Macroscopic three-mode squeezed and fully inseparable entangled beams from triply coupled intracavity Kerr nonlinearities

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The generation of macroscopic and spatially separated three-mode entangled light for triply coupled $\chi^{(3)}$ Kerr coupler inside a pumped optical cavity is investigated. With the linearized analysis for small perturbation around the steady state of the pumped cavity modes, the linearized Hamiltonian and the corresponding master equation are obtained. It is found that the bright three-mode squeezing and full inseparable entanglement can be achieved both inside and outside the cavity.

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I. INTRODUCTION

With the development of quantum information, multipartite entanglement becomes very important in multiparty quantum communication including quantum teleportation network [1], telecloning [2], and controlled dense coding [3]. By combining N single-mode squeezed light on appropriately coupled beam splitters, van Loock and Braunstein proposed to generate N-mode continuous-variable (CV) entangled light [1], and the three-mode case was demonstrated experimentally [3,4]. Alternatively, through concurrent optical parametric down-conversion (PDC) and up-conversion in a crystal, the generation of fully inseparable three-mode or multimode CV entangled states was studied extensively [5]. For example, Pfister *et al.* proposed to produce the multimode CV entanglement from the concurrent processes of PDC [6]. On the other hand, the generation of macroscopic entangled beams draws much attention because of its wide application in quantum cryptography [7], testing Bell's inequality [8], and the phase sensitive measurements and quantum optical lithography [9]. Recently, the bright tripartite entanglement in triply concurrent PDC inside a cavity was theoretically investigated by Bradley *et al.* [10]. Villar *et al.* [11] provided a protocol for the production of macroscopic tripartite entanglement in the above-threshold PDC, which was realized experimentally by Cassemiro *et al.* [12].

The nonlinear optical coupler is a device formed of two or more closely lying parallel nonlinear waveguides, whose guided modes are thus coupled by means of evanescent waves. This kind of device is widely investigated in all-optical switching and producing nonclassical effects of electromagnetic fields such as squeezing [13]. Recently, some schemes were proposed to generate entangled beams in the third-order Kerr nonlinear coupler [14]. Olsen, for instance, proposed to generate the macroscopic CV two-mode entangled light from the intracavity Kerr nonlinear coupler [14]. The advantages of these schemes are listed as follows: The nonlinear response of the third-order Kerr nonlinearity is fast and the Kerr effect does not require phase matching, and thus in this regard the Kerr nonlinear process becomes one of the basic methods producing the squeezed light, which is a base for the realization of the CV two-mode entangled beams via Kerr nonlinearity optical fiber [15]. In addition, the entangled output beams from the coupler are spatially separated, and thus they do not need to be separated by optical devices before measurement which may result in the unavoidable coherent losses. Finally, the nonlinear coupler incorporates generating the single-mode squeezed beams and mixing them by couplings of the evanescent waves, which avoids the complexity (especially the interferometric stabilization of the optical paths required for the quantum interference at the beam splitters) of the entanglement scheme proposed by van Loock and Braunstein [1] and thus the nonlinear coupling can be served as a compact source of the entangled light.

In the present work, based on the merits of producing entangled beams from the Kerr coupler mentioned above, we propose to generate macroscopic and spatially separated three-mode entangled light from an optical coupler with triply coupled $\chi^{(3)}$ Kerr nonlinearities in a cavity. With the linearized analysis for small perturbation around the steady state of the pumped cavity modes, the linearized Hamiltonian and the corresponding master equation are obtained. Then, based on the master equation, the Gaussian-type Wigner characteristic function of the intracavity field and the output field are obtained. It is found that the bright three-mode squeezing and fully inseparable entanglement can be achieved both inside and outside the cavity.

This paper is arranged as follows: In Sec. II, the model is introduced and the Hamiltonian is linearized around the steady state of the pumped cavity modes. In Sec. III, the three-mode squeezing and entanglement of the intracavity field and output field are investigated, respectively. Finally, in Sec. IV, we give our summary.

II. MODEL AND LINEARIZED HAMILTONIAN

We consider an optical resonator composed of three coupled Kerr nonlinearities. The coupled Kerr nonlinearities

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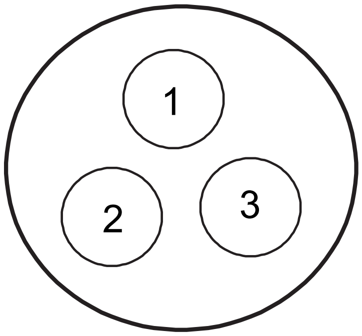

FIG. 1. Schematics of the triangular three-core fiber coupler.

are formed by Kerr nonlinear fiber with three cores in the triangular formation, and the guided modes in the cores are coupled by means of the evanescent field coupling (Fig. 1). This device was investigated for the all-optical switching [16,17], and it is the simplified case of a multicore fiber which is investigated theoretically and experimentally in Refs. [18,19], respectively. Each of the end faces of the coupler has a mirror and thus it can work as an optical resonant cavity [20]. The cavity modes are assumed to be resonantly driven by the external classical lasers. Including the damping of the cavity modes, the dynamics of the system is governed by the following master equation:

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{j=1}^{3} L_j\rho, \tag{1}$$

where

$$H = H_1 + H_2 + H_3, \tag{2}$$

$$H_1 = \sum_{j=1}^{3} \chi_j a_j^{\dagger 2} a_j^2, \tag{3}$$

$$H_2 = \sum_{j\neq j'=1}^{3} g_{jj'} a_j^\dagger a_{j'}, \tag{4}$$

$$H_3 = \sum_{j=1}^{3} \epsilon_j (a_j + a_j^\dagger), \tag{5}$$

$$L_j\rho = \kappa_j(2a_j\rho a_j^\dagger - a_j^\dagger a_j\rho - \rho a_j^\dagger a_j). \tag{6}$$

Here a_j are the annihilation operators of the cavity modes. The second term in the above Hamiltonian describes the Kerr interactions in the coupler, and $\chi_j = \frac{8\pi^2\omega_j^2\chi_j^{(3)}}{\epsilon_0 n^4(\omega_j)V_j}$ are the third-order Kerr nonlinearities, where ω_j are the frequencies of the cavity modes, $n(\omega_j)$ the refractive index, V_j the volume of quantization, and $\chi_j^{(3)}$ the third-order susceptibility [21]. The constants $g_{jj'}$ are the evanescent couplings between the guided modes, which are responsible for the energy exchanges between the waveguides and assumed to be real. ϵ_j are the real amplitudes of the pumping lasers on the cavity modes. The last term describes the damping of the cavity modes with damping rates κ_j.

From Eq. (1) the mean-value equations for the amplitudes of the cavity modes are found to be

$$\frac{d}{dt}\langle a_1\rangle = -i\epsilon_1 - i[g_{12}\langle a_2\rangle + g_{13}\langle a_3\rangle] - 2i\chi_1\langle a_1^\dagger a_1^2\rangle - \kappa_1\langle a_1\rangle,$$

$$\frac{d}{dt}\langle a_2\rangle = -i\epsilon_2 - i[g_{12}\langle a_1\rangle + g_{23}\langle a_3\rangle] - 2i\chi_2\langle a_2^\dagger a_2^2\rangle - \kappa_2\langle a_2\rangle,$$

$$\frac{d}{dt}\langle a_3\rangle = -i\epsilon_3 - i[g_{13}\langle a_1\rangle + g_{23}\langle a_2\rangle] - 2i\chi_3\langle a_3^\dagger a_3^2\rangle - \kappa_3\langle a_3\rangle. \tag{7}$$

For the sake of simplicity, here the case of the symmetric couplings, i.e., $\epsilon_j=\epsilon$, $g_{jj'}=g$ $(j\neq j')$, $\chi_j=\chi$, and $\kappa_j=\kappa$, are considered. With this assumption, we have $\langle a_j\rangle=\langle a\rangle$. Under the semiclassical approximation and letting $\alpha=\langle a\rangle$, Eq. (7) becomes

$$\frac{d}{dt}\alpha = -i\epsilon - 2ig\alpha - 2i\chi|\alpha|^2\alpha - \kappa\alpha. \tag{8}$$

The steady-state solution α^s of the above equation is obtained,

$$\alpha^s = -\frac{\epsilon}{2g + 2\chi I - i\kappa}, \tag{9}$$

where the steady intensity $I(=|\alpha^s|^2)$ meets the following equation:

$$\Xi(I) = 4\chi^2 I^3 + 8\chi g I^2 + (16g + \kappa^2)I - \epsilon^2 = 0. \tag{10}$$

The condition for the appearing optical bistability of the present system is that the equation $\frac{d}{dI}\Xi(I)=0$ has two positive roots. Since both of the roots $2\chi(-4g\pm\sqrt{4g^2-3\kappa^2})$ of the equation $\frac{d}{dI}\Xi(I)=0$ are negative, therefore the phenomenon of the optical bistability of the system of which the steady amplitudes obeyed by the equation (10) does not happen. The dependence of the intensity I of the cavity modes on the intensity of the pumping lasers $I_{\text{in}}(=|\epsilon|^2)$ are plotted in Fig. 2. It shows that, for the given input intensity, the steady intensity of the cavity field decreases as the increasing of the third-order Kerr nonlinearity χ and the coupling constant g between the cavity modes.

Now we will analyze quantum fluctuations of the cavity field so as to investigate the quantum properties of the system such as squeezing and entanglement. If the amplitudes of the quantum fluctuations are far smaller than the steady amplitudes α^s of the cavity modes, we can linearize the master equation (1) in the vicinity of these steady amplitudes α^s. Therefore, let us set

$$a_j = \alpha^s + b_j, \tag{11}$$

where the operators b_j represent the quantum fluctuations of the cavity modes. Then, if $\langle b_j\rangle \ll \alpha^s$, in the master equation (1) we may keep the quadratic terms in b_j and get the linearized Hamiltonian and the corresponding master equation

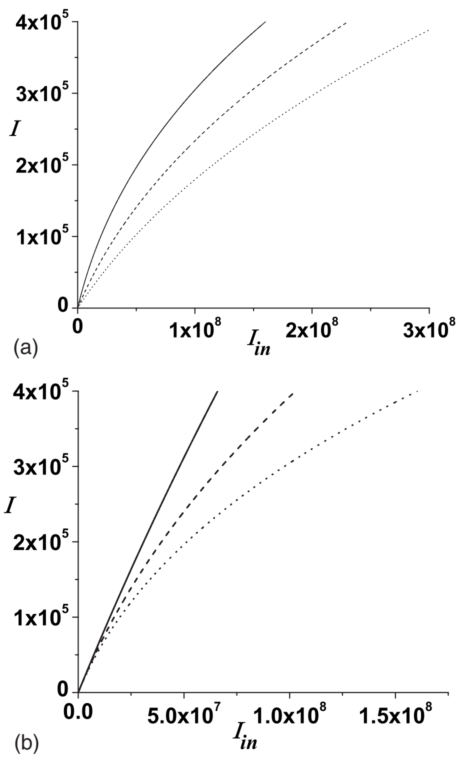

FIG. 2. The dependence of the intensity I of intracavity field on the input intensity I_{in} of pumping lasers. (a) $\chi=10^{-5}$, $\kappa=1$, $g=6$ (solid line), 8 (dashed line), 10 (dotted line); (b) $g=6$, $\kappa=1$, $\chi=10^{-6}$, (solid line), 5×10^{-6} (dashed line), 10^{-5} (dotted line).

governing the quantum fluctuations of the system

$$H_L=\sum_{j=1}^{3}(r_1 b_j^{\dagger}b_j+r_2 b_j^2+r_2^{*}b_j^{\dagger 2})+g\sum_{j\neq j'=1}^{3}b_j^{\dagger}b_{j'} \tag{12}$$

and

$$\frac{d\rho}{dt}=-i[H_L,\rho]+\sum_{j=1}^{3}\kappa(2b_j\rho b_j^{\dagger}-b_j^{\dagger}b_j\rho-\rho b_j^{\dagger}b_j), \tag{13}$$

where $r_1=4\chi|\alpha^s|^2$ and $r_2=\chi\alpha^{s*2}$. From the linearized Hamiltonian which characterizes Gaussian fluctuations of the system, the squeezing of the quantum noise of each cavity mode (single-mode squeezing) can be generated via the terms proportional to b_j^2 in the first part of Eq. (12) which is equivalent to three detuned degenerate PDCs of b_j modes. Physically, this kind of noise squeezing in the cavity modes originates from the fact that the quantum squeezing can be generated in Kerr interactions [each term in Eq. (3)]. Due to the beam-splitter-like linear couplings (terms proportional to $b_j^{\dagger}b_{j'}$) of the quantum noises, which stem from the evanescent couplings between the cavity modes a_j, the correlations between the quantum noises of the three cavity modes are established, and thus the three-mode entangled light can be generated in this system.

From Eq. (13), the equations of motion of the noise operators b_j can be easily obtained [see Eq. (28)], and the relevant mean values $\langle b_j\rangle$ are determined by the drift matrix $-A+\mathbf{I}\kappa$ in Eq. (28). The validity of the above linearization process depends on the eigenvalues of the drift matrix $-A+\mathbf{I}\kappa$, which are found as

$$\lambda_{1,2}=\kappa\pm\sqrt{4|r_2|^2-(r_1+2g)^2}, \tag{14}$$

$$\lambda_{3,4}=\kappa+\sqrt{4|r_2|^2-(r_1-g)^2}, \tag{15}$$

$$\lambda_{5,6}=\kappa-\sqrt{4|r_2|^2-(r_1-g)^2}. \tag{16}$$

If all the above eigenvalues have positive real parts, the system can reach its steady state and thus the linearization process is valid [22,23]. As a result, here the validity of the linearization process requires $\kappa>\text{Re}[\sqrt{4|r_2|^2-(r_1-g)^2}]$, which will be considered to be satisfied henceforth.

III. THREE-MODE SQUEEZING AND ENTANGLEMENT OF THE SYSTEM

The steady-state solution of Eq. (13) in phase space is a three-mode Gaussian state which can be described by the Wigner characteristic function with the following definition [24]:

$$\begin{aligned}\psi(\xi_1,\xi_2,\xi_3)&=\exp\left(\rho\sum_{j=1}^{3}(\xi_j b_j^{\dagger}-\xi^{*}b_j)\right)\\&=\exp\left(\rho\sum_{j=1}^{3}\sqrt{2}i(p_j\hat{X}_j-x_j\hat{P}_j)\right)\\&=\exp\left(-\frac{1}{4}XVX^T\right),\end{aligned} \tag{17}$$

where ρ is the density operator of the three-mode Gaussian state, phase space variables $X=(x_1,p_1,x_2,p_2,x_3,p_3)$ correspond to the quadrature operators of the optical modes $\hat{X}=(\hat{X}_1,\hat{P}_1,\hat{X}_2,\hat{P}_2,\hat{X}_3,\hat{P}_3)$ with the definitions of $\xi_j=x_j+ip_j$ and $b_j=(\hat{X}_j+i\hat{P}_j)/\sqrt{2}$. V is the correlation matrix of the three-mode Gaussian state which completely determines the properties of the quantum fluctuations of the state and is defined as $V_{lk}=\text{Tr}[\rho(\Delta\hat{X}_l\Delta\hat{X}_k+\Delta\hat{X}_k\Delta\hat{X}_l)]=\langle(\hat{X}_l\hat{X}_k+\hat{X}_k\hat{X}_l)\rangle$ [24]. The correlation matrix of the three-mode Gaussian state decided by the master equation (13) is found as

$$V=\begin{pmatrix}V_{11}&V_{12}&V_{13}&V_{14}&V_{13}&V_{14}\\V_{12}&V_{22}&V_{14}&V_{15}&V_{14}&V_{15}\\V_{13}&V_{14}&V_{11}&V_{12}&V_{13}&V_{14}\\V_{14}&V_{15}&V_{12}&V_{22}&V_{14}&V_{15}\\V_{13}&V_{14}&V_{13}&V_{14}&V_{11}&V_{12}\\V_{14}&V_{15}&V_{14}&V_{15}&V_{12}&V_{22}\end{pmatrix}, \tag{18}$$

where

$$V_{11}=2f_1-f_3-f_3^{*}+1,\quad V_{12}=-i(f_3-f_3^{*}),$$

$$V_{13}=-(f_4+f_4^{*}+2f_2),\quad V_{14}=-i(f_4-f_4^{*}),$$

$$V_{15}=f_4+f_4^{*}-2f_2,\quad V_{22}=2f_1+f_3+f_3^{*}+1,$$

$$f_1 = \frac{2|r_2|^2}{3}\left(\frac{2\Theta_2+\Theta_1}{\Theta_1\Theta_2}\right), \quad f_2 = \frac{2|r_2|^2}{3}\left(\frac{\Theta_2-\Theta_1}{\Theta_1\Theta_2}\right),$$

$$f_3 = -\frac{2r_2[(r_1-g)-i\kappa]}{3\Theta_1} - \frac{r_2[(r_1+2g)-i\kappa]}{3\Theta_2},$$

$$f_4 = -\frac{r_2[(r_1-g)-i\kappa]}{3\Theta_1} + \frac{r_2[(r_1+2g)-i\kappa]}{3\Theta_2},$$

$$\Theta_1 = [(r_1-g)^2+\kappa^2-4|r_2|^2],$$

$$\Theta_2 = [(r_1+2g)^2+\kappa^2-4|r_2|^2]. \qquad (19)$$

We see that the correlation matrix V is symmetric with respect to permutations between the three cavity modes as a consequence of our consideration of the symmetric couplings between the cavity modes. After obtaining the correlation matrix of the three-mode Gaussian state of the three-mode intracavity field, we can analyze the behavior of the squeezing and the entanglement of the cavity modes.

A. Intracavity three-mode squeezing

1. Quantify the optimal three-mode squeezing

First, let us discuss the three-mode squeezing of the intracavity field which is closely related to the three-mode entanglement. For a three-mode quantum state, defining the quadrature operator

$$X_c = \frac{1}{\sqrt{3}}\sum_{j=1}^{3}(\mu_j b_j + \mu_j^* b_j^\dagger), \qquad (20)$$

where $\Sigma_{j=1}^3|\mu_j|^2=3$. If the variance of the quadrature operator meets $V(X_c)<1$ then we can say the state exhibits ordinary three-mode squeezing [25]. The minimum variance corresponds to the optimal three-mode squeezing.

To find the optimal three-mode squeezing of the three coupled cavity modes, let us perform the following "tritter" transformation [2,6,26]:

$$c_1 = \sqrt{\tfrac{2}{3}}\left[b_1 - \tfrac{1}{2}(b_2+b_3)\right],$$

$$c_2 = \sqrt{\tfrac{1}{2}}(b_2-b_3),$$

$$c_3 = \sqrt{\tfrac{1}{3}}(b_1+b_2+b_3), \qquad (21)$$

where the new operators meet the commutation relation $[c_i,c_j^\dagger]=\delta_{ij}$. Then, the master equation (13) is transformed into

$$\frac{d\rho}{dt} = -i[H_L',\rho] + \sum_{j=1}^{3}\kappa(2c_j\rho c_j^\dagger - c_j^\dagger c_j\rho - \rho c_j^\dagger c_j), \qquad (22)$$

where

$$H_L' = (r_1-g)(c_1^\dagger c_1 + c_2^\dagger c_2) + (r_1+2g)c_3^\dagger c_3 + \sum_{j=1}^{3}(r_2 c_j^2 + r_2^* c_j^{\dagger 2}). \qquad (23)$$

So, after the "tritter" transformation, the three bosonic modes c_j are decoupled. The above master equation (22) equivalently describes three detuned degenerate PDCs of c_j modes. By defining the quadrature operators $X_{c_j}=(c_j e^{-i\theta_j/2}+c_j^\dagger e^{i\theta_j/2})$, according to Eq. (22), we readily find the minimal variances $\langle X_{c_j}^2\rangle_{\min}$ in the steady regime

$$V_{c_{1,2}} = \langle X_{c_{1,2}}^2\rangle_{\min} = 1 - \frac{r_1}{2\sqrt{(r_1-g)^2+\kappa^2}+r_1} \qquad (24)$$

and

$$V_{c_3} = \langle X_{c_3}^2\rangle_{\min} = 1 - \frac{r_1}{2\sqrt{(r_1+2g)^2+\kappa^2}+r_1}. \qquad (25)$$

Here the relation $r_1=4|r_2|$ is applied. Evidently, indicated by Eq. (21) and the definition of the three-mode squeezing Eq. (20), the single-mode squeezing $V_{c_{1,3}}$ of the c_1 and c_3 modes actually quantifies the three-mode squeezing of the intracavity coupled modes a_j. Since the relation $V_{c_1}<V_{c_3}$ is always held [comparing Eq. (24) to Eq. (25)], the squeezing V_{c_1} hence characterizes the optimal three-mode squeezing of the intracavity field under our consideration.

2. The properties of the intracavity three-mode squeezing

Due to the fact that the squeezing condition $V_{c_j}<1$ is satisfied, we therefore always have the steady intracavity three-mode squeezing. Revealed by Eq. (24), near the threshold, i.e., $\sqrt{4|r_2|^2-(r_1-g)^2}\to\kappa$, we have the optimal three-mode squeezing $V_{c_1}\to 0.5$. That is to say, the maximal degree of the steady intracavity three-mode squeezing cannot exceed the 50% squeezing below the vacuum level.

The dependence of the optimal three-mode squeezing V_{c_1} of the intracavity field on the input intensity I_{in} of the pumping lasers is plotted in Fig. 3. It shows that the three-mode squeezing increases with the increase of the pumping intensity, which can be inferred by Eq. (24) for $g>r_1$, together with the fact that r_1 increases as the intensity I_{in} increases for the given strength g and χ. With the same reason, for the same input intensity I_{in}, the three-mode squeezing of the intracavity field increases when the Kerr nonlinearity χ increases or the coupling g decreases. As shown in the insets of Fig. 3 where we plot the maximal pumping intensity $I_{\text{in}}^{\max}$ required for achieving the maximum intracavity three-mode squeezing, i.e., $V_{c_1}\approx 0.5$, versus the strength g and χ, respectively, we see that $I_{\text{in}}^{\max}$ increases on the increase of the coupling g but decreases with the increase of the Kerr nonlinearity χ.

B. Intracavity three-mode entanglement

1. Quantify CV genuine tripartite entanglement

Next let us discuss the entanglement of the three-mode intracavity field. Based on the nonpositive partial transpose

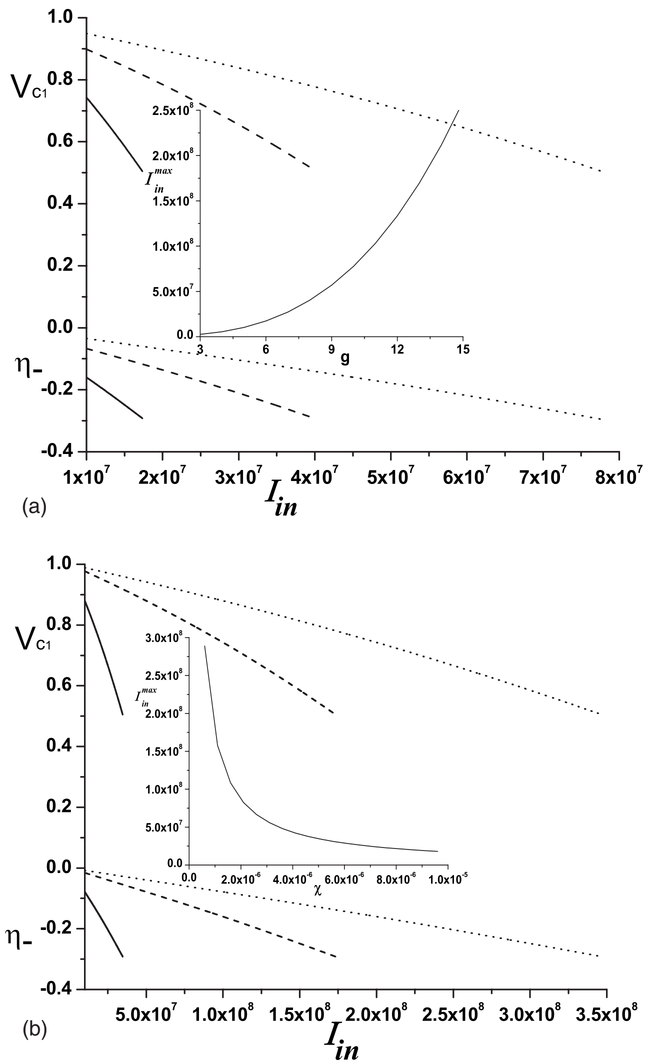

FIG. 3. The dependence of the intracavity three-mode entanglement η_- and squeezing V_{c_1} on the input intensity I_{in} of pumping lasers. (a) $\chi=10^{-5}$, $\kappa=1$, $g=6$ (solid line), 8 (dashed line), 10 (dotted line); (b) $g=6$, $\kappa=1$, $\chi=10^{-5}$ (solid line), 5×10^{-6} (dashed line), 10^{-6} (dotted line). The insets are for the maximal pumping intensity $I_{\text{in}}^{\max}$ required for achieving the maximal squeezing $V_{c_1}^{m}\approx0.5$ for the given couplings g and χ.

(NPT), which is a sufficient and necessary criterion for the inseparability of $1\times N$ bipartite Gaussian system [27], Giedke *et al.* put forth separability properties of three-mode Gaussian states [28]. For a physical three-mode Gaussian state with the Wigner characteristic function in Eq. (17), the direct consequence of the non-negativity of the density operator ρ is the uncertainty relation $V-i\Lambda\geqslant0$ [27]. Here the symplectic matrix Λ is block diagonal and defined as

$$\Lambda=\begin{pmatrix}\sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3\end{pmatrix},\quad \sigma_j=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\quad (j=1,2,3), \tag{26}$$

which results from the commutation rules $[\hat{X}_j,\hat{X}_{j'}]=i\Lambda_{jj'}$. Transposition Γ is equivalent to time reversal and corresponds in phase space to a sign change of the momentum variables, i.e., $X^T\to\Gamma X^T=(x_1,-p_1,x_2,-p_2,x_3,-p_3)^T$. In terms of the correlation matrix, we then have $V\to\Gamma V\Gamma$. For a three-mode Gaussian state, i.e., 1×2 bipartite Gaussian system, the NPT criterion states that the state is bipartite inseparable between the jth mode and the group of the rest two modes if and only if

$$\Gamma_j V\Gamma_j+i\Lambda<0, \tag{27}$$

where Γ_j denotes the partial transposition on the jth mode. According to Ref. [28], a three-mode Gaussian state is fully inseparable (genuine three-mode entanglement) if and only if the condition $\Gamma_j V\Gamma_j+i\Lambda<0$ is simultaneously satisfied for $j=1,2,3$.

2. Entanglement properties of the three-mode cavity field

For the present system, due to the fact that the correlation matrix V in Eq. (18) is permutational symmetry among the three cavity modes, the negative eigenvalues of the matrix $\Gamma_j V\Gamma_j+i\Lambda$ are the same for $j=1,2,3$ and denoted by η_-. The negativity η_- characterizes the entanglement properties of the three-mode Gaussian state, and hence we utilize the negativity η_- to quantify the genuine three-mode entanglement of the present system [29].

The negativity η_- is also plotted in Fig. 3.It is shown that the intracavity three-mode entanglement η_- has the same dependence on the Kerr nonlinearity χ and the coupling constant g as the intracavity three-mode squeezing V_{c_1} does. As a matter of fact, from the "tritter" transformation described by Eq. (21), the entanglement of the three-mode cavity field can be considered as the output of a "tritter" [formed by a 1:1 beam splitter (BS) and a 1:2 BS which are coupled to each other by letting an output from the 1:2 BS be an input state of the 1:1 BS] for the three input states being the single-mode squeezed states of c_j. The increases (or decreases) of the single-mode squeezing of modes c_j will lead to the relevant increase (or decrease) of the three-mode entanglement from the outport of the "tritter." Therefore, the dependence of the intracavity three-mode entanglement on the coupling g and the Kerr nonlinearity χ is very similar to that of the intracavity three-mode squeezing. Figure 3 shows that, for the given Kerr nonlinearity χ, linear coupling g, and the input intensity $I_{\text{in}}<I_{\text{in}}^{\max}$, the three-mode squeezing and three-mode entanglement can be simultaneously generated.

C. Output squeezing and entanglement

We proceed to discuss the properties of the squeezing and the entanglement of the field outside the cavity. The squeezing and entanglement spectra of the output field should be investigated since the output field from the cavity has a range of frequency. From the master equation (13), the Langevin equations of motion for the operators b_j are obtained as

$$\frac{d}{dt}B(t)=(A-\mathbf{I}k)B(t)+\sqrt{2}\kappa B_{\text{in}}(t), \tag{28}$$

where

$$A = \begin{pmatrix} -ir_1 & -2ir_2^* & -ig & 0 & -ig & 0 \\ 2ir_2 & ir_1 & 0 & ig & 0 & ig \\ -ig & 0 & -ir_1 & -2ir_2^* & -ig & 0 \\ 0 & ig & 2ir_2 & ir_1 & 0 & ig \\ -ig & 0 & -ig & 0 & -ir_1 & 2ir_2^* \\ 0 & ig & 0 & ig & -2ir_2 & ir_1 \end{pmatrix}, \tag{29}$$

$B(t)=[b_1(t),b_1^\dagger(t),b_2(t),b_2^\dagger(t),b_3(t),b_3^\dagger(t)]^T$, and $B_{\rm in}=[b_1^{\rm in}(t),b_1^{\rm in\dagger}(t),b_1^{\rm in}(t),b_2^{\rm in\dagger}(t),b_2^{\rm in}(t),b_2^{\rm in\dagger}(t)]^T$ with $b_j^{\rm in}(t)$ being the vacuum noise operators that meet the commutation relations

$$[b_i^{\rm in}(t),b_j^{\rm in\dagger}(t')]=\delta_{ij}\delta(t-t'). \tag{30}$$

Using the input-output relationship $B^{\rm out}+B^{\rm in}=\sqrt{2\kappa}B$ [30], the operators of the output field in the frequency domain can be expressed in terms of the input noise operators

$$B^{\rm out}(\omega)=-(A+\mathbf{I}k+i\omega)(A-\mathbf{I}k+i\omega)^{-1}B^{\rm in}(\omega)=MB^{\rm in}(\omega), \tag{31}$$

with $B^{\rm out}(\omega)=[b_1^{\rm out}(\omega),b_1^{\rm out\dagger}(-\omega),b_2^{\rm out}(\omega),b_2^{\rm out\dagger}(-\omega),b_3^{\rm out}(\omega),b_3^{\rm out\dagger}(-\omega)]^T$, and $B^{\rm in}(\omega)=[b_1^{\rm in}(\omega),b_1^{\rm in\dagger}(-\omega),b_2^{\rm in}(\omega),b_2^{\rm in\dagger}(-\omega),b_3^{\rm in}(\omega),b_3^{\rm in\dagger}(-\omega)]^T$. The matrix M has the form

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{13} & M_{14} & M_{11} & M_{12} & M_{15} & M_{16} \\ M_{23} & M_{24} & M_{12} & M_{22} & M_{15} & M_{26} \\ M_{13} & M_{14} & M_{15} & M_{16} & M_{11} & M_{12} \\ M_{23} & M_{24} & M_{25} & M_{26} & M_{21} & M_{22} \end{pmatrix}, \tag{32}$$

and the elements of the matrix will be given numerically. By defining the quadrature operators of the output modes in the continuum representation $\hat{X}_j(\omega)=[b_j^{\rm out}(\omega)+b_j^{\rm out\dagger}(-\omega)]/\sqrt{2}$, $\hat{Y}_j(\omega)=-i[b_j^{\rm out}(\omega)-b_j^{\rm out\dagger}(-\omega)]/\sqrt{2}$ [31] and using the commutation relation $[b_i^{\rm in}(\omega),b_j^{\rm in\dagger}(\omega')]=\delta_{ij}\delta(\omega-\omega')$, the correlation matrix of the output field with frequency shift ω has the same form as that of the intracavity field (18) and can be calculated as

$$V'(\omega) = \begin{pmatrix} V'_{11} & V'_{12} & V'_{13} & V'_{14} & V'_{13} & V'_{14} \\ V'_{12} & V'_{22} & V'_{14} & V'_{15} & V'_{14} & V'_{15} \\ V'_{13} & V'_{14} & V'_{11} & V'_{12} & V'_{13} & V'_{14} \\ V'_{14} & V'_{15} & V'_{12} & V'_{22} & V'_{14} & V'_{15} \\ V'_{13} & V'_{14} & V'_{13} & V'_{14} & V'_{11} & V'_{12} \\ V'_{14} & V'_{15} & V'_{14} & V'_{15} & V'_{12} & V'_{22} \end{pmatrix}, \tag{33}$$

where

$$V'_{11(22)}=|A_{1x(y)}|^2+|C_{1x(y)}|^2+|E_{1x(y)}|^2,$$

$$V'_{12(4)}=-2\,{\rm Im}[A_{1x}A_{1(2)y}^*+C_{1x}C_{1(2)y}^*+E_{1x}E_{1y}^*],$$

$$V'_{13}=2\,{\rm Re}[A_{1x}C_{1x}^*+A_{1x}^*C_{1x}+|E_{1x}|^2],$$

$$V'_{15}=2\,{\rm Re}[A_{1y}C_{1y}^*+A_{1y}^*C_{1y}+|E_{1y}|^2], \tag{34}$$

and $A_{1x(y)}=M_{11}\pm M_{21}$, $C_{1x(y)}=M_{13}\pm M_{23}$, $E_{1x(y)}=M_{15}\pm M_{25}$, $A_{2y}=C_{1y}$, and $C_{2y}=A_{1y}$. On obtaining the correlation matrix of the output field, we can discuss the entanglement of the output field.

Since we have shown that $V_{c_1}[=\langle X_{c_1}^2\rangle_{\min}]$ is the optimal three-mode intracavity squeezing, it is evident that the optimal output squeezing of the three-mode intracavity field is the output squeezing from the effective detuned PDC of c_1 mode governed by the master equation (22). Following the method mentioned above, the operator $C_1^{\rm out}$ of the output field from the effective detuned PDC of the c_1 mode is found in the frequency domain as

$$C_1^{\rm out}(\omega)=\frac{D_1c_1^{\rm in}(\omega)+4i\kappa r_2c_1^{\rm in\dagger}(-\omega)}{(\kappa-i\omega)^2+\Delta^2-4|r_2|^2}, \tag{35}$$

where $D_1=4|r_2|^2+\omega^2+(\kappa-ig)^2$, $\Delta=r_1-g$, and the noise operator $c_1^{\rm in}(\omega)$ satisfies $[c_1^{\rm in}(\omega),c_1^{\rm in\dagger}(\omega')]=\delta(\omega-\omega')$. With the definition of the amplitude operator $X_{c_1}(\omega)=[C_1^{\rm out}(\omega)e^{-i\theta_1/2}+C_1^{\rm out\dagger}(-\omega)e^{i\theta_1/2}]$, the output squeezing $V_{c_1}^{\omega}[=\langle X_{c_1}^2\rangle(\omega)]$ is optimized with regard to θ_1 as

$$V_{c_1}^{\omega}=1-\frac{8\kappa|r_2|\sqrt{(\kappa^2+D_2)^2+4\kappa^2\Delta^2}-32\kappa^2|r_2|^2}{(\kappa^2-D_2)^2+4\kappa^2\omega^2}, \tag{36}$$

where $D_2=\omega^2-\Delta^2+4|r_2|^2$. Here $V_{c_1}^{\omega}$ characterizes the optimal output squeezing of the three-mode intracavity field. According to Eq. (36), at zero frequency $\omega=0$ and near the threshold, i.e., $\sqrt{4|r_2|^2-(r_1-g)^2}\to\kappa$, we have $V_{c_1}^{\omega=0}\to 0$, which means that the output squeezing at the cavity resonant frequencies becomes nearly perfect when the system operates near the threshold. As shown in Fig. 4, for the given Kerr nonlinearity $\chi=10^{-5}$, the evanescent coupling $g=8.0$ and the cavity damping $\kappa=1.0$, the input intensity $I_{\rm in}$ near the threshold equals approximately 4.64×10^7, and the almost perfect output squeezing is obtained. According to Eq. (36), when the system operates below the threshold, the maximum of the output squeezing $V_{c_1}^{\omega m}=\frac{2g-3r_1}{2g-r_1}$ for $g>\frac{3r_1}{2}$ occurring at the frequency $\omega_{c_1}=\pm\frac{1}{2}\sqrt{(2g-r_1)(2g-3r_1)-2k^2}$, and the maximal squeezing $V_{c_1}^{\omega m}$ decreases and moves further away from the cavity resonant frequencies with the increase of the evanescent coupling g and the decrease of the Kerr nonlinearity χ (shown in Fig. 4). Moreover, because here the maximal output squeezing $V_{c_1}^{\omega m}<1$, the output squeezing can always be achievable. In addition, indicated by Eq. (22), the squeezing spectrum $V_{c_3}^{\omega}$ corresponding to nonoptimal intracavity three-mode squeezing V_{c_3} can be obtained just through replacing Δ in Eq. (36) by r_1+2g. The maximal output squeezing $V_{c_3}^{\omega m}=\frac{4g+r_1}{4g+3r_1}$ and appears at the frequency $\omega_{c_3}=\pm\frac{1}{2}\sqrt{(4g+r_1)(4g+3r_1)-2k^2}$. Figure 4 shows that, comparing to the output squeezing $V_{c_1}^{\omega}$, the output squeezing $V_{c_3}^{\omega}$ has smaller maximum which arises at the higher frequency, since we have the relations $V_{c_1}^{\omega m}<V_{c_3}^{\omega m}$ and $\omega_{c_1}<\omega_{c_3}$.

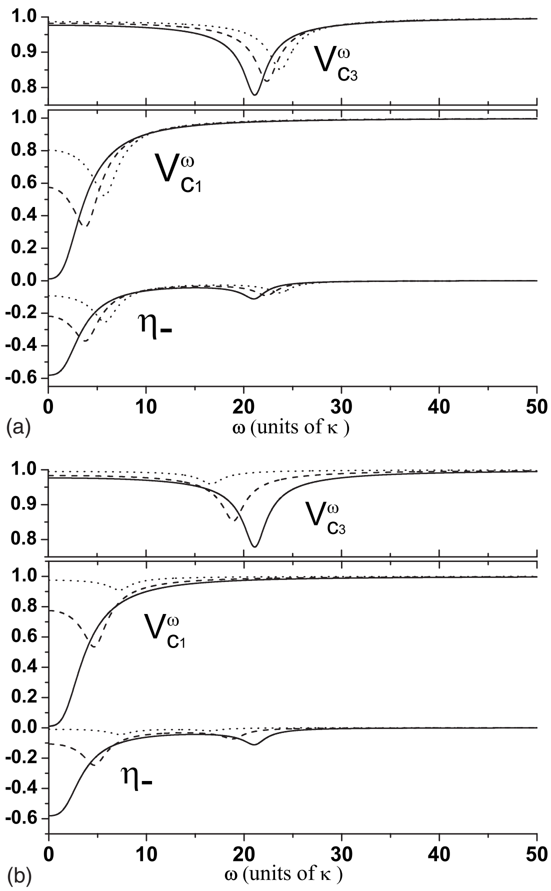

FIG. 4. The spectra of the entanglement and squeezing of the output field for $I_{\rm in}=4.64\times10^7$ (a) $\chi=10^{-5}$, $\kappa=1$, $g=8$ (solid line), 9 (dashed line), 10 (dotted line); (b) $g=8$, $\kappa=1$, $\chi=10^{-5}$ (solid line), 5×10^{-6} (dashed line), 10^{-6} (dotted line).

The entanglement spectrum of the output field is also plotted in Fig. 4. From it, we see that the variation of the output entanglement with the Kerr nonlinearity χ and linear coupling g is very similar to that of the output squeezing, which is in agreement with the intracavity case. This is because the entanglement of the output field can be considered as the consequence of mixing the outputs of three degenerate PDCs of modes c_j on a "tritter," indicated by Eqs. (21) and (22), and hence the entanglement property of the output field is determined by the output squeezing of modes c_j. Moreover, because the maximal output squeezing $V_{c_1}^{\omega m}$ and $V_{c_3}^{\omega m}$ arise at different frequencies ($\omega_{c_3}>\omega_{c_1}$), the output entanglement spectrum has two local maximums which appear at the frequencies ω_{c_3} and ω_{c_1}, respectively. Therefore, together with Fig. 1, here we see that the bright and robust three-mode squeezed and full inseparable entangled beams can be achievable outside the cavity in the present model.

IV. CONCLUSIONS

In this paper, the scheme for generating macroscopic three-mode CV entanglement in triply coupled $\chi^{(3)}$ Kerr coupler inside a pumped optical cavity is investigated. With the linearized analysis for small perturbation around the steady state of the pumped cavity modes, the linearized Hamiltonian and the corresponding master equation are obtained. We found that the maximal three-mode intracavity squeezing is limited to 50% squeezing with respect to the vacuum fluctuations, and the output squeezing transferred from the cavity can exceed this limit and becomes nearly perfect at the cavity resonant frequencies and near the threshold. Also it is found that the bright three-mode highly squeezed and fully inseparable entangled light beams can be achieved outside the cavity.

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