Optical nonlinear waves in semiconductor quantum dots: Solitons and breathers

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A general theory of optical resonance solitons and breathers in the presence of single-excitonic and biexcitonic transitions in an ensemble of inhomogeneously broadened semiconductor quantum dots is constructed. Optical plane wave solitons (2π pulses) are formed in stacked layer structures (many-layered systems) of semiconductor quantum dots. Optical small-amplitude bright breathers (0π pulses) in semiconductor quantum dot waveguides are considered. Explicit analytical expressions for the shape and parameters of the solitons and breathers in the regime of self-induced transparency are obtained as well as simulations of the space-time dynamics of two-dimensional breathers presented with realistic parameters which can be reached in current experiments. It is shown that, unlike for plane wave breathers, the parameters additionally depend on the waveguide mode. In the special case of plane wave breathers in semiconductor quantum dots, known analytical results are recovered.

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I. INTRODUCTION

The propagation of intense optical pulses in nonlinear media can result in waves with invariant profiles. Depending on the carrier frequency with respect to the optical transitions, these waves can be on- or off-resonant. Resonant nonlinear waves can be formed within the McCall-Hahn mechanism if the conditions for self-induced transparency (SIT) are fulfilled:

$$\omega T \gg 1, \quad T \ll T_{1,2}, \tag{1}$$

where T and ω are the width and frequency of the pulse and T_1 and T_2 are the longitudinal and transverse relaxation times of the resonant atoms, respectively. Depending on the area of the pulse Θ which is a measure of the light-matter interaction strength, different waves can form: for $\Theta > \pi$, a soliton (2π pulse) is generated, and for $\Theta \ll 1$, an optical small-amplitude resonance breather (0π pulse) can be formed [1–5], which can be viewed as a pulsing soliton with an internal frequency Ω and a "wave number" Q with $\Omega \ll \omega$ and $Q \ll k$, where k is the wave number of the pulse. The breathers [6] which we are considering here are equivalent to the pulsating 0π pulse from the McCall-Hahn theory of SIT and the original breather of the sine-Gordon equation [7]. Thus, they are like bound states of two kinklike solitons, but unlike single solitons, they can be excited with relatively low intensities of the input pulses. It can be shown that these breathers are related to the *single* soliton solution of the nonlinear Schrödinger equation (NSE) [8–10], since the NSE is the small amplitude limit of both the sine-Gordon and the SIT (Maxwell-Bloch) equations [8–12].

In addition to the conditions for the existence of SIT, optical resonance breathers must fulfill the condition

$$\omega \gg \Omega \gg \frac{1}{T} \gg \frac{1}{T_2}, \tag{2}$$

which further limits the media in which breathers can form (in solids, typically $T_1 \gg T_2$). For instance, these conditions are satisfied for optical waves in solid dielectrics containing a small concentration of the resonant impurity atoms [13,14]. The relation between Ω and Q must be determined in each specific case.

The existence of resonance breathers is one of the most interesting and important manifestations of optical nonlinearity of the medium. Since they have many solitonlike properties and, unlike solitons, can be excited for relatively small pulse intensities in comparison to solitons which makes the experimental realization easier, resonance breathers are of particular interest for the study of nonlinear optical phenomena. For these reasons, the study of the properties and the formation of breathers in different media counts as one of the principal problems in the physics of nonlinear waves. In addition, a recent study [15] of nonresonant soliton solutions revealed many interesting aspects, among which is that most solitons are unstable, and when unstable, the instability almost always leads to the formation of a breather. Thus, in some nonlinear systems, breathers seem to be more stable and numerous than solitons. Consequently, one can anticipate that in some systems, breathers may generally be of more utility than single soliton solutions (2π pulses).

In view of this, the physical properties of optical resonance solitons and breathers of SIT in dielectrics containing small concentrations of the resonant impurity atoms and bulk semiconductors have been investigated in detail [13,16]. For theoretical investigations of this phenomenon, the Maxwell-Bloch equations in $(1+1)$ dimensions are considered. The

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system of equations is integrable in terms of the inverse scattering transform techniques, and the exact solutions of the equations can be obtained [17–19]. The situation in limited media is different, and propagating pulses which possess a transverse structure, for instance, surface waves or waveguide modes [2], need to be considered. Here, for the description of waves with a transverse structure, it is necessary to use the Maxwell-Bloch equations in (2+1) or (3+1) dimensions and to take into account the boundary conditions of the different media which requires special considerations.

Semiconductor quantum dots (SQDs) have been studied extensively in the last years with respect to their nonlinear optical properties [20]. SQDs, also referred to as zero-dimensional systems, are nanostructures with confinement of the charge carriers in all three spatial dimensions, resulting in atomlike discrete energy spectra with strongly enhanced carrier lifetimes. Such features make quantum dots similar to atoms in many respects (artificial atoms).

Observation of optical coherence effects in ensembles of quantum dots is usually spoiled by the inhomogeneous line broadening due to dot size fluctuations, with typical broadenings comparable to electronic level splittings. Quantum dots often have a base length in the range 50–400 Å. Size fluctuations in the quantum dot ensemble lead to an inhomogeneous single-exciton and biexciton level broadening, with a full width at half maximum of typically more than several tens of meV. In addition, semiconductors, as opposed to atomic systems which can be modeled by noninteracting two-level systems, experience many-body effects (such as exciton-exciton interactions leading to the formation of biexcitonic states) which are unknown in atomic systems. Thus the situation is more involved in quantum dots as compared to atoms, because a strong optical pulse cannot only excite single electron-hole pairs (excitons) but also multiple pairs (multiexcitons).

On the other hand, due to the large dipole moments, the nonlinear interaction between the quantum dots and the optical excitation is strongly enhanced in comparison with atomic systems. Hence, from a fundamental standpoint, nonlinear optical effects, among them the effect of SIT, will be substantially different in SQDs. Due to very long relaxation times at low temperatures [21], nonlinear optical experiments can be performed with pulses of several tens of picoseconds where the influence of phonon dephasing is not of major importance [22,23]. In addition, great progress has been made in the exploitation of SQDs for technical devices and their design and control. Experimental observations of self-induced transmission on a free exciton resonance in CdSe [24] and SIT in InGaAs quantum dot waveguides [25] have been reported. For these reasons, it is interesting to consider nonlinear optical effects in SQDs. On the theoretical side, the effect of SIT for solitons in a sample of inhomogeneously broadened SQDs in the presence of single-excitonic and biexcitonic transitions has been investigated numerically in Ref. [26]. The results have demonstrated that intense optical pulses have properties of McCall-Hahn's 2π pulse and can propagate in realistic dot samples without suffering strong losses. Numerical methods were used to integrate the semiconductor Maxwell-Bloch equations, with typical values for the pulse and quantum dot parameters.

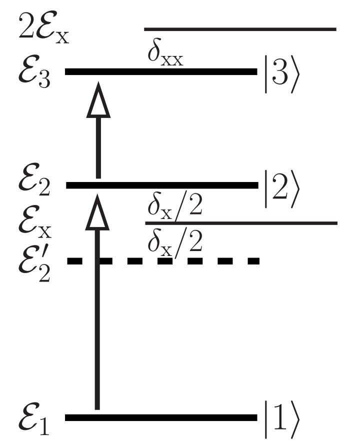

FIG. 1. Schematic of the SQD energetic levels.

Rabi oscillations as a prerequisite for resonance solitons have been discussed numerically in Ref. [27,28].

Recently, theoretical and numerical investigations of breathers of SIT have been considered for plane waves in SQDs, and explicit analytical expressions for the parameters of the resonance breathers have been presented [29]. The purpose of the present work is to theoretically investigate the processes of the formation of optical resonance solitons and breathers under the condition of SIT in SQD many-layered systems and waveguides. A comparison with work in spatially homogeneous systems [29] (plane wave propagation) is made.

II. BASIC EQUATIONS

We consider the formation of optical nonlinear waves of SIT in many-layered SQD systems for linearly polarized plane waves as well as in SQD waveguides in the case when a linearly polarized plane wave TE mode with width T and frequency $\omega \gg T^{-1}$ with an electric field strength $\vec{E}(x,y,z,t) = \vec{e}E(x,y,z,t)$ is propagating along the positive z axis, where $\vec{e}$ is the vector of polarization directed along the y axis. We investigate in detail waveguide modes which are limited in one transverse spatial dimension (here, in x direction) and are independent of the other spatial dimension.

The pulse is tuned to transitions from the ground state $|1\rangle$ of the SQD to the states $|2\rangle$ and $|3\rangle$, with energies $\mathcal{E}_1=0$, $\mathcal{E}_2=\hbar\omega_0=\mathcal{E}_x+\delta_x/2$, and $\mathcal{E}_3=\hbar\Omega_0=2\mathcal{E}_x+\delta_{xx}$, respectively. The quantities $\mathcal{E}_x=(\mathcal{E}_2+\mathcal{E}_2')/2$ and $\mathcal{E}_3$ are the energies of the single-excitonic and biexcitonic states, respectively. $\delta_x=\mathcal{E}_2-\mathcal{E}_2'$ and δ_{xx} are the energies of the exciton fine structure splitting and biexcitonic binding energy (negative if bound), respectively (Fig. 1); $\hbar$ is Planck's constant. In order that $\delta_x/2 \ll \hbar\omega_0$ and $\delta_{xx} \ll \hbar(\Omega_0-\omega_0)$, the 1 to 2 transition and the 2 to 3 transition are very close to each other and to the pulse frequency ω. To avoid the influence of electron-phonon scattering [22,23], we assume that the pulse excites the system mostly within the zero-phonon line [30]. The energetic spectrum of the quantum dots can be considered as a quasi-equidistant three-level system in a cascade configuration ($\mu_{13}=0$) under off-resonant excitation $\Omega_0-\omega_0-\omega \neq 0$ and

$\omega_0 - \omega \neq 0$. We assume that the detunings from the resonance $\Omega_0 - \omega_0 - \omega$ and $\omega_0 - \omega$ lie within the bandwidth of the pulse.

The Hamiltonian of the system is given by

$$H = H_0 + V, \tag{3}$$

where

$$H_0 = \hbar\omega_0|2\rangle\langle 2| + \hbar\Omega_0|3\rangle\langle 3|$$

describes the kinetics of the single-excitonic and biexcitonic states and

$$V = -\vec{P}\cdot\vec{E}$$

is the Hamiltonian of the light-quantum dot interaction. The vector of polarization

$$\vec{P}(x,z,t) = n_0 \int g(\Delta')(\vec{\mu}_{12}\rho_{21} + \vec{\mu}_{23}\rho_{32})(\Delta',x,z,t)d\Delta' + \text{c.c.}$$

is determined by the interband transitions occurring in the quantum dots between the three energetic levels. The quantities $\vec{\mu}_{12}$ and $\vec{\mu}_{23}$ are the dipole moments for the corresponding transitions which we assume to be parallel to each other; $g(\Delta)$ is the inhomogeneous broadening function which arises due to quantum dot size fluctuations. Because the energy levels depend on the size of the dots, the exciton frequency ω_0 and the quantity $\Delta = \omega_0 - \omega$ are also size dependent; n_0 is the constant quantum dot density. Since we investigate the situation of a small concentration of quantum dots, interaction of one quantum dot with another is neglected in the Hamiltonian (3).

The wave equation for the strength of the electric field of an optical pulse $E(x,z,t)$ in media has the form

$$-c^2\frac{\partial^2 E}{\partial z^2} + \eta^2\frac{\partial^2 E}{\partial t^2} - c^2\frac{\partial^2 E}{\partial x^2} = -4\pi\frac{\partial^2 P}{\partial t^2}, \tag{4}$$

where

$$P(x,z,t) = n_0 \int g(\Delta')(\mu_{12}\rho_{21} + \mu_{23}\rho_{32})(\Delta',x,z,t)d\Delta' + \text{c.c.} \tag{5}$$

is the y component of the vector of polarization, $\mu_{12} = \vec{\mu}_{12}\cdot\vec{e}$, $\mu_{23} = \vec{\mu}_{23}\cdot\vec{e}$, c is the velocity of light in vacuum, and η the semiconductor refractive index.

The quantities ρ_{nm} are the elements of the density matrix ρ which are determined by the Liouville equation

$$i\hbar\frac{\partial\rho_{nm}}{\partial t} = \sum_l (\langle n|H|l\rangle\rho_{lm} - \rho_{nl}\langle l|H|m\rangle),$$

where $n,m,l = 1,2,3$. Substituting in this equation the expression for the Hamiltonian (3), we obtain a system of equations for the elements of the density matrix for the quantum dot ensemble:

$$i\hbar\frac{\partial\rho_{11}}{\partial t} = (-\mu_{12}\rho_{21} + \mu_{12}^*\rho_{12})E,$$

$$i\hbar\frac{\partial\rho_{22}}{\partial t} = (\mu_{12}\rho_{21} - \mu_{23}\rho_{32} - \mu_{12}^*\rho_{12} + \mu_{23}^*\rho_{23})E,$$

$$i\hbar\frac{\partial\rho_{33}}{\partial t} = (-\mu_{23}^*\rho_{23} + \mu_{23}\rho_{32})E,$$

$$i\hbar\frac{\partial\rho_{21}}{\partial t} = \hbar\omega_0\rho_{21} - \mu_{12}^*E(\rho_{11} - \rho_{22}) - \mu_{23}E\rho_{31},$$

$$i\hbar\frac{\partial\rho_{32}}{\partial t} = \hbar(\Omega_0 - \omega_0)\rho_{32} + \mu_{12}E\rho_{31} - \mu_{23}^*E(\rho_{22} - \rho_{33}),$$

$$i\hbar\frac{\partial\rho_{31}}{\partial t} = \hbar\Omega_0\rho_{31} - \mu_{23}^*E\rho_{21} + \mu_{12}^*E\rho_{32}. \tag{6}$$

We can simplify Eqs. (4) and (6) using the method of slowly changing profiles. For this purpose, we represent the functions E and ρ_{nm} in the form

$$E = \sum_{l=\pm 1}\hat{E}_l Z_l,$$

$$\rho_{21} = \hat{\rho}_{21}Z_1, \quad \rho_{32} = \hat{\rho}_{32}Z_1, \quad \rho_{31} = \hat{\rho}_{31}Z_2, \tag{7}$$

where $\hat{E}_l$ and $\hat{\rho}_{nm}$ are the slowly varying complex amplitudes of the optical electric field and the elements of the density matrix, respectively, and $Z_n = e^{in(kz-\omega t)}$. To guarantee that the quantity E is real, we set $\hat{E}_l = \hat{E}_{-l}^*$. We note that such a representation of the solutions of a nonlinear wave equation for pulses in semiconductors is widely used in the theory of nonlinear waves [1,13,16,29,31–33].

On substituting the expressions (7) in the nonlinear equations (4) and (6), we obtain for the slowly varying complex amplitudes of the optical electric field $\hat{E}_l$ the nonlinear wave equation

$$\sum_{l=\pm 1} Z_l\left[l^2(c^2k^2 - \kappa\omega^2)\hat{E}_l - 2ilkc^2\frac{\partial\hat{E}_l}{\partial z} - 2il\kappa\omega\frac{\partial\hat{E}_l}{\partial t} - c^2\frac{\partial^2\hat{E}_l}{\partial z^2} + \kappa\frac{\partial^2\hat{E}_l}{\partial t^2} - c^2\frac{\partial^2\hat{E}_l}{\partial x^2}\right] = -4\pi\frac{\partial^2 P}{\partial t^2} \tag{8}$$

and the system of equations for the quantities $\hat{\rho}_{nm}$,

$$i\hbar\frac{\partial\rho_{11}}{\partial t} = -\mu_{12}\hat{E}_{-1}\hat{\rho}_{21} + \mu_{12}^*\hat{E}_1\hat{\rho}_{21}^*,$$

$$i\hbar\frac{\partial\rho_{22}}{\partial t} = \mu_{12}\hat{E}_{-1}\hat{\rho}_{21} - \mu_{23}\hat{E}_{-1}\hat{\rho}_{32} - \mu_{12}^*\hat{E}_1\hat{\rho}_{21}^* + \mu_{23}^*\hat{E}_1\hat{\rho}_{32}^*,$$

$$i\hbar\frac{\partial\rho_{33}}{\partial t} = -\mu_{23}^*\hat{E}_1\hat{\rho}_{32}^* + \mu_{23}\hat{E}_{-1}\hat{\rho}_{32},$$

$$i\hbar\frac{\partial\hat{\rho}_{21}}{\partial t} = \hbar(\omega_0 - \omega)\hat{\rho}_{21} - \mu_{12}^*\hat{E}_1(\rho_{11} - \rho_{22}) - \mu_{23}\hat{E}_{-1}\hat{\rho}_{31},$$

$$i\hbar\frac{\partial\hat{\rho}_{32}}{\partial t} = \hbar(\Omega_0 - \omega_0 - \omega)\hat{\rho}_{32} - \mu_{23}^*\hat{E}_1(\rho_{22} - \rho_{33}) + \mu_{12}\hat{E}_{-1}\hat{\rho}_{31},$$

$$i\hbar\frac{\partial\hat{\rho}_{31}}{\partial t}=\hbar(\Omega_0-2\omega)\hat{\rho}_{31}-\mu_{23}^{*}\hat{E}_1\hat{\rho}_{21}+\mu_{12}^{*}\hat{E}_1\hat{\rho}_{32}, \qquad (9)$$

where $\kappa=\eta^2$. Here, the rotating wave approximation for the density matrix elements has been used [13,16,32]. Equations (9) are exact only in the limit of infinite relaxation times. To take into account that we consider a coherent interaction of the optical pulse with quantum dots, i.e., $T \ll T_{1,2}$, the influence of relaxation on the nonlinear wave processes are neglected in the present work. For this purpose, we consider a pulse which is short compared to the radiative lifetime of the system, but long compared to the electron-phonon dephasing time in order to neglect electron-phonon interaction [27].

In the absence of phase modulation, $\hat{E}_l=\hat{E}_{-l}=\hat{E}_l^{*}=\hat{E}$, we obtain for the real envelope $\hat{E}$:

$$\sum_{l=\pm1}Z_l\left[l^2(c^2k^2-\kappa\omega^2)\hat{E}-2ilkc^2\frac{\partial\hat{E}}{\partial z}-2il\kappa\omega\frac{\partial\hat{E}}{\partial t}-c^2\frac{\partial^2\hat{E}}{\partial z^2}+\kappa\frac{\partial^2\hat{E}}{\partial t^2}-c^2\frac{\partial^2\hat{E}}{\partial x^2}\right]=-4\pi\frac{\partial^2 P}{\partial t^2}, \qquad (10)$$

and for the slowly varying amplitudes $\hat{\rho}_{nm}$:

$$i\hbar\frac{\partial\rho_{11}}{\partial t}=(-\mu_{12}\hat{\rho}_{21}+\mu_{12}^{*}\hat{\rho}_{21}^{*})\hat{E},$$

$$i\hbar\frac{\partial\rho_{22}}{\partial t}=(\mu_{12}\hat{\rho}_{21}-\mu_{23}\hat{\rho}_{32}-\mu_{12}^{*}\hat{\rho}_{21}^{*}+\mu_{23}^{*}\hat{\rho}_{32}^{*})\hat{E},$$

$$i\hbar\frac{\partial\rho_{33}}{\partial t}=(-\mu_{23}^{*}\hat{\rho}_{32}^{*}+\mu_{23}\hat{\rho}_{32})\hat{E},$$

$$i\hbar\frac{\partial\hat{\rho}_{21}}{\partial t}=\hbar(\omega_0-\omega)\hat{\rho}_{21}-\mu_{12}^{*}\hat{E}(\rho_{11}-\rho_{22})-\mu_{23}\hat{E}\hat{\rho}_{31},$$

$$i\hbar\frac{\partial\hat{\rho}_{32}}{\partial t}=\hbar(\Omega_0-\omega_0-\omega)\hat{\rho}_{32}-\mu_{23}^{*}\hat{E}(\rho_{22}-\rho_{33})+\mu_{12}\hat{E}\hat{\rho}_{31},$$

$$i\hbar\frac{\partial\hat{\rho}_{31}}{\partial t}=\hbar(\Omega_0-2\omega)\hat{\rho}_{31}-\mu_{23}^{*}\hat{E}\hat{\rho}_{21}+\mu_{12}^{*}\hat{E}\hat{\rho}_{32}. \qquad (11)$$

When the transitions between energetic states of the quantum dots correspond to a $\Delta m=0$ transition, we may take μ_{21} and μ_{23} to be real vectors, $\mu_{21}=\mu_{21}^{*}$, $\mu_{23}=\mu_{23}^{*}$; such transitions might be induced by linearly polarized light which is investigated in detail. Analogously, we can consider excitations in quantum dots when we are dealing with $\Delta m=\pm 1$ transitions, such as might be induced by circularly polarized light; $\vec{\mu}_{21}$ and $\vec{\mu}_{23}$ are then necessarily complex vectors.

The systems of nonlinear equations for the slowly varying envelopes (8), (9) and (10), (11) are in sufficiently general form to describe various processes of the formation and propagation of optical resonance solitons and breathers in SQDs as well as for plane waves in many-layered systems and waveguide modes with transverse structure.

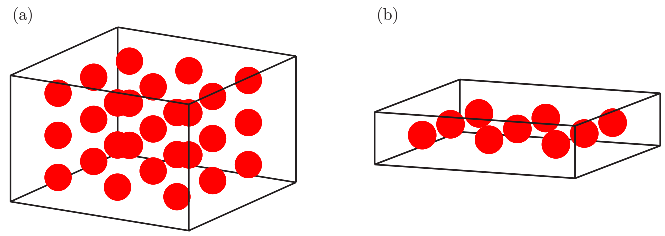

FIG. 2. (Color online) Schematics of the geometries in which the soliton and breather solutions are considered: (a) a many-layered SQD system and (b) a SQD waveguide structure with confinement in one spatial dimension.

III. SELF-INDUCED TRANSPARENCY IN STACKED LAYER STRUCTURES (MANY-LAYERED SYSTEMS) OF SEMICONDUCTOR QUANTUM DOTS

For the study of SIT in stacked layer structures, for example, of $In_xGa_{1-x}As$ SQDs embedded in a GaAs host material, it is sufficient to consider plane waves, i.e., all quantities characterizing the wave process depend only on one spatial coordinate z and time [see Fig. 2(a)]. In the interaction of an optical pulse with a resonantly absorbing medium, the most significant effects are usually observed at exact resonance and no inhomogeneous broadening of the spectral line $g(\Delta)=\delta(\Delta)$. Therefore, for simplicity, we first consider the system of equations (10) and (11) for plane waves in the absence of phase modulation. Using the slowly varying amplitude approximation which is based on the consideration that the pulse envelopes $\hat{E}_l$ vary sufficiently slowly in space and time as compared to the carrier wave parts, i.e.,

$$\left|\frac{\partial\hat{E}_l}{\partial t}\right| \ll \omega|\hat{E}_l|, \qquad \left|\frac{\partial\hat{E}_l}{\partial z}\right| \ll k|\hat{E}_l|,$$

the wave equation (10) and the polarization (5) take the form

$$\sum_{l=\pm1}Z_l\left[l^2(c^2k^2-\kappa\omega^2)\hat{E}-2ilkc^2\frac{\partial\hat{E}}{\partial z}-2il\kappa\omega\frac{\partial\hat{E}}{\partial t}\right]=4\pi l^2\omega^2 P \qquad (12)$$

and

$$P=n_0(\mu_{12}\hat{\rho}_{21}+\mu_{23}\hat{\rho}_{32})Z_1+\text{c.c.}, \qquad (13)$$

respectively. We should note that such a situation can be realized only in dielectrics with optically active impurity atoms, not in SQD systems (see, for example, Refs. [1,13,16,31]). Nevertheless, the consideration of this situation is useful for comparison with and generalization to the more complicated situation which is realized in SQDs.

In the case of exact resonance and an equidistant energy spectrum, we have

$$\omega_0=\omega, \quad \Omega_0=2\omega_0, \quad \Omega_0-\omega_0=\omega. \qquad (14)$$

Then the system of equations (11) reduces to

$$i\frac{\partial\rho_{11}}{\partial t} = \dot{\Theta}(\hat{\rho}_{12} - \hat{\rho}_{21}),$$

$$i\frac{\partial\rho_{22}}{\partial t} = \dot{\Theta}(\hat{\rho}_{21} - \tau\hat{\rho}_{32} - \hat{\rho}_{12} + \tau\hat{\rho}_{23}),$$

$$i\frac{\partial\rho_{33}}{\partial t} = \dot{\Theta}\tau(\hat{\rho}_{32} - \hat{\rho}_{23}),$$

$$i\frac{\partial\hat{\rho}_{21}}{\partial t} = \dot{\Theta}(-\rho_{11} + \rho_{22} - \tau\hat{\rho}_{31}),$$

$$i\frac{\partial\hat{\rho}_{32}}{\partial t} = \dot{\Theta}(-\tau\rho_{22} + \tau\rho_{33} + \hat{\rho}_{31}),$$

$$i\frac{\partial\hat{\rho}_{31}}{\partial t} = \dot{\Theta}(-\tau\hat{\rho}_{21} + \hat{\rho}_{32}), \tag{15}$$

where

$$\Theta(z,t) = \frac{\mu_{12}}{\hbar}\int_{-\infty}^{t}\hat{E}(z,t')dt'$$

is the area of the pulse and $\tau = \mu_{23}/\mu_{12}$.

The solution of these equations allows the determination of the polarization (13). Under the initial condition of the quantum dots in the ground state, i.e., $\rho_{11}=1$, $\rho_{22}=\rho_{33}=0$ for $t\to-\infty$ (attenuating medium), the solutions of Eqs. (15) have the form

$$\hat{\rho}_{21} = \frac{i}{2b^3}(\sin 2b\Theta + 2\tau^2 \sin b\Theta),$$

$$\hat{\rho}_{32} = -\frac{i\tau}{2b^3}(\sin 2b\Theta - 2\sin b\Theta), \tag{16}$$

where

$$b^2 = 1 + \tau^2.$$

Substituting Eqs. (16) in (13), the polarization obtains the following form:

$$P = in_0\frac{\mu_{12}}{2b^3}\sum_{l=\pm1} lZ_l[(1-\tau^2)\sin 2b\Theta + 4\tau^2 \sin b\Theta]. \tag{17}$$

On substituting Eq. (17) in the nonlinear equation (12) and taking into account the dispersion relation $c^2k^2=\eta^2\omega^2$ for the optical plane waves, we obtain for the real quantity $\Psi = b\Theta$ the nonlinear wave equation

$$\frac{\partial^2\Psi}{\partial t^2} + \frac{c}{\eta}\frac{\partial^2\Psi}{\partial t\partial z} = -\frac{\pi\omega\mu_{12}^2 n_0}{\hbar\eta^2(1+\tau^2)}[(1-\tau^2)\sin 2\Psi + 4\tau^2\sin\Psi]. \tag{18}$$

This is the well-known double-sine-Gordon equation [32,34]. It is not integrable by means of the inverse scattering transform and not completely integrable in general [34], but we can consider the two special cases when $\tau=0$ and $\tau=1$.

In the first case ($\tau=0$) which corresponds to a two-level system, Eq. (18) reduces to

$$\frac{\partial^2 2\Psi}{\partial t^2} + \frac{c}{\eta}\frac{\partial^2 2\Psi}{\partial t\partial z} = -\frac{2\pi\omega\mu_{12}^2 n_0}{\hbar\eta^2}\sin 2\Psi, \tag{19}$$

and in the second case ($\tau=1$) corresponding to a three-level system with equal transition probabilities, Eq. (18) reduces to

$$\frac{\partial^2\Psi}{\partial t^2} + \frac{c}{\eta}\frac{\partial^2\Psi}{\partial t\partial z} = -\frac{2\pi\omega\mu_{12}^2 n_0}{\hbar\eta^2}\sin\Psi. \tag{20}$$

Equations (19) and (20) are the well-known sine-Gordon equation for the quantities 2Θ and $\sqrt{2}\Theta$, respectively [32,34–36].

The simplest way to investigate this equation is to use a local time coordinate $\zeta = t - z/V$, where V is the constant pulse velocity. In this case, Eq. (19) transforms to

$$\frac{d^2\Theta}{d\zeta^2} = \frac{1}{T^2}\sin\Theta, \tag{21}$$

where the pulse width T is determined by the expression

$$T^2 = \frac{\hbar\eta^2}{2\pi\omega\mu_{12}^2 n_0}\left(\frac{c}{\eta V} - 1\right).$$

Equation (21) has McCall-Hahn's "2π pulse" or soliton solution

$$\hat{E} = \frac{2\hbar}{\mu_{12}T}\,\mathrm{sech}\,\frac{t - \frac{z}{V}}{T}.$$

Analogously, we find a soliton solution of Eq. (20).

Now we consider SIT for plane waves in SQDs. From the energetic structure of the quantum dots, we can see that the conditions of perfect resonance (14) are exact only in the limits $\delta_x \to 0$ and $\delta_{xx}\to 0$. In real quantum dots, these quantities are not equal to zero and therefore result in errors by the amounts of the order δ_x/ω_0 and $\delta_{xx}/(\Omega_0-\omega_0)$. To achieve $\delta_x/2 \ll \hbar\omega_0$ and $\delta_{xx} \ll \hbar(\Omega_0-\omega_0)$, the 1 to 2 transition and the 2 to 3 transition are assumed to be very close to each other and to the pulse frequency ω, and the energy spectrum of the quantum dots is considered as a quasiequidistant three-level system in a cascade configuration ($\mu_{13}=0$) under off-resonant excitation $\Omega_0-\omega_0-\omega\neq 0$ and $\omega_0-\omega\neq 0$. We assume that the detunings from the resonance $\Omega_0-\omega_0-\omega$ and $\omega_0-\omega$ lie within the bandwidth of the pulse.

Otherwise, it is necessary to apply an excitation with two pulses of different frequencies, under which condition only the existence of simultonlike excitations are possible. A simulton is a nonlinear two-frequency pulse characterized by two different carrier wave frequencies which can propagate in a medium without profile distortion [32].

In general, the detunings from the resonance $\Omega_0-\omega_0-\omega$ and $\omega_0-\omega$, which describe the SQDs, are different. Under the assumption of off-resonant excitation with a constant detuning $\Omega_0-\omega_0-\omega\approx\omega_0-\omega=\Delta$ and $\mu_{12}=\mu_{23}$, i.e. $\tau=1$, Eqs. (11) are simplified considerably to

$$i\frac{\partial p}{\partial t}=\Delta p+\dot{\Theta}N,\quad i\frac{\partial N}{\partial t}=\dot{\Theta}(p-p^*),\qquad(22)$$

where

$$p=\hat{\rho}_{21}+\hat{\rho}_{32},\quad N=\rho_{33}-\rho_{11}.$$

Assuming the validity of the simple factorization for the imaginary part of the quantity p, which is usually applied in the theory of SIT [1,13,16,32], we obtain

$$\mathrm{Im}\,p(\Delta,z,t)=F(\Delta)\mathrm{Im}\,p(0,z,t)=F(\Delta)\frac{1}{\sqrt{2}}\sin\sqrt{2}\Theta,\qquad(23)$$

where $F(\Delta)$ is the dipole spectral response function [1,13,37–39].

Substituting Eq. (23) in the system of equations (22) and taking into account that for attenuating media $\Theta\to 0$ and $N\to -1$ for $t\to-\infty$, we find that the function $F(\Delta)$ is determined by the pulse width T and the detuning Δ in the form

$$F(\Delta)=\frac{1}{1+\Delta^2T^2}$$

and, as in the case of the two-level system, is Lorentzian [13,37,38]. From Eqs. (22) and (23), we obtain the real part of the quantity p and N:

$$\mathrm{Re}\,p=\frac{1-F}{\Delta}\dot{\Theta},\quad N=-F(\Delta)\cos\sqrt{2}\Theta+F(\Delta)-1.$$

Using these results, the polarization of the SQDs, under the condition $\tau=1$, obtains the form

$$P=in_0\frac{\mu_{12}}{\sqrt{2}}\sum_{l=\pm1}lZ_l\int g(\Delta')F(\Delta')d\Delta'\sin\sqrt{2}\Theta.\qquad(24)$$

We should note that the solutions (16) and (23) describe two different physically interesting situations. Equations (16) are the solutions of Eqs. (11) under the condition of exact resonance (14) and for an arbitrary value of the quantity τ; for example, for the two-level and three-level atom realized for $\tau=0$ and $\tau=1$, respectively. Equation (23) is the solution of Eqs. (11) for off-resonant excitation $\Omega_0-\omega_0-\omega=\omega_0-\omega\neq 0$ and $\tau=1$; this describes, for example, the situation in SQDs. The two solutions coincide for the case $\tau=1$ and $\Omega_0-\omega_0-\omega=\omega_0-\omega=0$ $[F(0)=1]$.

Substituting the polarization (24) in the wave equation (10), we obtain the nonlinear wave equation

$$\left(\frac{d\hat{E}}{d\zeta}\right)^2=T^{-2}\hat{E}^2-\frac{\mu_{12}^{\ 2}}{2\hbar^2}\hat{E}^4,\qquad(25)$$

where the width of the pulse is determined by the equation

$$\frac{1}{T^2}=\frac{2\pi\omega\mu_{12}^2n_0}{\left(\frac{1}{V}-\frac{1}{C}\right)C\eta^2\hbar}\int g(\Delta')F(\Delta')d\Delta'-\mathcal{O}(\Delta^2);\qquad(26)$$

$C=c/\eta$ is the velocity of light in the medium. The solution of Eq. (25) for the envelope function has the form [1,13]

$$\hat{E}=\frac{2}{\mu_0T}\,\mathrm{sech}\,\frac{t-\frac{z}{V}}{T},\qquad(27)$$

where $\mu_0=\sqrt{2}\mu_{12}/\hbar$. From Eq. (26), we can determine the delay of the 2π pulse on a unit length in the resonance medium

$$\frac{1}{V}-\frac{1}{C}=\frac{2\pi\omega\mu_{12}^2n_0}{C\eta^2\hbar}\int\frac{g(\Delta')d\Delta'}{T^{-2}+\Delta'^2}-\mathcal{O}(\Delta^2).\qquad(28)$$

Such optical solitons in SQDs have been investigated numerically in Ref. [26].

IV. OPTICAL BREATHERS IN SEMICONDUCTOR QUANTUM DOT WAVEGUIDES

The plane wave breather solution of the system of equations for the SQDs (10) and (11) in the (many-layered system) stacked layer structure of SQDs was considered analytically in a recent work [29]. Here, we investigate optical breathers in SQD waveguides. We consider a breather solution of Eq. (8) for one TE mode of a planar waveguide which is limited to $-h/2\leqslant x\leqslant h/2$ in one transverse spatial dimension and is independent of the other spatial coordinate y. For TE modes, only the components E_y of the electric and H_x and H_z of the magnetic field of the waveguide modes do not vanish. We should note that waveguides which are limited in both x and y directions will be approximately described by means of a planar model of waveguide if the limitation in one direction is much stronger than in the other, i.e., $\Delta L_x\ll\Delta L_y$, where $\Delta L_x,\Delta L_y$ are the extension of the waveguide in the x and y direction, respectively.

In the absence of phase modulation, the envelope of the electric field $\hat{E}_l(x,z,t)$ and the function of the transverse structure of the waveguide mode $\xi(x)$ will be real functions, i.e., $\xi(x)=\xi^*(x)$ and $\hat{E}_l=\hat{E}_l^*$. Then $\hat{E}_l$ can be factorized and written in the form $\hat{E}_l(x,z,t)=\xi(x)\tilde{E}_l(z,t)$ [1,2,40,41]. The functions $\xi(x)$ for different waveguide modes are orthogonal to each other [40,41].

Multiplying Eq. (8) by the function $\xi(x)$ and integrating, we obtain

$$\int \xi(x)\sum_{l=\pm1} Z_l\Bigg[l^2(c^2k^2-\kappa\omega^2)\hat{E}_l - 2ilkc^2\frac{\partial \hat{E}_l}{\partial z} - 2il\kappa\omega\frac{\partial \hat{E}_l}{\partial t} - c^2\frac{\partial^2 \hat{E}_l}{\partial z^2} + \kappa\frac{\partial^2 \hat{E}_l}{\partial t^2} - c^2\frac{\partial^2 \hat{E}_l}{\partial x^2}\Bigg]dx = 4\pi l^2\omega^2 \int \xi(x)P dx. \tag{29}$$

For simplicity, the index l of the quantity $\hat{E}_l$ will be neglected in the following:

$$\int \xi(x)\sum_{l=\pm1} Z_l\Bigg[l^2(c^2k^2-\kappa\omega^2)\hat{E} - 2ilkc^2\frac{\partial \hat{E}}{\partial z} - 2il\kappa\omega\frac{\partial \hat{E}}{\partial t} - c^2\frac{\partial^2 \hat{E}}{\partial z^2} + \kappa\frac{\partial^2 \hat{E}}{\partial t^2} - c^2\frac{\partial^2 \hat{E}}{\partial x^2}\Bigg]dx = 4\pi l^2\omega^2 \int \xi(x)P dx. \tag{30}$$

Unlike for situations considered in the previous section for solitons and in Ref. [29] for breathers, which are valid only for plane waves, for waves with transverse structure it is important to keep the second order derivatives for the transverse coordinate x in the wave equations (29) and (30).

In the further considerations, it will be more convenient to transform this equation to the following form:

$$\int \xi(x)\sum_{l=\pm1} Z_l\left(W_l\frac{\partial \Theta}{\partial t} - iA_l\frac{\partial^2 \Theta}{\partial t^2} - iB_l\frac{\partial^2 \Theta}{\partial z\partial t} - c^2\frac{\partial^3 \Theta}{\partial z^2\partial t} + \kappa\frac{\partial^3 \Theta}{\partial t^3}\right)dx = \frac{4\pi l^2\omega^2\mu_{12}}{\hbar}\int \xi(x)P dx, \tag{31}$$

where

$$W_l = l^2(c^2k^2-\kappa\omega^2) - c^2\frac{\partial^2}{\partial x^2},\quad B_l = 2lkc^2,\quad A_l = 2l\kappa\omega,$$

$$\Theta(x,z,t) = \frac{\mu_{12}}{\hbar}\int_{-\infty}^{t}\hat{E}(x,z,t')dt' = \xi(x)\frac{\mu_{12}}{\hbar}\int_{-\infty}^{t}\tilde{E}(z,t')dt' = \xi(x)\vartheta(z,t); \tag{32}$$

the polarization P is determined from Eq. (5).

Taking into account that for waveguide modes with $l=\pm1$,

$$W_{\pm1}\xi(x) = 0, \tag{33}$$

and hence that the transverse equation

$$\frac{\partial^2 \xi}{\partial x^2} + \gamma_m\xi = 0 \tag{34}$$

is satisfied for the waveguide mode, where $\kappa\omega^2/c^2 - k^2 = \gamma_m$ is the mode eigenvalue, Eq. (31) reduces to

$$\sum_{l=\pm1} Z_l\left(-iA_l\frac{\partial^2 \vartheta}{\partial t^2} - iB_l\frac{\partial^2 \vartheta}{\partial z\partial t} - c^2\frac{\partial^3 \vartheta}{\partial z^2\partial t} + \kappa\frac{\partial^3 \vartheta}{\partial t^3}\right) = \frac{4\pi l^2\omega^2\mu_{12}}{\hbar}\frac{\int \xi(x)P dx}{\int \xi^2(x)dx} \tag{35}$$

with the polarization

$$P(x,z,t) = i\frac{n_0\mu_{12}}{\sqrt{2}}\sum_{l=\pm1} lZ_l\int g(\Delta')F(\Delta')d\Delta' \sin\sqrt{2}\Theta(x,z,t). \tag{36}$$

We consider a breather solution of Eq. (35) under the condition $\Theta \ll 1$. For this purpose, we make use of the perturbative reduction method, in which we use the following expansion for the area of the nonlinear pulse [35,42]:

$$\Theta(x,z,t) = \xi(x)\sum_{\alpha=1}^{\infty}\sum_{n=-\infty}^{\infty}\varepsilon^{\alpha}Y_n f_n^{(\alpha)}(\zeta,\tau), \tag{37}$$

where

$$Y_n = e^{in(Qz-\Omega t)},\quad \zeta = \varepsilon Q(z - v_g t),\quad \tau = \varepsilon^2 t,\quad v_g = \frac{\partial \Omega}{\partial Q};$$

ε is a small parameter. Such a representation allows us to separate from Θ the still more slowly changing quantities $f_n^{(\alpha)}$. Consequently, it is assumed that the quantities Ω, Q, and $f_n^{(\alpha)}$ satisfy the inequalities

$$\omega \gg \Omega,\quad k \gg Q,\quad \left|\frac{\partial f_n^{(\alpha)}}{\partial t}\right| \ll \Omega|f_n^{(\alpha)}|,\quad \left|\frac{\partial f_n^{(\alpha)}}{\partial z}\right| \ll Q|f_n^{(\alpha)}|,$$

and Eq. (2).

On substituting Eqs. (36) and (37) in Eq. (35), we obtain the nonlinear wave equation

$$\sum_{l=\pm1}\sum_{\alpha=1}^{\infty}\sum_{n=-\infty}^{\infty}\varepsilon^{\alpha}Y_n Z_l\left(\tilde{W}_{l,n} + \varepsilon J_{l,n}\frac{\partial}{\partial \zeta} + \varepsilon^2 h_{l,n}\frac{\partial}{\partial \tau} + \varepsilon^2 iH_{l,n}\frac{\partial^2}{\partial \zeta^2}\right)f_n^{(\alpha)} = -i\frac{\tilde{R}}{3}\sum_{l=\pm1}\sum_{\alpha,\alpha',\alpha''=1}^{\infty}\sum_{n,n',n''=-\infty}^{\infty} lZ_l Y_n\varepsilon^{\alpha+\alpha'+\alpha''}f_{n-n'-n''}^{(\alpha)}f_{n'}^{(\alpha')}f_{n''}^{(\alpha'')}, \tag{38}$$

where

$$\tilde{W}_{l,n} = -in\Omega\left(-A_l n\Omega + B_l nQ - \kappa n^2\Omega^2 + c^2n^2Q^2 + \frac{l}{n}\frac{R}{\Omega}\right),$$

$$J_{l,n} = A_l 2n\Omega Q v_g - B_l nQ(\Omega + Qv_g) + 3n^2\kappa\Omega^2 Q v_g - c^2n^2Q^2(2\Omega + Qv_g),$$

$$H_{l,n} = -A_l Q^2 v_g^2 + B_l Q^2 v_g - 3\kappa n\Omega Q^2 v_g^2 + c^2 nQ^2(\Omega + 2Qv_g),$$

$$h_{l,n} = -A_l 2n\Omega + B_l nQ - 3\kappa n^2\Omega^2 + c^2n^2Q^2,$$

$$R=\frac{4\pi\omega^2\mu_{12}^2n_0}{\hbar}\int g(\Delta')F(\Delta')d\Delta',\quad \widetilde{R}=R\frac{\int \xi^4(x)dx}{\int \xi^2(x)dx}. \tag{39}$$

The constant R characterizes the nonlinear interaction of the optical pulse with the quantum dots and depends on the inhomogeneous broadening of the spectral line. The coefficient $\widetilde{R}$ additionally takes into account the transverse structure of the waveguide mode.

To determine the values of $f_n^{(\alpha)}$, we equate the various terms corresponding to the same powers of ε. As a result, we obtain a chain of equations. In first order,

$$\widetilde{W}_{l,n}f_n^{(1)}=0. \tag{40}$$

In the following, we are interested in a breather which vanishes for $t\to\pm\infty$. Consequently, according to Eq. (40), only the quantities $f_{\pm1}^{(1)}$ differ from zero. The relation between Ω and Q, for fixed values of l and n $(l,n=\pm1)$, is determined from Eq. (40):

$$A_l n\Omega-B_l nQ+\kappa n^2\Omega^2-c^2n^2Q^2-\frac{l}{n}\frac{R}{\Omega}=0. \tag{41}$$

We have to consider the situation when $n=l=\pm1$, where $f_l^{(1)}\neq0$ and $f_{-l}^{(1)}=0$ separate from the situation when $n=-l=\pm1$, where $f_{-l}^{(1)}\neq0$ and $f_l^{(1)}=0$. The former case is considered in detail. Substituting Eq. (41) in Eqs. (39) with $n=l=\pm1$, one easily sees that the following equations hold:

$$J_{l,l}=0,$$

$$v_g=\frac{c^2(k+Q)}{\kappa\left(\omega+\Omega+\frac{R}{2\kappa\Omega^2}\right)},$$

$$H_{l,l}=lQ^2\Omega\left(c^2-\kappa v_g^2+v_g^2\frac{R}{\Omega^3}\right),$$

$$h_{l,l}=-2\kappa\Omega\left(\omega+\Omega+\frac{R}{2\kappa\Omega^2}\right). \tag{42}$$

To second order in ε, we obtain

$$f_l^{(2)}=0. \tag{43}$$

Substituting Eqs. (41)–(43) in Eq. (38) and taking into account that from the condition $\hat{E}_l=\hat{E}_{-l}^*$ follows that $f_{-l}^{(1)}=(f_l^{(1)})^*$, we obtain, using the definition $H_{l,l}=lH_0$, to third order in ε:

$$-ilh_{l,l}\frac{\partial f_l^{(1)}}{\partial\tau}+H_0\frac{\partial^2 f_l^{(1)}}{\partial\zeta^2}+\widetilde{R}|f_l^{(1)}|^2f_l^{(1)}=0. \tag{44}$$

From Eq. (44), after simple transformations, we obtain the well-known nonlinear Schrödinger equation (NSE) for the quantity $\lambda_j=\varepsilon f_j^{(1)}$ $(j=\pm1)$:

$$ij\frac{\partial\lambda_j}{\partial t}+p\frac{\partial^2\lambda_j}{\partial y^2}+q|\lambda_j|^2\lambda_j=0, \tag{45}$$

where

$$p=\frac{c^2-\kappa v_g^2+v_g^2\frac{R}{\Omega^3}}{2\kappa\left(\omega+\Omega+\frac{R}{2\kappa\Omega^2}\right)},\quad q=\frac{\widetilde{R}}{2\kappa\Omega\left(\omega+\Omega+\frac{R}{2\kappa\Omega^2}\right)},$$

$$y=z-v_gt. \tag{46}$$

The quantity q contains $\widetilde{R}$ and hence depends on the waveguide mode. Under the condition $pq>0$, i.e., stabilization between "dispersion" and nonlinear compression of the pulse, the NSE has a fundamental or $N=1$ soliton, a so-called bright NSE soliton, solution,

$$\lambda_j=K\frac{e^{ij\phi_1}}{\cosh\phi_2}, \tag{47}$$

where

$$\phi_1=\frac{V_b}{2p}z-\left(\frac{V_bv_g}{2p}+\frac{V_b^2}{4p}-\frac{q}{2}K^2\right)t,$$

$$\phi_2=K\sqrt{\frac{q}{2p}}[z-(v_g+V_b)t]. \tag{48}$$

K and V_b are the amplitude and velocity of the nonlinear wave, respectively. Substituting the soliton solution for the superenvelope Eq. (47) in Eq. (37) and taking into account that the function $\cosh\phi_2$ varies more slowly than $\cos(Qz-\Omega t+\phi_1)$, we obtain for the envelope of the electric field the bright breather solution [2,9–11,43,44]

$$\hat{E}(x,z,t)=\frac{2K\hbar\Omega}{\mu_{12}}\frac{\sin[Qz-\Omega t-\phi_1(z,t)]}{\cosh\phi_2(z,t)}\xi(x). \tag{49}$$

The appearance in expression (49) of the factor $\sin[Qz-\Omega t-\phi_1(z,t)]$ indicates the presence of periodic beats (slow in comparison with the spatial coordinates and time, with characteristic parameters Ω and Q), as a result of which the soliton solution (47) for $f_l^{(1)}$ is transformed into the solution (49) for the real envelope $\hat{E}$. This is a closed-form time- and space-periodic solution of the nonlinear wave equation (10). In the framework of the NSE, it is called simply a soliton, but in the framework of SIT or the sine-Gordon equation, it would be called a breather (pulsing soliton), with the NSE soliton being the envelope. In either framework, to the order that these equations have been expanded, the solutions will propagate through the medium without any loss of energy and be stable against infinitesimal perturbations of the inital data [34]. Equation (49) is a breather in the above sense with the NSE soliton as the envelope and the phase modulation given by $\phi_1(z,t)\neq0$.

The stability of the breather solution as it propagates in a solid is a question which has not been addressed in this paper. This would require one to carry out the expansion done herein to one additional order. Doing so would yield a collection of various terms involving damping, anisotropy, and

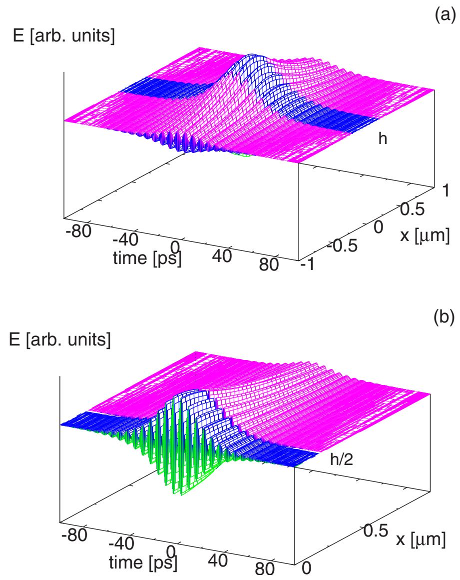

FIG. 3. (Color online) In (a), the general form of the two-dimensional breather is shown for a fixed value of z. Along the x axis, the dark zone corresponds to the thickness of the waveguide. Outside of the waveguide, the breather amplitude decays exponentially as one moves away from the boundaries. (b) shows a cut of the breather at $x=0$. The breather oscillates with the frequency Ω along the t axis. This cut corresponds to the one-dimensional breather result of Ref. [29].

diffraction which would act on the breather as it propagates. Once such terms are obtained, there are standard methods for determining their effects on a breather or soliton solution [17,45].

Unlike for solitons where the pulse velocity is related to the pulse width T [see Eq. (26)], in the case of the NSE soliton (breather) the pulse velocity and the pulse width are independent from each other [46]. Figure 3 shows the space-time dynamics of the breather solution for realistic quantum dot and pulse parameters [52]. In Fig. 3(a), the evolution of the breather solution inside and outside the waveguide with thickness h is shown. In Fig. 3(b), a cross section of the breather at $x=0$ is given, showing the breather oscillating with frequency Ω.

V. CONCLUSION

In the present paper, we have shown that in the propagation of optical pulses through SQD media under the condition of SIT, optical solitons (2π pulses) and small-amplitude breathers (0π pulses) can arise. The soliton of SIT is investigated analytically which up to now has only been considered numerically [26]. The explicit forms of the shape and velocity of the plane wave soliton are given by Eqs. (27) and (28). We have obtained the small-amplitude breather solution for the sine-Gordon equation (19) and (20), which is well known to be the soliton solution of the NSE [11,12,43,47], using the expansion (37). Another way of transforming the small-amplitude wave packets of the sine-Gordon equation to the NSE can be found in the monograph [12].

The breathers of SIT which are considered in SQD waveguides have a transverse structure and are characterized by the function $\xi(x)$ which is determined by Eq. (34). The explicit form of the breather shape and parameters determined via the waveguide mode are given by Eqs. (46), (48), and (49). The relation between the breather beat quantities Ω and Q is given by Eq. (41). The breather solution is valid if the condition $qp>0$ is satisfied. In analogy to solitons, we can call the solution (49) fundamental or bright breather.

It should be noted that these results and their interpretation are applicable to pulses with sufficiently smooth envelopes under the condition that the size of the pulse is large in comparison with the wavelength, i.e., $kL_0 \gg 1$, where L_0 is the length of the soliton or breather. Moreover, the length of the breather should be much greater than the characteristic length of change of the periodic beats, $QL_0 \gg 1$.

It should also be noted that the NSE contains not only the soliton solution (47), but also N-soliton solutions with a more complicated behavior. In particular, for N-soliton solutions of the NSE, there are characteristic oscillations of the envelope and strong compression of the pulse peaks already in the initial stage of propagation of the wave. Under these conditions, we cannot always use the slowly varying envelope approximation, nor the separation from $\hat{E}_l$ of the more slowly varying parts $f_l^{(\alpha)}$ [Eq. (37)]. Therefore, the scheme presented above is not valid for such solutions, and for them a completely different method is needed (see, for example, Ref. [48]).

We considered the situation when a phase modulation is absent, $\hat{E}_l=\hat{E}_l^*$, and thus the quantities $\hat{E}_l$ and Θ are real functions. In order to find the breather (NSE soliton) solution, we have used the expansion (37) to separate from the real quantity Θ the more slowly varying complex functions $f_l^{(\alpha)}$ which satisfy the NSE and contain the phase function $\phi_1(z,t)$. Consequently, the existence of the phase function $\phi_1(z,t)$ does not mean that we take into account a phase modulation. In order to consider the more general situation with phase modulation (which is not the goal of this article), it is necessary to consider a more general form of Eq. (29) under the condition that the $\hat{E}_l$ is a complex function and contains a phase function which depends on the time and space variables.

Finally, it should be noted that the results for breathers of the waveguide mode are generalizations of the situation considered in Ref. [29] in which plane wave SIT breathers, for example, in a stacked layer structure (many-layered system) of SQDs, are investigated. Unlike for the plane wave breather solution, the parameters ϕ_1 and ϕ_2 of the breather in the waveguide depend on the quantity $\tilde{R}$ and consequently on the waveguide mode. However, the relation between Ω and Q characterizing the oscillations of the breather is independent of the waveguide mode. We have considered here the breather solution (49) to the continuous Maxwell-Bloch equations, but we should note that breathers can also appear in different physical situations such as, for instance, discrete breather solutions [49,50] as well as quantum breathers in electron-phonon coupled finite chain systems [51], which have similar properties.

In conclusion, we have predicted the existence of optical breathers in a three-level SQD system for experimentally relevant parameters. In particular, we predict optical breathers with a temporal length of a few tens of picoseconds and peak intensities of several MW/cm^2. We hope that the presented results will initiate a search for breathers in quantum dot nanostructures. Breathers as stable wave solutions are of importance for the lossless and shape-invariant transport of information on nanoscales. Technically, the different approximations presented here could be used in the future for a search for other types of 0π pulses than investigated here.

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