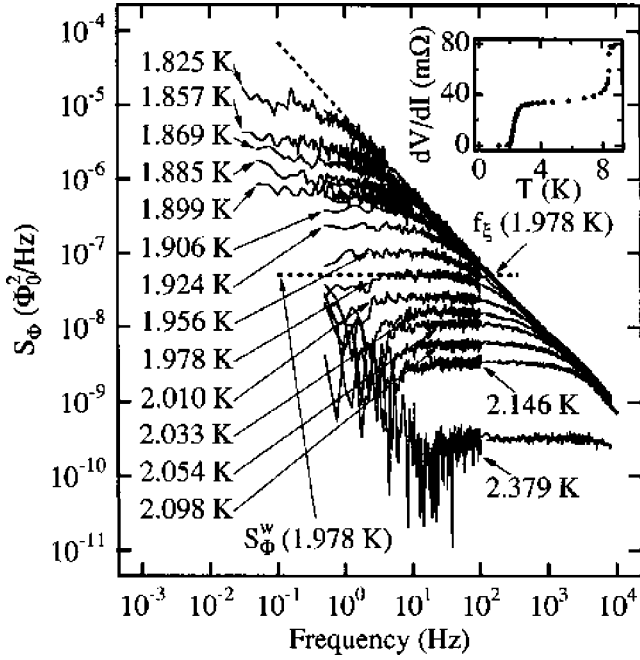


FIG. 1. Reproduced from Ref. [14]. Spectral density of magnetic flux noise,  $S_\phi(f)$ , versus frequency  $f = \omega/2\pi$  for 15 temperatures above the Kosterlitz-Thouless-Berezinskii transition. Scatter at higher temperatures is due to subtraction of SQUID noise. Dashed lines have slope  $-1$  and  $0$ . Inset shows  $dV/dI$  versus  $T$ .

Comparing Eqs. (13) and (12) we see that the spectral density of the flux noise measured in a large loop is also proportional to  $A^2\Gamma_z$  but is suppressed by an additional factor of order  $(z/R)^3$ .

For the purpose of quantitative comparison, we take the magnetic dipole matrix element to be  $\mu_{12} = \mu_B/2$ , corresponding to a transition between Zeeman sublevels  $|i\rangle = |F=2, m_F=2\rangle$  and  $|f\rangle = |F=2, m_F=1\rangle$  of a ground-state rubidium atom. Equation (11) then gives

$$S_\phi(f) = \frac{16m_e^2}{e^2} \Gamma_z A^2 f(z, R), \quad (14)$$

where  $e$  and  $m_e$  are the charge and mass of the electron and  $f(z, R)$  denotes the function in Eq. (12) or Eq. (13). In order to make contact with the flux measurements reported in Ref. [14] and reproduced in Fig. 1, we write  $S_\phi(f) = x\Phi_0^2$ , where  $\Phi_0 = h/(2e)$  is the flux quantum. This gives

$$\Gamma_z = x \frac{\pi^2 \hbar^2}{16m_e^2 A^2 f(z, R)} \stackrel{z \ll R}{\approx} x \frac{3\pi^3 \hbar^2}{256m_e^2 A^2} \left(\frac{R}{z}\right)^3. \quad (15)$$

The SQUID loop that measured the flux in [14] had an effective area of  $A = 180 \times 900 \mu\text{m}^2 \approx 2 \times 10^{-7} \text{m}^2$  and had  $R/z \geq 2.3$ , giving a spin flip rate of  $\Gamma_z \geq 2 \times 10^6 x$ . At spin-flip frequencies above 10 kHz (corresponding to a quantisation magnetic field stronger than  $1 \mu\text{T}$ ), the value of  $x$  given in Ref. [14] is below  $10^{-9}$ , corresponding to a trap lifetime in excess of 500 s. This rather slow  $T_1$  relaxation rate is very promising from the point of view of keeping atoms trapped near a superconducting surface. At the same time it may well

be fast enough to be measured in the very benign environment of a cryostat.

As well as inducing atomic spin flips, the magnetic field fluctuations can generate noise in the relative phase between Zeeman sublevels. In the presence of a static field  $B_0$  normal to the surface, the variance of the phase between levels 1 and 2 after time  $T$  is given by

$$\sigma[\phi(T)]^2 = \frac{(\mu_{22} - \mu_{11})^2}{\hbar^2} \int_0^T dt \int_0^T dt' \langle \hat{B}_z(t) \hat{B}_z(t') \rangle, \quad (16)$$

where  $\hat{B}_z(t)$  is the noise field and does not include the constant field  $B_0$ . This phase noise can be related to the spin flip rate. We connect  $\hat{B}_z(t)$  to  $\hat{B}_z(\omega)$  through

$$\hat{B}_z(t) = \int_0^\infty d\omega [\hat{B}_z(\omega) e^{-i\omega t} + \text{H.c.}], \quad (17)$$

and we use Eqs. (2) and (3) to obtain

$$\mu_{12}^2 \langle \hat{B}_z(\omega) \hat{B}_z^\dagger(\omega') \rangle = \frac{\hbar^2}{2\pi} \Gamma_z(\omega) \delta(\omega - \omega'). \quad (18)$$

Substitution of Eqs. (17) and (18) into Eq. (16) then yields the result

$$\sigma[\phi(T)]^2 = \frac{(\mu_{22} - \mu_{11})^2}{\mu_{12}^2} \frac{2}{\pi} \int_0^\infty d\omega \Gamma_z(\omega) \frac{1 - \cos(\omega T)}{\omega^2}. \quad (19)$$

Now,  $\Gamma_z(\omega)$  is proportional to the measured flux noise  $S_\phi(f)$  [Eq. (15)], which we know is constant up to a characteristic frequency  $f_x$ , as shown in Fig. 1. Let us call this low frequency rate  $\Gamma(0)$ . Above that,  $\Gamma_z(\omega)$  drops off as roughly  $1/\omega$ . The integral in Eq. (19) is completely dominated by the low frequency range between  $0 \leq \omega \leq 2\pi/T$ , so we can make the approximation  $\Gamma_z(\omega) = \Gamma(0)$  provided the observation time satisfies  $T > 1/f_x$ . Since  $f_x$  exceeds 100 Hz, this is the case for  $T > 10$  ms. Indeed, for large enough observation times the integral kernel approximates the  $\delta$  function,

$$\frac{2}{\pi} \frac{(1 - \cos \omega T)}{\omega^2} \mapsto T \delta(\omega). \quad (20)$$

Equation (19) then reads simply

$$\sigma[\phi(T)]^2 = \frac{(\mu_{22} - \mu_{11})^2}{\mu_{12}^2} \Gamma_z(0) T. \quad (21)$$

Supposing once again that states 1 and 2 are the ground states  $|F=2, m_F=2\rangle$  and  $|F=2, m_F=1\rangle$  of a rubidium atom, the ratio of magnetic matrix elements squared has the value of unity and we obtain the particularly simple result  $\sigma[\phi(T)]^2 = \Gamma_z(0) T \approx 2 \times 10^6 x(0) T$ . Since the value of  $x(0)$  reported in [14] is in the range  $10^{-9} - 10^{-5}$ , the corresponding dephasing lifetime  $T_2$  is in the range 50 ms – 500 s. This provides a very convenient time scale for the study of vortex noise using Ramsey interferometry, in which atoms prepared in a coherent superposition of Zeeman sublevels are later interrogated to measure the time-evolution of the coherence. The vortex field noise would be manifest as a loss of Ramsey

fringe visibility with time, which could be studied as a function of the atom-surface distance. It should also be possible to explore the transverse coherence of the noise by varying the transverse extent of the cloud and measuring the loss of Ramsey fringe visibility as the cloud length increases.

Measurements using cold atoms may also be able to image the vortices. The typical vortex separation of  $\xi \sim 2 \mu\text{m}$  [14], could be resolved by bringing the atom cloud to a similar distance from the surface, where the field of each vortex is of order  $\Phi_0/(\pi\xi^2) \sim 1\text{G}$ . At this close approach, the spin-flip lifetime is strongly reduced, but lifetimes approaching 100 ms may nevertheless be achieved. One imaging method would be to study the density distribution of the atoms, which is altered by the presence of the vortices through the effect of the vortex fields on the trapping potential. This approach is used to image classical current distributions in wires on an atom chip [3]. The motion of the vortices could be tracked through the motion of the density patterns in the atom cloud.

To conclude, we have shown that it is feasible to detect

vortex dynamics in two-dimensional superconducting films by means of trapped cold neutral atoms. In particular we have considered the rate of atomic spin flips due to the magnetic field noise from the vortex motion. At 100  $\mu\text{m}$  from the surface we find that this lifetime can be in excess of 500 s, giving ample time to study the vortices. We have also considered the dephasing time for superpositions of Zeeman sublevels and find that this is short enough to be a sensitive measure of the vortex field noise. Finally, we have noted that spatial imaging of the vortices should be possible using cold atoms trapped close enough to the surface. These estimates show that cold atom clouds offer a sensitive new probe for the study of vortex dynamics in superconducting thin films.

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