Analog to multiple electromagnetically induced transparency in all-optical drop-filter systems

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We theoretically study a parallel optical configuration which includes N periodically coupled whispering-gallery-mode resonators. The model shows an obvious effect which has a direct analogy with the phenomenon of multiple electromagnetically induced transparency in quantum systems. The numerical simulations illuminate that the frequency transparency windows are sharp and highly transparent. We also briefly discuss the experimental feasibility of the current scheme in two practical systems, microrings and microdisks.

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Electromagnetically induced transparency (EIT) [1], which is based on the destructive quantum interference, is an interesting phenomenon that the absorption of a probe-laser field which is resonant with an atomic transition can be reduced or even eliminated by applying a strong driving laser beam at a different frequency. Since experimental observation in atomic vapors [2], this effect is playing an essential role in a variety of physical processes, ranging from lasing without inversion [3], enhanced nonlinear optics [4] to quantum computation and communication [5].

Recent theoretical analysis of optical coupled resonators (or cavities) without the use of atomic resonance have revealed that coherent effects in coupled resonator systems are remarkably similar to those in atoms. In Ref. [6], it was shown that the EIT-like effect can be established in directly coupled optical resonators due to mode splitting and classical destructive interference [6]. In Ref. [7], it was pointed out that the existence of a classical analogue of the electromagnetically induced transparency in coupled optical resonators is crucial for on-chip coherent manipulation of light at room temperatures, including the capabilities of stopping, storing and time reversing an incident optical pulse. More recently, some experiments have been reported for observing the structure tuning of the EIT-like spectrum in a compound glass waveguide platform using relatively large resonators [8], coupled fused-silica microspheres [9,10] and integrated micron-size silicon optical resonator systems [11]. These experiments open up the new possibility for optical communication and simulation of coherent effect in quantum optics using multiple coupled optical resonators [12].

In this paper, we study multiple EIT-like transmission spectrum by means of N indirectly coupled resonators (via two parallel waveguides, namely, bus and drop) which is a generalization of the two coupled resonators system [13]. N indirectly coupled resonators result in $N-1$ frequency transparency windows with equal separation. The numerical simulations indicate that these frequency windows are ultrasharp and highly transparent with practical parameters. The experimental feasibility of the current scheme in two practical systems, microrings and microdisks, will be finally discussed briefly.

Consider an array of resonators in which each resonator is coupled to the adjacent resonators by means of two parallel waveguides, as shown in Fig. 1. The cavities are labeled in ascending order from left to right, and it is assumed that there are N cavities altogether. In the case of slowly varying field amplitudes, the modes of this system can be described by the coupled harmonic oscillator model. The motion of the ith ($i=1,2,\ldots,N$) cavity mode a_i (with the center frequency ω_i) is

$$\frac{da_i}{dt}=i(\omega-\omega_i)a_i-\frac{\kappa_0^{(i)}+\kappa_1^{(i)}+\kappa_2^{(i)}}{2}a_i-\sqrt{\kappa_1^{(i)}}a_i^{\mathrm{in}}-\sqrt{\kappa_2^{(i)}}e^{i\phi_i}a_{i+1}^{\mathrm{out}}. \tag{1}$$

Here ω denotes the carrier frequency of the input laser field; $\kappa_0^{(i)}$, $\kappa_1^{(i)}$, and $\kappa_2^{(i)}$ represent the linewidth associated with the intrinsic cavity losses, the coupling to waveguides 1 and 2, respectively; a_i^{in} and a_{i+1}^{out} describe the input fields of the ith cavity from waveguides 1 and 2, respectively, as shown in Fig. 1. The two output fields (transmission and reflection) are related to the input fields by the output-input relations $a_i^{\mathrm{in}}=\exp(i\phi_{i-1})(a_{i-1}^{\mathrm{in}}+\sqrt{\kappa_1^{(i)}}a_{i-1})$ and $a_i^{\mathrm{out}}=\exp(i\phi_i)a_{i+1}^{\mathrm{out}}+\sqrt{\kappa_2^{(i)}}a_i$. Here ϕ_i stands for the phase delay along the waveguides between the adjacent cavities i and $i+1$. For simplicity and convenience, we set $\exp(i\phi_i)=1$ by choosing appropriate resonator-resonator distance L.

We are interested in the steady-state regime of the current system. To facilitate the discussion, we also suppose that each cavity has the same dissipation, which originate from the intrinsic loss and the two waveguides, i.e., $\kappa_0^{(i)}=\kappa_0$, $\kappa_1^{(i)}=\kappa_1$, $\kappa_2^{(i)}=\kappa_2$. Neglecting all the fluctuations, setting $da_i/dt=0$, and taking the expectation value with respect to the steady state of Eq. (1), it is easy to find that

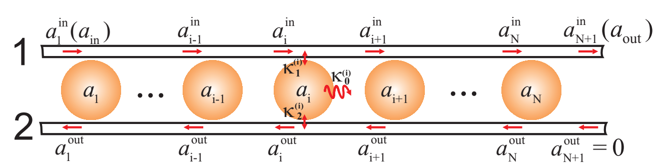

FIG. 1. (Color online) Coupled microcavity resonators sharing two waveguides 1 and 2, namely, bus and drop. As a typical example, loss parameters in cavity i are described.

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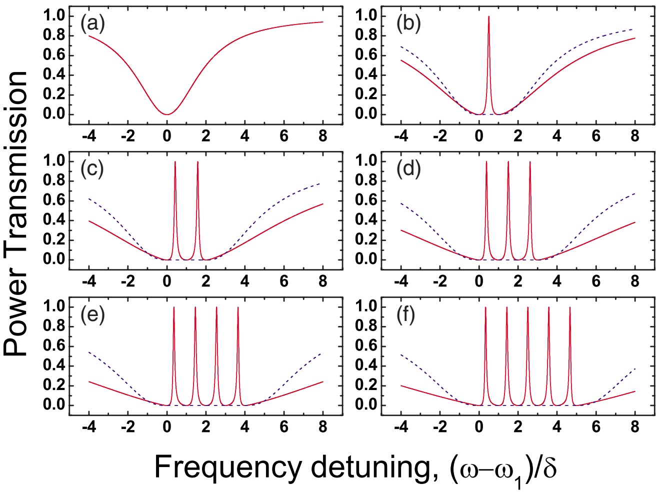

FIG. 2. (Color online) Overall power transmission coefficient $|\mathcal{T}(\omega)|^2$. Red solid (blue dotted) lines in (a)–(f) describe the system including one, two, three, four, five, six cavities, respectively, and there are (no) side coupling among them. Other parameters, $\kappa_0/\kappa_1=10^{-3}$, $\kappa_1/\delta=2$.

$$\left(i\Delta_i-\frac{\kappa}{2}\right)\langle a_i\rangle-\sqrt{\kappa_1}\langle a_i^{\mathrm{in}}\rangle-\sqrt{\kappa_2}\langle a_{i+1}^{\mathrm{out}}\rangle=0, \tag{2}$$

where $\kappa=\kappa_0+\kappa_1+\kappa_2$ denotes the total dissipation of each cavity mode, and Δ_i denotes the detuning $\omega-\omega_i$. Also, using simple recurrence relations, we obtain $\langle a_i^{\mathrm{in}}\rangle=\langle a_{\mathrm{in}}\rangle+\sqrt{\kappa_1}\Sigma_{j=1}^{i-1}\langle a_j\rangle$ and $\langle a_i^{\mathrm{out}}\rangle=\sqrt{\kappa_2}\Sigma_{j=i}^{N}\langle a_j\rangle$, where we have used $\langle a_1^{\mathrm{in}}\rangle=\langle a_{\mathrm{in}}\rangle$ and $\langle a_{N+1}^{\mathrm{out}}\rangle=0$. Therefore, Eq. (2) reduces to

$$\left(i\Delta_i-\frac{\kappa}{2}\right)\langle a_i\rangle-\sqrt{\kappa_1}\langle a_{\mathrm{in}}\rangle-\kappa_1\sum_{j=1}^{i-1}\langle a_j\rangle-\kappa_2\sum_{j=i+1}^{N}\langle a_j\rangle=0. \tag{3}$$

If $\kappa_1=\kappa_2$, Eq. (3) can be further simplified to

$$\langle a_i\rangle=\frac{\sqrt{\kappa_1}\langle a_{\mathrm{in}}\rangle+\kappa_1\sum_{j=1}^{N}\langle a_j\rangle}{i\Delta_i-\kappa_0/2}. \tag{4}$$

Through simple algebra, we achieve

$$\sum_{i=1}^{N}\langle a_i\rangle=\frac{\sqrt{\kappa_1}c}{1-\kappa_1 c}\langle a_{\mathrm{in}}\rangle, \tag{5}$$

where the constant $c=\Sigma_{i=1}^{N}(i\Delta_i-\kappa_0/2)^{-1}$. According to the output-input relations, the final output field which is our interest, can be expressed as

$$\langle a_{\mathrm{out}}\rangle=\langle a_{N+1}^{\mathrm{in}}\rangle=\frac{1}{1-\kappa_1 c}\langle a_{\mathrm{in}}\rangle, \tag{6}$$

so the overall power transmission coefficient $|\mathcal{T}(\omega)|^2=|\langle a_{\mathrm{out}}\rangle/\langle a_{\mathrm{in}}\rangle|^2$ for this system reads

$$|\mathcal{T}(\omega)|^2=|1-\kappa_1 c|^{-2}. \tag{7}$$

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Obviously, the transmission $|\mathcal{T}(\omega)|^2$ has N local minima $|\mathcal{T}(\omega)|^2_{\min}$ $(\simeq 0)$ at $\omega\simeq\omega_i$ and $N-1$ local maxima $|\mathcal{T}(\omega)|^2_{\max}$ at $\omega=(\omega_i+\omega_{i+1})/2$. For the simplicity of numerical simulation, we further assume that the resonant frequencies are equally (or periodically) spaced between the adjacent cavities' modes, i.e., $\omega_{i+1}-\omega_i=\delta$.

The overall transmission is shown in the red solid lines of Figs. 2(a)–2(f), which describe the cases of $N=1$, 2, 3, 4, 5, 6, respectively. When $N=1$, i.e., the system only includes one cavity, Fig. 2(a) actually depicts the well-known transmission response curve of a single cavity mode [14]. When $N>1$, obviously, there exist some sharp peaks in the middle of the adjacent cavities' modes, which is a direct analogy to the phenomenon of electromagnetically induced transparency in atomic vapors [15] or semiconductors [16]. These narrow peaks originate from the interference effect of the cavities' delay. The output a_{i+1}^{out} of the $i+1$th cavity couples back into the ith cavity as the second input port besides a_i^{in}. The transmission is nearly canceled when the light is resonant with one of the cavities' modes due to the overcoupling regime $(\kappa_1,\kappa_2\gg\kappa_0)$ between the waveguides and resonators [14]. However, in the middle the adjacent modes, the destructive interference results in a very narrow transmission resonance [13]. We will discuss these in the following part.

For comparison, the blue dotted lines in Figs. 2(b)–2(f) describe the cases that the output a_{i+1}^{out} of the $i+1$th cavity directly decays into free space instead of coupling back into the ith cavity. Therefore there is no interference effect between the two input fields, and the whole output transmission describes the collective response of all the cavities' modes.

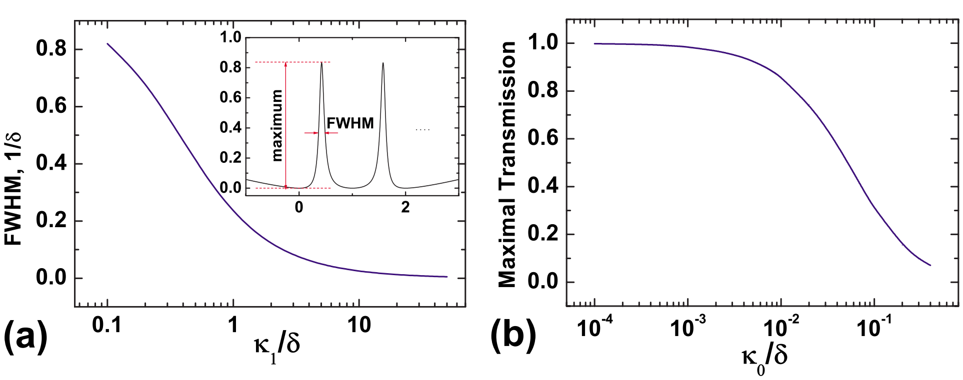

FIG. 3. (Color online) (a) FWHM of the EIT windows vs κ_1. Other parameter, $\kappa_0/\delta=10^{-4}$. (b) Maximal power transmission (peak value) $|\mathcal{T}(\omega)|^2_{\max}$ vs κ_0. Other parameter, $\kappa_1/\delta=2$.

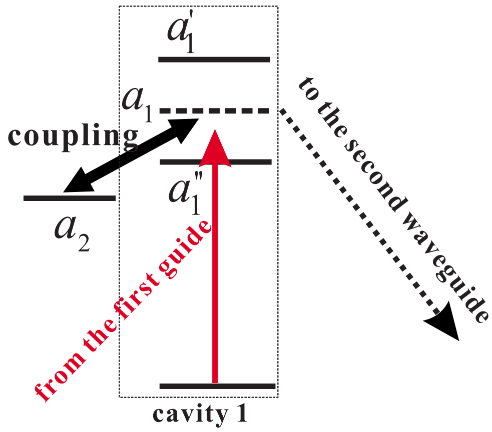

FIG. 4. (Color online) Sketched level diagram of the first cavity coupled with the second. Double-arrow bold line describes the strong coupling between the adjacent cavities via two waveguides. a_1' and a_1'' are two dressed modes with different energies.

To quantitatively characterize the above optical EIT, Figs. 3(a) and 3(b) depict the full-width at half-maximum (FWHM) and maximal transmission rate of the EIT windows, respectively. As shown in Fig. 3(a), FWHM mostly depends on the coupling strength κ_1, that is, larger κ_1 leads to a narrower peak for the given intrinsic cavity loss κ_0 and the mode spacing δ. In other words, once the parameters κ_0 and κ_1 have been given, the smaller δ causes a sharper peak. It agrees with the experiental prediction that there is no EIT phenomenon if $\kappa_1 \ll \delta$, because at this time the spectrum differences between the adjacent cavities' modes are so large that interference does not work significantly, and the total transmission will represent the resonant absorptions of N cavities' modes. Figure 3(b) shows how the maximal transmission $|\mathcal{T}(\omega)|^2_{\max}$ depends on the intrinsic cavity loss κ_0 for a given coupling strength κ_1. When the magnitude of κ_0 is of the order of δ, $|\mathcal{T}(\omega)|^2_{\max}$ will decrease rapidly to zero; when κ_0 can be arbitrarily small, $|\mathcal{T}(\omega)|^2_{\max}$ almost maintains unitary since there is no external loss for the optical resonator-waveguide system here.

It is necessary to give some brief remarks on the analogy and difference between the electromagnetically and coupled-resonator induced transparency. On one hand, the coupled-resonator induced transparency can be described in the language which is used in the thoroughly documented field of EIT, where the role of the "atom" is played by the cavity, the "atomic transition" is acted by the cavity mode, and the "strong driving laser beam" is represented by the strong coupling between the adjacent cavities. As shown in Fig. 4, we illustrate the sketched level diagram of two coupled cavities, i.e., $N=2$. Due to the overcoupling regime among the waveguides and cavities, a probe beam tuned on near resonance with the first cavity mode a_1 will be strongly coupled into the second waveguide. The application of the second cavity mode a_2 and the strong coupling between a_1 and a_2 will split the first cavity mode into two dressed modes a_1' and a_1'' with different energies. In this case, the input field with frequency ω_1 can enter into the second waveguide via two intermediate cavity modes, a_1' and a_1'', whose detunings are of equal magnitudes and opposite signs. As a result, their contributions to the second waveguide process cancel out in second-order perturbation theory and the first cavity becomes transparent to the probe beam. Remarkably, this is analogous to the conventional EIT in the atomic medium.

On the other hand, unlike the conventional atomic EIT, the coupled-resonator induced transparency needs the condition that the probe beam be tuned on near resonance but not exact resonance with the cavity mode. This is because the system works in the overcoupling regime which results in the light transferring from waveguide 1 to 2 with near unit transfer efficiency. Thus the power transmission from waveguide 1 to the second cavity is very small and the returned power from the second to the first cavity is even smaller. As a result, the transmission will be nearly canceled when the light is resonant with the first cavity mode since the destructive interference does not play a significant role in the light transmission. For more cavities, similar treatments can be done, which lead to the above-mentioned multiple EIT-like transmission spectrum.

We now turn to analyze the experimental feasibility of the present multiple optical EIT. For this we briefly discuss two systems, microrings [11,17] and microdisks [18]. Recently, such cavities have been considered as candidates for quantum information processing [19–23]. For microring resonators, the adjacent rings can be linked by two etched waveguides, so the resonators (microrings) and data channels (waveguides) can be integrated on a chip. Modes in microrings possess high quality factors (up to 1.4×10^5 [24]) and ultrasmall mode volumes. The current silicon-on-insulator (SOI) technology allows for a high etching precision (better than 0.02 μm) by designing the mask and controlling inductively coupled plasma reactive ion etching, so that the resonant detuning δ of the microrings and the condition $\exp(i\phi)=1$ are easily satisfied. For microdisk resonators, quality factors in excess of 1 million have been demonstrated in micron scale silicon nitride (SiN_x) [25] and silica [18] at near visible wavelengths and in the 1550-nm band, respectively. In-and-out coupling can be achieved by use of two fiber tapers [14]. To achieve and modulate the resonant detuning δ, the microdisk modes can be first positioned with an accuracy of 0.5 nm using standard lithographic techniques. For a more accurate modulation, wet chemical etching introduced in Refs. [25,26] (better than 0.2 pm) and temperature tuning discussed in Ref. [27] can be applied.

In conclusion, we theoretically present an all-optical scheme to directly simulate multiple EIT using periodically side-coupled N resonators. The highly sharp EIT-like transparency windows only rely on the frequencies of the cavities' modes, so that it is selective through the design of resonator array. This is of importance for applications in optical communications (e.g., channel-selective bandpass filters) and quantum information processing (e.g., slow light systems).

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[1] E. Arimondo, Prog. Opt. **35**, 259 (1996); S. E. Harris, Phys. Today **50**, 37 (1997); J. P. Marangos, J. Mod. Opt. **45**, 471 (1998); M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005), and references therein.

[2] S. E. Harris, J. E. Field, and A. Kasapi, Phys. Rev. A **46**, R29 (1992); Y. Q. Li and M. Xiao, *ibid.* **51**, R2703 (1995); S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999); A. V. Sokolov, D. R. Walker, D. D. Yavuz, G. Y. Yin, and S. E. Harris, *ibid.* **85**, 562 (2000).

[3] A. S. Zibrov, M. D. Lukin, D. E. Nikonov, L. Hollberg, M. O. Scully, V. L. Velichansky, and H. G. Robinson, Phys. Rev. Lett. **75**, 1499 (1995).

[4] S. E. Harris, J. E. Field, and A. Imamoglu, Phys. Rev. Lett. **64**, 1107 (1990); H. Schmidt and A. Imamoglu, Opt. Lett. **21**, 1936 (1996); S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999); M. D. Lukin and A. Imamoglu, *ibid.* **84**, 1419 (2000).

[5] M. D. Lukin, S. F. Yelin, and M. Fleischhauer, Phys. Rev. Lett. **84**, 4232 (2000); M. D. Lukin and A. Imamoglu, Nature (London) **413**, 273 (2001); Z. Ficek and S. Swain, J. Mod. Opt. **49**, 3 (2002).

[6] D. D. Smith, H. Chang, K. A. Fuller, A. T. Rosenberger, and R. W. Boyd, Phys. Rev. A **69**, 063804 (2004).

[7] M. F. Yanik, W. Suh, Z. Wang, and S. Fan, Phys. Rev. Lett. **93**, 233903 (2004).

[8] S. T. Chu, B. E. Little, W. Pan, T. Kaneko, and Y. Kokubun, IEEE Photonics Technol. Lett. **11**, 1426 (1999).

[9] A. Naweed, G. Farca, S. I. Shopova, and A. T. Rosenberger, Phys. Rev. A **71**, 043804 (2005).

[10] K. Totsuka, N. Kobayashi, and M. Tomita, Phys. Rev. Lett. **98**, 213904 (2007).

[11] Q. Xu, S. Sandhu, M. L. Povinelli, J. Shakya, S. Fan, and M. Lipson, Phys. Rev. Lett. **96**, 123901 (2006); Q. Xu, J. Shakya, and M. Lipson, Opt. Express **14**, 6463 (2006).

[12] J. K. Poon, L. Zhu, G. A. DeRose, and A. Yariv, Opt. Lett. **31**, 456 (2006).

[13] L. Maleki, A. B. Matsko, A. A. Savchenkov, and V. S. Ilchenko, Opt. Lett. **29**, 626 (2004).

[14] H. Rokhsari and K. J. Vahala, Phys. Rev. Lett. **92**, 253905 (2004).

[15] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).

[16] M. Phillips and H. Wang, Phys. Rev. Lett. **89**, 186401 (2002).

[17] V. R. Almeida, C. A. Barrios, R. R. Panepucci, and M. Lipson, Nature (London) **431**, 1081 (2004).

[18] T. J. Kippenberg, S. M. Spillane, D. K. Armani, and K. J. Vahala, Appl. Phys. Lett. **83**, 797 (2003).

[19] J. R. Buck and H. J. Kimble, Phys. Rev. A **67**, 033806 (2003); S. M. Spillane, T. J. Kippenberg, K. J. Vahala, K. W. Goh, E. Wilcut, and H. J. Kimble, *ibid.* **71**, 013817 (2005); D. W. Vernooy and H. J. Kimble, *ibid.* **55**, 1239 (1997); T. Aoki, B. Dayan, E. Wilcut, W. P. Bowen, A. S. Parkins, H. J. Kimble, T. J. Kippenberg, and K. J. Vahala, Nature (London) **443**, 671 (2006).

[20] T. A. Brun and H. Wang, Phys. Rev. A **61**, 032307 (2000).

[21] W. Yao, R. B. Liu, and L. J. Sham, Phys. Rev. Lett. **95**, 030504 (2005).

[22] Y.-F. Xiao, X. M. Lin, J. Gao, Y. Yang, Z. F. Han, and G. C. Guo, Phys. Rev. A **70**, 042314 (2004); Y.-F. Xiao, Z.-F. Han, and G.-C. Guo, *ibid.* **73**, 052324 (2006).

[23] E. Waks and J. Vuckovic, Phys. Rev. Lett. **96**, 153601 (2006).

[24] J. Niehusmann *et al.*, Opt. Lett. **29**, 2861 (2004).

[25] P. E. Barclay, B. Lev, K. Srinivasan, O. Painter, and H. Mabuchi, Appl. Phys. Lett. **89**, 131108 (2006).

[26] I. M. White, N. M. Hanumegowda, H. Oveys, and X. Fan, Opt. Express **13**, 10754 (2005).

[27] M. L. Gorodetsky and I. S. Grudinin, J. Opt. Soc. Am. B **21**, 697 (2004).