

Quantum phase gate through a dispersive atom-field interaction

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We propose a scheme for realizing a two-qubit quantum phase gate between atoms inside a high- Q optical cavity. In the scheme, the dispersive interaction between atoms and the cavity field induces photon-number Stark shift, which can yield a resonant interaction inside one selected atom-field subspace and leave the others in a highly dispersive regime. Due to the selective atom-cavity interaction, (1) the scheme only requires commonly addressing the atoms inside the optical cavity during the gate operation, and (2) it can be directly used to implement an N -qubit conditional phase gate. These properties can significantly reduce the number of operations for implementing quantum computation and quantum algorithms.

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The existence of quantum algorithms for some specific problems shows that a quantum computer can in principle provide a tremendous speedup compared to classical computers [1,2]. This discovery has motivated a lot of studies in recent years on the theoretical and practical aspects of quantum computation. It is well known that the two-qubit controlled phase gate and one-qubit gate are universal in the construction of a quantum computer, i.e., any unitary transformation can be decomposed into these elementary gates [3]. In order to implement a quantum computer in a real physical system, a quantum system is needed that makes possible the storage and readout of quantum information and the implementation of the required set of quantum gates. The isolation of the quantum system from the environment should be very well in order to suppress decoherence processes [4].

Among a variety of systems being explored for hardware implementations of quantum computers, cavity quantum electrodynamics (QED) is favored because of its demonstrated advantages when subjected to coherent manipulation [5]. Recently, many experiments have been made with atoms trapped in a high- Q optical cavity, ranging from the deterministic generation of single-photon states [6] and trapping of single atoms inside a small optical cavity in a regime of strong coupling [7,8], to the observation of atom-photon quantum entanglement [9] and implementation of quantum-communication protocols [10]. Theoretically, several proposals have been proposed for realizing conditional quantum logic operations [11–17]. In most of these schemes, two different kinds of atom-field coupling are used for their purposes, in which the dispersive and resonant interactions apply for all initial atom-field states. In particular, in Ref. [16], a dispersive interaction was used to realize a multiqubit controlled phase gate. Since the cavity field is only virtually excited, the cavity decay is suppressed, but the effect of atomic spontaneous emission in the scheme cannot be neglected. In Ref. [17], a scheme was proposed for implementing a two-qubit conditional phase gate with a single resonant interaction, which can be easily generalized to realize an N -qubit controlled phase. However, the scheme requires individually addressing each atom during the gate operation when all atoms are inside the cavity, which is difficult to

achieve with the current experimental technology.

In this paper, we propose an alternative scheme to implement a quantum phase gate among atoms trapped in a high- Q optical cavity. The present protocol is based on a highly controlled and selective atom-field interaction, which can yield resonant interaction inside a selected atom-field subspace and leave the others in a highly dispersive regime. It is noted that in the system of a single atom interacting with a single-mode cavity field, the selective interaction has been suggested for conditional large Fock state preparation [18] and deterministic manipulation of cavity states [19]. In those works, the authors considered the selectivity of desired Fock states and could not be directly used to selectively manipulate the quantum states of many atoms trapped in optical cavity. Recently, we proposed a scheme for generating Dicke states [20] with selective interaction in the cavity QED [21]. This scheme cannot be used to realize quantum logic operation between atoms. In this paper, we study a simple cavity QED system to realize highly controlled and selective atom-field interactions. Using the selective interaction, we demonstrate the realization of quantum phase gate between many atoms. The scheme presented here has the following advantages. First, atomic spontaneous emission can be neglected in our model because the large atom-field detuning suppresses the spontaneous decay of the internal electronic states of the atoms. Second, the scheme requires only commonly addressing the atoms inside the optical cavity during the gate operation. Third, it can be directly used to implement an N -qubit conditional phase gate [23–27]. These properties can significantly reduce the number of operations for implementing quantum computation and quantum algorithms.

We first demonstrate the two-qubit quantum phase gate. The system consists of two five-level atoms trapped in a high- Q optical cavity. The energy level configuration of the atoms is depicted in Fig. 1. It includes three stable ground states $|g\rangle$, $|s\rangle$, and $|a\rangle$, and two excited states $|e\rangle$ and $|h\rangle$. In this paper, we encode the states $|a\rangle$ and $|s\rangle$ as logic zero and one states, i.e., $|a\rangle \equiv |0\rangle$ and $|s\rangle \equiv |1\rangle$. The single cavity mode couples with the atomic transitions $|g\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |h\rangle$ with the coupling constants g_{1c} and g_{2c} (both are assumed real), respectively. The two classical fields dispersively drive the atomic transitions $|s\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |h\rangle$ with the cou-

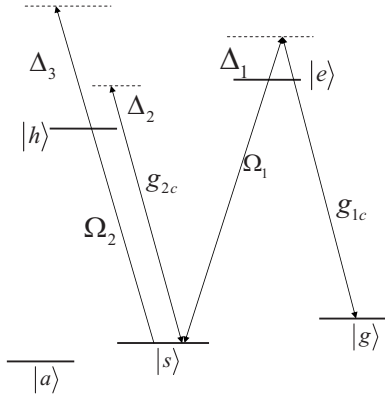


FIG. 1. Involved atomic energy level configuration and the corresponding atomic transitions. The cavity mode couples with the transitions $|g\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |h\rangle$. The two classical fields dispersively drive the atomic transitions $|s\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |h\rangle$, respectively. Δ_i ($i=1,2,3$) is the corresponding one-photon detuning. The energy of state $|s\rangle$ can be effectively adjusted by the classical field with amplitude Ω_2 .

pling constants Ω_1 and Ω_2 (both are assumed real), respectively. $\Delta_{i=1,2,3}$ represents the corresponding one-photon detuning. In the interaction picture, the Hamiltonian describing the system is given by

$$H = \sum_{j=1}^2 (g_{1c}a|e_j\rangle\langle g_j|e^{-i\Delta_1 t} + \Omega_1|e_j\rangle\langle s_j|e^{-i\Delta_1 t} + g_{2c}a|h_j\rangle\langle s_j|e^{-i\Delta_2 t} + \Omega_2|h_j\rangle\langle s_j|e^{-i\Delta_3 t} + \text{H.c.}), \quad (1)$$

where the subscript j represents the j th atom, and a^\dagger and a are the creation and annihilation operators associated with the quantized cavity mode. With the large-detuning conditions $|\Delta_1| \gg |\Omega_1|$, $|g_{1c}|$, $|\Delta_2| \gg |g_{2c}|$, and $|\Delta_3| \gg |\Omega_2|$, i.e., the one-photon detuning Δ_i is much larger than the associated coupling strengths, we can adiabatically eliminate the two excited states $|e\rangle$ and $|h\rangle$, and obtain the effective Hamiltonian [22]

$$H_{\text{eff}} = \sum_{j=1}^N \left[\frac{g_{1c}^2}{\Delta_1} a^\dagger a |g_j\rangle\langle g_j| + \left(\delta + \frac{\Omega_1^2}{\Delta_1} + \frac{g_{2c}^2}{\Delta_2} a^\dagger a \right) |s_j\rangle\langle s_j| + \frac{g_{1c}\Omega_1}{\Delta_1} (a^\dagger |g_j\rangle\langle s_j| + a |s_j\rangle\langle g_j|) \right], \quad (2)$$

where $\delta = \Omega_2^2/\Delta_3$ can be tuned by the external classical field Ω_2 . The first two terms represent the dynamical energy shifts of atomic levels $|g\rangle$ and $|s\rangle$. The last two terms describe the dynamical transitions between these levels.

The cavity field is initially prepared in the vacuum state $|0_c\rangle$, and the atoms 1 and 2 are prepared in the computational basis states $\{|a_1\rangle|a_2\rangle, |a_1\rangle|s_2\rangle, |s_1\rangle|a_2\rangle, |s_1\rangle|s_2\rangle\}$. It is obvious that the states $|a_1\rangle|a_2\rangle|0_c\rangle$ do not evolve with time since $H|a_1, a_2\rangle|0_c\rangle = 0$. Therefore, we can only consider the time evolution of states $|a_1\rangle|s_2\rangle|0_c\rangle$, $|s_1\rangle|a_2\rangle|0_c\rangle$, and $|s_1\rangle|s_2\rangle|0_c\rangle$. We first demonstrate that if the system is initially in the state $|s_1\rangle|s_2\rangle|0_c\rangle$, it acquires an $e^{i\pi}$ phase shift after a Rabi period governed by the selective atom-cavity interaction. According

to the interaction in Eq. (2), the state $|s_1\rangle|s_2\rangle|0_c\rangle$ couples to the one cavity photon state $|+\rangle|1_c\rangle$ with the coupling constant $\Omega_{1,\text{eff}} = \sqrt{2}g_{1c}\Omega_1/\Delta_1$ and the detuning $\Delta_{1,\text{eff}} = \delta + (\Omega_1^2 - g_{1c}^2)/\Delta_1 - g_{2c}^2/\Delta_2$, where $|+\rangle = (|g_1, s_2\rangle + |s_1, g_2\rangle)/\sqrt{2}$. Then the state $|+\rangle|1_c\rangle$ is coupled to the state $|g_1\rangle|g_2\rangle|2_c\rangle$ with the coupling constant $\Omega_{2,\text{eff}} = \sqrt{2}\Omega_{1,\text{eff}}$ and the detuning $\Delta_{2,\text{eff}} = \delta + (\Omega_1^2 - 3g_{1c}^2)/\Delta_1 + g_{2c}^2/\Delta_2$. We adjust the experimental parameter Ω_1 , Ω_2 , Δ_1 , Δ_2 , and Δ_3 to satisfy

$$\Delta_{1,\text{eff}} = 0, \quad (3a)$$

$$|\Delta_{2,\text{eff}}| = 2 \left| \frac{g_{2c}^2}{\Delta_2} - \frac{g_{1c}^2}{\Delta_1} \right| \gg |\Omega_{2,\text{eff}}|. \quad (3b)$$

The physical idea behind these equations is that Eq. (3a) results in the resonant transition between the states $|s_1\rangle|s_2\rangle|0_c\rangle$ and $|+\rangle|1_c\rangle$, while Eq. (3b) causes other transitions to be largely detuned so as to be neglected. Thus the dynamics of the system is reduced to an effective two-level model

$$H_{\text{eff},2} = \Omega_{1,\text{eff}}|s_1\rangle|s_2\rangle|0_c\rangle\langle 1_c| + \text{H.c.} \quad (4)$$

Through such a selective resonant interaction (4), we choose the interaction time $t = \pi/\Omega_{1,\text{eff}}$ and the state $|s_1\rangle|s_2\rangle|0_c\rangle$ acquires an $e^{i\pi}$ phase factor, i.e.,

$$|s_1\rangle|s_2\rangle|0_c\rangle \rightarrow -|s_1\rangle|s_2\rangle|0_c\rangle. \quad (5)$$

In the following, we show that, if the system is initially in the state $|a_1\rangle|s_2\rangle|0_c\rangle$, it does not acquire a phase shift due to the selective atom-cavity interaction. According to Eq. (2), the state $|a_1\rangle|s_2\rangle|0_c\rangle$ couples to the one cavity photon state $|a_1\rangle|g_2\rangle|1_c\rangle$ with the coupling constant $\Omega_{3,\text{eff}} = g_{1c}\Omega_1/\Delta_1 = \Omega_{1,\text{eff}}/\sqrt{2}$ and the detuning $\Delta_{3,\text{eff}} = \delta + \Omega_1^2/\Delta_1 - g_{1c}^2/\Delta_1$. If the experimental parameters Ω_1 , Ω_2 , Δ_1 , Δ_2 , and Δ_3 are chosen to satisfy

$$|\Delta_{3,\text{eff}}| \gg |\Omega_{3,\text{eff}}|, \quad (6)$$

we can neglect the transition between the states $|a_1\rangle|s_2\rangle|0_c\rangle$ and $|a_1\rangle|g_2\rangle|1_c\rangle$ at the time scale $\pi/\Omega_{1,\text{eff}}$, so the state $|a_1\rangle|s_2\rangle|0_c\rangle$ does not acquire a phase shift. According to Eqs. (3a) and (3b) we have $\Delta_{3,\text{eff}} = g_{2c}^2/\Delta_2$, which satisfies Eq. (6). Thus the state $|a_1\rangle|s_2\rangle|0_c\rangle$ does not acquire a phase shift on the time scale $\pi/\Omega_{1,\text{eff}}$. The time evolution of the state $|s_1\rangle|a_2\rangle|0_c\rangle$ is similar to that of state $|a_1\rangle|g_2\rangle|1_c\rangle$, and the difference is that the coupled state is $|g_1\rangle|a_2\rangle|1_c\rangle$. We can repeat the above-mentioned procedure to demonstrate that at the time $t = \pi/\Omega_{1,\text{eff}}$ the state $|s_1\rangle|a_2\rangle|0_c\rangle$ does not acquire a dynamical phase shift. Therefore, through the selective atom-cavity interaction, we can implement the two-qubit controlled phase gate

$$|a_1, a_2\rangle \rightarrow |a_1, a_2\rangle, \quad (7a)$$

$$|a_1, s_2\rangle \rightarrow |a_1, s_2\rangle, \quad (7b)$$

$$|s_1, a_2\rangle \rightarrow |s_1, a_2\rangle, \quad (7c)$$

$$|s_1, s_2\rangle \rightarrow -|s_1, s_2\rangle. \quad (7d)$$

In the above analysis we did not take into account the

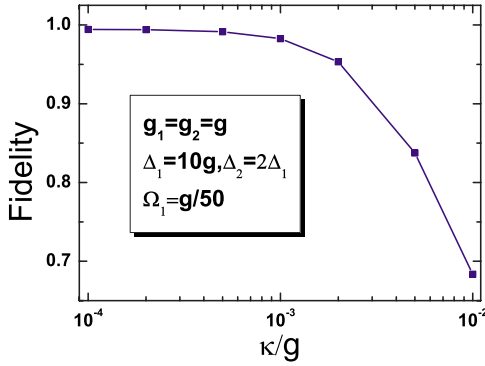


FIG. 2. (Color online) Two-qubit phase gate fidelity F vs the cavity decay rate κ . Other parameters: $g_{1c}=g_{2c}=g$, $\Delta_1=10g$, $\Delta_2=2\Delta_1$, $\Omega_1=g/50$, $\delta=g^2/\Delta_2-(\Omega_1^2-g^2)/\Delta_1$.

influence of decoherence on the scheme. The dissipation channels include the atomic spontaneous emission (at the rate γ_s) and the cavity decay (at the rate κ). First, in the limit of large atom-cavity detuning, two excited states are adiabatically eliminated so that atomic spontaneous emission can be strongly suppressed. Second, to acquire a phase shift on the given state $|s_1, s_2\rangle|0\rangle_c$, the cavity field is excited effectively and there is a quantum-information transfer between the atoms and the cavity, so that cavity decay should play a dominant role in the gate implementation.

We confirm this point by numerically calculating the evolution of the system density matrix governed by the Hamiltonian H . The quality of a two-qubit controlled phase gate can be described by a fidelity, which is defined by $F = \langle \varphi(0) | U^\dagger \rho(t=\tau) U | \varphi(0) \rangle$ where $\rho(t)$ represents the temporal density matrix reduced by tracing out the cavity mode part, $|\varphi(0)\rangle$ denotes the initial atomic state, and U describes the quantum phase gate (7). Here, for simplicity, we choose $|\varphi(0)\rangle = \otimes_{j=1}^2 (|s_j\rangle + |g_j\rangle) / \sqrt{2}$. From Fig. 2, it can be seen that the cavity decay rate κ is the dominant noise source in the gate operation, and the fidelity F rapidly decreases to 0.85 from 0.985 when κ increases to 0.005 g from 0.001 g , while there is not much change when the atomic spontaneous emission is taken into an account for a given cavity decay rate. This is in excellent agreement with the above-mentioned theoretical analysis. As a possible implementation using ^{87}Rb , the above atomic level structures can be obtained. For example, the states $|a\rangle$, $|s\rangle$, and $|g\rangle$ are $|F=1, m=0\rangle$, $|F=2, m=0\rangle$, and $|F=2, m=2\rangle$ of $5S_{1/2}$, respectively; $|e\rangle$ and $|h\rangle$ are $|F=2, m=1\rangle$ and $|F=2, m=-1\rangle$ of $5P_{1/2}$, respectively. The cavity QED parameters $(g, \kappa, \gamma_s)/2\pi = (16, 1.4, 3)\text{MHz}$ have been reported [28]. It is also noted that we need a very small cavity decay (smaller than the atom-field coherent coupling strength by approximately two orders of magnitude) for the sake of a high gate fidelity, which still cannot fully match the current experimental technique, but may be reached in the near future.

Now, we show that the above-mentioned procedure can be directly generalized to realize the N -qubit conditional phase gate. Before a detailed description, we briefly introduce the Goto and Ichimura scheme of Ref. [24], which is similar to the present proposal. Goto and Ichimura's scheme utilizes an

adiabatic passage via dark states, while the present scheme works in the dispersive interaction. Both are immune to spontaneous emission due to excited states, but suffer from decoherence due to cavity photon loss.

We assume that the cavity field is initially in the vacuum state $|0\rangle_c$ and atoms are prepared in the computational basis states $\{|i_1, i_2, \dots, i_{N-1}, i_N\rangle, i=a, s\}$. Since the state $|a_1, a_2, \dots, a_N\rangle|0\rangle_c$ does not experience any dynamics, we consider only the time evolution of states of the form $|\psi\rangle_{i_1, i_2, \dots, i_m} = |\dots s_{i_1} \dots s_{i_2} \dots s_{i_m} \dots\rangle|0\rangle_c$ where the qubits i_1, i_2, \dots, i_m ($1 \leq m \leq N$) are in the state $|s\rangle$, and other states are in the state $|a\rangle$. To simplify description, we introduce a series of states $\{|\phi_{m,n}\rangle \equiv |C_{m,n}\rangle|n\rangle, n=0, 1, \dots, m\}$, where $|C_{m,n}\rangle \equiv \sqrt{n!(m-n)!/m!} J_{i_1, i_2, \dots, i_m}^n |\psi\rangle_{i_1, i_2, \dots, i_m}$ and $J_{i_1, i_2, \dots, i_m}^+ = \sum_{j=i_1, i_2, \dots, i_m} |g_j\rangle\langle s_j|$. The state $|\phi_{m,n}\rangle$ couples to the state $|\phi_{m,n+1}\rangle$ with the coupling constant $\Omega_{m,n,eff} = \sqrt{(n+1)(m-n)} g_{1c} \Omega_1 / \Delta_1$ and the detuning $\Delta_{m,n,eff} = (\delta + \Omega_1^2 / \Delta_1) - (2n+1)(g_{1c}^2 / \Delta_1 - g_{2c}^2 / \Delta_2) - m g_{2c}^2 / \Delta_2$. We choose the parameters to satisfy the conditions

$$\delta + \frac{\Omega_1^2}{\Delta_1} - \left(\frac{g_{1c}^2}{\Delta_1} - \frac{g_{2c}^2}{\Delta_2} \right) - \frac{g_{2c}^2}{\Delta_2} N = 0 \quad (8)$$

and

$$2 \left| \frac{g_{1c}^2}{\Delta_1} - \frac{g_{2c}^2}{\Delta_2} \right|, \left| \frac{g_{2c}^2}{\Delta_2} \right| \gg \left| \frac{\sqrt{2(N-1)} g_{1c} \Omega_1}{\Delta_1} \right|. \quad (9)$$

Equations (8) and (9) have a similar physical essence as Eq. (3). Thus we obtain the result that, if all atoms are initially in the state $|s\rangle$, the dynamics of the system reduces to an effective two-level model

$$H_{eff,N} = \Omega_{N,0,eff} (|\phi_{N,0}\rangle\langle\phi_{N,1}| + |\phi_{N,1}\rangle\langle\phi_{N,0}|), \quad (10)$$

and at the time $t = \pi / |\Omega_{N,0,eff}|$, the state $|s_1, s_2, \dots, s_N\rangle|0\rangle_c$ acquires an $e^{i\pi}$ phase factor.

For the case that at least one atom is initially in the state $|a\rangle$, the detuning between states $|\phi_{m,0}\rangle$ and $|\phi_{m,1}\rangle$ is much larger than effective coupling $\Omega_{m,0,eff}$ according to Eqs. (8) and (9). Thus we can neglect the transition between these states at the time scale $\pi / \Omega_{N,0,eff}$, and state $|\psi\rangle_{i_1, i_2, \dots, i_m}$ ($m < N$) does not acquire a dynamical phase shift. Therefore, through selective atom-cavity interaction, we can carry out the following unitary transformation:

$$U = e^{i\pi |s_1, s_2, \dots, s_N\rangle\langle s_1, s_2, \dots, s_N|} \quad (11)$$

with only one-step operation. This is the N -qubit controlled phase gate that we desire.

Finally, we show that our interaction model can prepare the Dicke state $|C_{N,n}\rangle$ [20] of N trapped atoms. Unlike our previous scheme [21], where the preparation of the Dicke state $|C_{N,n}\rangle$ is based on the success of $|C_{N,n-1}\rangle$, the current scheme can directly obtain $|C_{N,n}\rangle$ from $|C_{N,0}\rangle$. To generate $|C_{N,n}\rangle$, we adjust the parameters $\Delta_{1,2,3}, \Omega_{1,2}, g_{1c,2c}$ to satisfy the condition

$$\frac{ng_{1c}^2}{\Delta_1} + \frac{(N-n)g_{2c}^2}{\Delta_2} = \delta + \frac{\Omega_1^2}{\Delta_1}, \quad (12)$$

and the corresponding coupling strength relations. In this case, the state $|\phi_{N,0}\rangle$ resonantly couples to $|\phi_{N,n}\rangle$, while other transitions are dispersive. With a determined interaction time, the initial state $|\phi_{N,0}\rangle$ can be transferred to the desired Dicke state $|\phi_{N,n}\rangle$.

In summary, we have proposed a method to directly implement a two-qubit controlled phase gate with neutral atoms trapped in a high- Q optical cavity. The scheme is based on a highly controlled and selective atom-field interaction that can yield resonant interaction inside a selected atom-field subspace and leave the others in a highly dispersive regime. The scheme is insensitive to atomic spontaneous

emission, and it is also possible to obtain a required cavity decay rate in near-future experimentals. Due to selective atom-cavity interaction, the scheme can be directly generalized to implement an N -qubit conditional phase gate and prepare a Dicke state, and requires only commonly addressing the atoms inside optical cavity. Therefore, our scheme can significantly reduce the number of operations for realizing quantum computation and quantum algorithms.

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