Coexistence of superfluid and Mott phases of lattice bosons

R. A. Barankov,[1] C. Lannert,[2] and S. Vishveshwara[1]
[1]*Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green Street Urbana, Illinois 61801, USA*
[2]*Department of Physics, Wellesley College, Wellesley, Massachusetts 02481, USA*
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Recent experiments on strongly interacting bosons in optical lattices have revealed the coexistence of spatially separated Mott-insulating and number-fluctuating phases. We explore the condensate properties of the number fluctuating phase trapped between the Mott-insulating regions and derive the associated collective-mode structure. We discuss the crossover of the interlayer properties between two- and three-dimensional behavior as a function of the lattice parameters and estimate the critical temperatures for the transition from the normal to superfluid state.

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I. INTRODUCTION

Dilute gases of ultracold bosons on a lattice present a model system for exploring quantum phases of matter. Experiments in optical lattice traps have demonstrated controlled tunability through a quantum phase transition between a Mott-insulating phase which has fixed particle number on each lattice site and a phase exhibiting number fluctuations [1,2]. As has been recently observed in radially symmetric traps [3,4], for sufficiently deep optical lattice potentials the system arranges itself into layered structure in which Mott-insulating phases of bosons commensurate with the lattice alternate with interlayers of incommensurate bosons with fluctuating site occupancy [5–8]. Various questions concerning such inhomogeneous systems have so far been unanswered in both theory and experiment: Can particles in the number fluctuating interlayer form a stable condensate? If so, in what temperature range is the condensate robust? Is there collective behavior in the interlayer, analogous to that seen in bulk superfluids? How does the system cross over from three-dimensional to two-dimensional behavior as the interlayer width is varied? Understanding these issues would also be important for related avenues in cold atomic physics, such as the interplay of spatial inhomogeneity and quantum criticality [9], realization of robust states for quantum computation [10], and the physics of interacting fermions on a lattice [11].

It has been established that dilute gases of bosons in optical lattice potentials are well described by the Bose-Hubbard Hamiltonian [5,12] in which the bosons' movement between sites is characterized by a tunneling term $\mathcal{H}_J = -J\Sigma_{\langle ij\rangle} a_i^\dagger a_j$ related to the overlap of the single-particle wave functions between neighboring sites i and j, and the on-site interaction is modeled by $\mathcal{H}_U = (U/2)\Sigma_i n_i(n_i - 1)$. It is the external trapping potential $V(\mathbf{r}_i)$ which is responsible for breaking the uniformity of the system and promoting spatial coexistence of the Mott insulating and superfluid phases at large interactions [5–8]. Analytical treatments of the inhomogeneous system are complicated by the fact that no single approximation of the Hamiltonian can faithfully describe the entire phase space. The Bogoliubov approximation [13,14] captures the condensed phase for large J/U but breaks down close to the Mott regions. The decoupled-site approximation [9,13,15,16], valid when $J \ll U$ and the boson density is close to a commensurate value, works well within and close to the Mott regions but fails deep within the incommensurate phase. A third possibility is presented by the "pseudospin" approximations [17–19], valid for intermediate values of J/U, which bypass these shortcomings by treating kinetic energy and interactions on comparable footing.

In this work, we employ a pseudospin approximation of the Bose-Hubbard model to describe inhomogeneous systems where the density of bosons varies as a result of a confining trap. Concentrating on a single interlayer trapped between two Mott-insulating phases, we identify a well-defined order parameter for the interlayer and derive the dynamical equations governing the system. We obtain the collective excitation spectrum of the interlayer condensate and show that in the homogeneous limit, it properly reproduces the known properties of bulk superfluids. We explore the behavior of the collective modes as a function of the width of the interlayer and show that they provide a signature of dimensional crossover in the condensate, which can be achieved by tuning experimental parameters. We conclude with a discussion of the expected mean-field critical temperature T_c of an interlayer superfluid and its behavior as a function of interlayer width.

II. MODEL HAMILTONIAN

Focusing our attention on the Mott phases with integer boson filling n and $n+1$ and the superfluid phase at intermediate fillings, we consider a Hilbert space restricted to the number-basis states $|n\rangle$ and $|n+1\rangle$ at each site. Considering the excluded states $|n-1\rangle$ and $|n+2\rangle$, we find that their contribution to the energy is of order of J^2/U. We note that the number fluctuations on sites are driven by the incommensurability of bosons with the lattice in the presence of the trapping potential. The truncated Hilbert space in the limit $J/U \ll 1$ may be represented by the spin-1/2 states [15,18] $|n+1\rangle = |\uparrow\rangle$ and $|n\rangle = |\downarrow\rangle$, the eigenstates of the operator s^z with eigenvalues $\pm 1/2$. The tunneling term in the Bose-Hubbard Hamiltonian can be identified with raising and lowering spin-1/2 operators s^+ and s^-, such that $a_i^\dagger a_j \rightarrow (n+1)s_i^+ s_j^-$, where $a_i^\dagger$ is the boson creation operator on site i. The interaction and the potential energy terms are diagonal

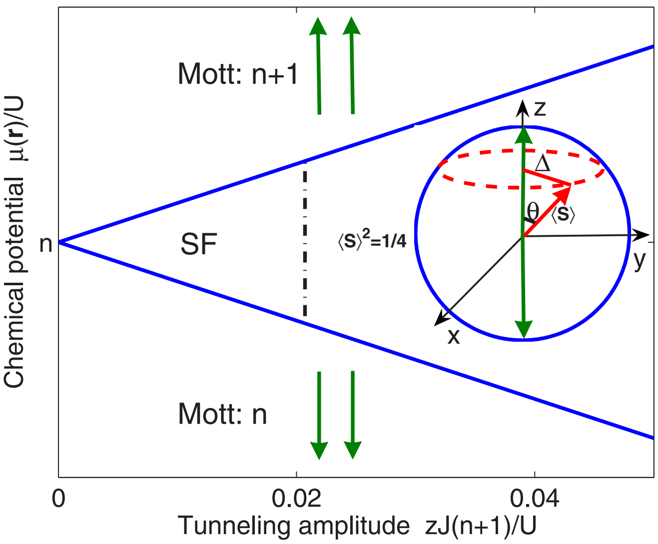

FIG. 1. (Color online) The mean-field phase diagram for the Bose-Hubbard Hamiltonian. The dash-dotted line corresponds to the interlayer with fluctuating site occupancy. Spontaneous symmetry breaking in the ground state is shown on the sphere $\langle \mathbf{s}\rangle^2=1/4$: the equilibrium configuration $\langle \mathbf{s}\rangle$ is degenerate on the circle (dashed line) with nonzero order parameter $|\Delta|=(1/2)\sin\theta_0$. North and south poles of the sphere correspond to Mott states with $n+1$ and n bosons per site, respectively.

in the number basis at each site and the boson number operator ($\hat{n}_i=a_i^\dagger a_i$) can be expressed in terms of the spin-1/2 matrix s^z, $\hat{n}=n+1/2+s^z$. Thus, in the truncated Hilbert space, one obtains an effective Hamiltonian identical to the spin-1/2XY model in the external "magnetic" field:

$$\mathcal{H}=-J(n+1)\sum_{\langle ij\rangle}(s_i^x s_j^x+s_i^y s_j^y)+\sum_i (Un-\mu_i)s_i^z. \qquad (1)$$

Here, $\langle ij\rangle$ denotes a summation over nearest-neighbor sites, and $\mu_i=\mu-V(\mathbf{r}_i)$ defines the chemical potential offset by the external trapping potential, $V(\mathbf{r}_i)$. The chemical potential μ is set by the total number of particles in the system, $\langle N\rangle=\Sigma_i\langle\hat{n}_i\rangle$.

The pseudospin operators are coupled ferromagnetically in the x-y plane and, therefore, at low temperatures can form an ordered state with broken U(1) symmetry in the plane. At the mean-field level, in the ground-state configuration, pseudospins are aligned with the local "magnetic" field, $\mathbf{B}_i^0=zJ(n+1)\ [2\langle s_i^x\rangle,2\langle s_i^y\rangle,\cos\theta_i]$, where $\cos\theta_i=(\mu_i-Un)/[zJ(n+1)]$ with z the coordination number of the lattice, and for simplicity we have assumed $\langle\mathbf{s}_i\rangle\approx\langle\mathbf{s}_j\rangle$ for nearest neighbors. The equilibrium components of the pseudospin at site i are parametrized by angles on the sphere:

$$\langle s_i^z\rangle=(1/2)\cos\theta_i,\quad \langle s_i^+\rangle=(1/2)e^{i\varphi}\sin\theta_i, \qquad (2)$$

where the angle φ independent of site index expresses the phase coherence in the system. The continuous degeneracy in the ground state is illustrated in Fig. 1. In the Mott phase, the pseudospins are completely polarized along the z direction—i.e., $\langle s_i^z\rangle=\pm1/2$—allowing the identification of $\mu_\pm=Un\pm zJ(n+1)$, the values of the chemical potential at the boundaries of the Mott states with n and $n+1$ bosons per site (see Fig. 1). In the xy-symmetry broken phase, $\langle a^\dagger\rangle=\langle s^+\rangle/\sqrt{n+1}\neq0$, and we have a condensate with local order parameter, $\Delta_i=\langle s_i^+\rangle$, as illustrated in Fig. 1.

In our discussion of the equilibrium pseudpospin configuration we have neglected the spatial variation of the pseudospin components, which is analogous to the well-known Thomas-Fermi approximation usually employed for a description of the bulk properties of bosonic condensates [14]. To estimate the range of applicability of such a description, one can derive the equation defining the equilibrium configuration in the continuum limit by minimizing the Hamiltonian (1):

$$\ell^2\cos\theta\nabla^2\sin\theta=z\sin\theta(\cos\theta_0-\cos\theta), \qquad (3)$$

where

$$\cos\theta_0=[\mu-V(\mathbf{r})-Un]/[zJ(n+1)] \qquad (4)$$

defines the Thomas-Fermi profile when one neglects the gradient term. It follows from Eq. (3) that the gradient term in Eq. (3) becomes important in the vicinity of the Mott states at $\mathbf{r}\approx\mathbf{r}_\pm$ where $\sin\theta_0\approx0$. The equilibrium configuration starts to deviate from the Thomas-Fermi profile at $\delta r\lesssim\max\{\ell,[J(n+1)\ell^2 dr_\pm/dV]^{1/3}\}$. In the following, we shall concentrate on the bulk properties of interlayers neglecting the boundary effects.

III. INTERLAYER GEOMETRY

The locations and sizes of the interlayers can be determined by the relationship $\mu_\pm=\mu-V(\mathbf{r}_\pm)$, where the chemical potential μ is obtained self-consistently by fixing the total number of particles in the system, N. For radially symmetric traps, a simplification occurs in the limit of a thin interlayer, $\delta r_n\ll r_n$ (where r_n is the radius at the center of the interlayer between two Mott states with particle occupation n and $n+1$ and δr_n is its width). In this case, the trapping potential can be linearized around r_n and we find that the number of particles in the interlayer is the same as in the case $J/U=0$, where the interlayer region would be filled with n and $n+1$ Mott phases. Hence, the chemical potential at small J/U can be found by setting $N=(4\pi/3)\Sigma_{n=0}^{m-1}(r_n/\ell)^3$, where m is the total number of Mott states in the trap and ℓ is the lattice spacing [8]. We find that for a three-dimensional parabolic trapping potential, $V(r)=\alpha r^2$, the interlayer parameters are given by $r_n=(\mu/\alpha)^{1/2}[1-nU/\mu]^{1/2}$, $\delta r_n=6J(n+1)/(\alpha r_n)$, when $n>0$, and $\delta r_0=3J/(\alpha r_0)$. These results show that it is possible to tune the width of the interlayers from $\delta r_n\simeq\ell$ to $\delta r_n\gg\ell$, effectively changing the dimensionality of the layers. As a characteristic example in the range of recent experiments [3,4], a system with trap curvature $\alpha\approx h\times24$ Hz/μm^2, total particle number $N\approx10^6$, lattice spacing 0.43 μm, interparticle interaction $U\approx h\times10$ kHz, and tunneling strength $J\approx h\times120$ Hz hosts two Mott regions with $n=1$ and $n=2$ and two interlayers. The corresponding interlayer parameters are $r_0\approx25$ μm, $\delta r_0\approx0.5$ μm and $r_1\approx14$ μm, $\delta r_1\approx4$ μm. We remark that for a harmonic trap, when gravity is taken into account, the system experiences a shift along the direction of the gravitational field, but is otherwise unaffected.

IV. COLLECTIVE MODES

Having identified the interlayer region, we now turn to the low-energy collective modes within the interlayer. These modes can be calculated using the Heisenberg equations of motion for the pseudospin operators, $\partial_t \mathbf{s}_i = i[\mathcal{H}, \mathbf{s}_i]$. In the mean-field approximation, one obtains the Bloch equations $\partial_t \langle \mathbf{s}_i \rangle = \langle \mathbf{s}_i \rangle \times \mathbf{B}_i$, where the effective magnetic field is given by $B_i^+ = J(n+1)\Sigma_j 2\langle s_j^+ \rangle$ (summation is over the nearest neighbors of site i) and $B_i^z = zJ(n+1)\cos\theta_i$. Assuming that the characteristic wavelength of the excitations is much larger than the lattice spacing ℓ, we approximate the sums entering the effective magnetic field $\mathbf{B}$ by their continuum limit, $\Sigma_j \langle \mathbf{s}_j \rangle \approx z\langle \mathbf{s} \rangle + \ell^2 \nabla^2 \langle \mathbf{s} \rangle$. The resulting Bloch equations can be analyzed as follows. The equilibrium density of bosons (number of bosons per lattice site) in the interlayer is $\rho_0 = n + 1/2 + \cos\theta_0/2$ and the number density $\rho = \rho_0 + \delta\rho$ obeys the continuity equation $\partial_t \rho + \nabla \mathbf{j} = 0$ where the current density is $\mathbf{j} = [J(n+1)/2]\sin^2\theta_0 \nabla \varphi$. As follows from the Bloch equations, there is a relation between the canonically conjugate density deviation $\delta\rho$ and the phase, $\partial_t \varphi = -2zJ(n+1)\delta\rho$. Using this relationship, one obtains the following differential equation for density fluctuations around equilibrium:

$$\partial_t^2 \delta\rho = 4z[J(n+1)\ell]^2 \nabla [\Delta_0^2 \nabla \delta\rho], \tag{5}$$

where $\Delta_0 = (1/2)\sin\theta_0$ is the local value of the order parameter vanishing at the boundaries of the Mott states. The form of Eq. (5) is identical to that governing a trapped Bose-Einstein condensate in the absence of a lattice [14] with Δ_0^2 playing the role of an equilibrium density of the condensate confined between two Mott states to an interlayer with radius r_n and width δr_n. It must be noted that while the equations governing density distortions are identical to those derived from the standard Gross-Pitaevskii formalism [14] for a condensate in the absence of a lattice, the equations of motion for the order parameter $\langle s^+ \rangle$ in general do not correspond to the Gross-Pitaevskii form, but reproduce it in the limit of small density distortions.

For the uniform case, the excitation spectrum can be obtained by treating the order parameter Δ_0 as spatially independent. The eigenvalue equation (5) is solved by the Fourier transformation $\delta\rho \propto \exp(i\mathbf{p}\cdot\mathbf{r} - i\omega_{\mathbf{p}} t)$, where $\mathbf{p}$ is the wave vector. The resulting sound mode

$$\omega_{\mathbf{p}} = cp, \quad c = \sqrt{z}J(n+1)\ell|\sin\theta_0|, \tag{6}$$

is related to the spontaneously broken symmetry in the ordered state. According to the Landau criterion, the soundlike spectrum of Eq. (6) makes the ordered state a superfluid. One notices that the speed of sound, c, goes to zero as one approaches the Mott phases at $\sin\theta_0 = 0$.

In the trapped geometry, an estimate of the excitation spectrum can be obtained from the quantization conditions imposed on the wave vector p in Eq. (6), with the speed of sound approximated by its value in the center of the layer, $c_0 = \sqrt{z}J(n+1)\ell$. For a spherically symmetric trap, the excitation modes are confined within an interlayer centered at radius r_n with width $\delta r_n = 2a_n$. The excitation modes terminate at the boundaries of the Mott regions—i.e., $p_j = j/a_n$ with $j = 0, 1, \ldots$—which gives a radial mode spectrum $\omega_j \simeq \Omega_r j$ with characteristic frequency $\Omega_r = J(n+1)\ell/a_n$. The quantization of the surface modes is related to the angular momentum $L = 0, 1, \ldots$ through $p_L = L/r_n$ which leads to the spectrum $\omega_L \simeq \Omega_a L$ with characteristic frequency $\Omega_a = \Omega_r a_n / r_n$. The degeneracy of the surface modes is $(2L+1)$ for each value of L. The perturbative calculation of the modes in Eq. (5) in the limit $a_n/r_n \ll 1$ confirms these estimates and gives the following result:

$$\left(\frac{\omega_{Lj}}{\Omega}\right)^2 \approx j(j+1) + \frac{a_n^2}{r_n^2}\left[1 + \frac{3}{(2j-1)(2j+3)}\right] + L(L+1)\frac{a_n^2}{2r_n^2}\left[1 - \frac{1}{(2j-1)(2j+3)}\right], \tag{7}$$

where $\Omega = \sqrt{6}\Omega_r$, $j = 0, 1, \ldots$, $L = 0, 1, \ldots$, and $j + L \neq 0$. In the continuum approximation, the wavelength of the modes should be much larger than the lattice spacing, which sets upper bounds on the quantum numbers: $L \ll r_n/\ell$ and $j \ll a_n/\ell$. The second term in Eq. (7) is independent of L and is associated with the curvature of the interlayer; it vanishes at $j = 0$. The lowest-energy modes for thin interlayers, $a_n/r_n \ll 1$, correspond to angular excitations ($j = 0$) given by $\omega_L = 2\Omega_a \sqrt{L(L+1)}$, $L = 1, 2, \ldots$.

We note that the mode spectrum of Eq. (7) corresponds to that of a condensate confined with an explicitly shell-shaped trap (for instance, a "bubble trap" in Ref. [20]) since the "effective confining potential" in Eq. (5) has the form $V_{eff} \propto (r - r_n)^2 / a_n^2$ for thin interlayers. The calculation of the radial ($L = 0$) modes with $j = 1, 2$ in Ref. [21] confirms this connection for the lowest-lying radial modes (analogous to "breathers" in spherical condensates).

V. FINITE TEMPERATURES

The low-energy excitations of the system are described by the effective Hamiltonian derived from Eq. (1) using equilibrium configuration in Eq. (2). The resulting Hamiltonian involves phase and density variations φ and $\delta\rho$ of the order parameter:

$$\mathcal{H} = \frac{J(n+1)}{4\ell} \int d\mathbf{r}[\sin^2\theta_0 (\nabla\varphi)^2 + 4z(\delta\rho)^2]. \tag{8}$$

We quantize the low-energy modes by expanding the phase variable in the Heisenberg representation in the basis of eigenfunctions of Eq. (5):

$$\varphi(\mathbf{r}, t) = \sum_k A_k [\alpha_k e^{-i\omega_k t} f_k(\mathbf{r}) + \alpha_k^\dagger e^{i\omega_k t} f_k^*(\mathbf{r})], \tag{9}$$

where $A_k = \sqrt{zJ(n+1)/\omega_k}$ are the coherence factors, the excitation spectrum ω_k is obtained in Eq. (7) with $k = \{L, M, j\}$, and f_k are the normalized eigenfunctions of Eq. (5). The creation and annihilation operators obey the Bose statistics condition $[\alpha_{k'},\alpha_k^\dagger]=\delta_{k,k'}$. Using relation $\partial_t\varphi=-2zJ(n+1)\delta\rho$ one also finds an expression for the density operator. Substituting φ from Eq. (9) and $\delta\rho$ into Eq. (8), for the specific case of a spherically symmetric interlayer, one obtains the energy of the system at low temperatures $T\ll J(n+1)$

$$E(T) = \sum_{Lj} (2L+1)\omega_{Lj}n_{Lj}, \tag{10}$$

where factor $2L+1$ takes into account the degeneracy of radial modes and $n_{Lj}=1/[\exp(\omega_{Lj}/T)-1]$ is the thermal occupation of bosonic excitation modes.

The characteristic frequencies Ω_r and Ω_a of the radial and angular modes set temperature scales at which the spectrum in Eq. (7) becomes quasiclassical, $j,L\gg 1$. For the aforementioned experimental parameters, the corresponding energy scales are of the order $\Omega_r\simeq 5$ nK and $\Omega_a\simeq 0.5$ nK. There are three temperature regimes in this case. In the extreme low-temperature limit $T\ll\Omega_a$, thermal excitations are gapped—i.e., $E(T)\propto\Omega_a\exp(-2\sqrt{2}\Omega_a/T)$. At intermediate temperatures $\Omega_a\ll T\ll\Omega_r$, the radial modes are frozen and only the two-dimensional angular modes contribute to the energy, $E(T)\propto T^3/\Omega_a^2$. At higher temperatures $T\gg\Omega_r$, both radial and angular modes are excited, and the energy has a three-dimensional phononlike temperature dependence, $E(T)\propto T^4/\Omega_r^3$. The separation of temperature scales achieved by changing the interlayer width from $\delta r_n\simeq\ell$ to $\delta r_n\gg\ell$ tunes the effective dimensionality of the system from two to three dimensions.

At finite temperatures, the order parameter is depleted from its zero temperature value by the collective modes. In the low-temperature regime $T\ll J(n+1)$ for wide interlayers $\delta r_n\gg\ell$, the order parameter depletion is similar to the case of a three-dimensional weakly interacting Bose-Einstein condensate (BEC), $\delta\Delta(T)\propto[T/J(n+1)]^2$. At higher temperatures, the long-range order is destroyed by the quasiparticle excitations whose wavelength is of the order of the lattice spacing. The mean-field critical temperature is obtained by introducing the effective "magnetic" field $\mathbf{B}$ in the pseudospin Hamiltonian, $\mathcal{H}=\Sigma_i\mathcal{H}_i=-\Sigma_i\mathbf{B}_i\mathbf{s}_i$ [17]. In the local density approximation, the "magnetic" field is defined self-consistently:

$$B^+ = [zJ(n+1)B^+/|\mathbf{B}|]\tanh(\beta|\mathbf{B}|/2), \tag{11}$$

where $\beta=1/T$, $B^+=2zJ(n+1)\langle s^+\rangle_T$, $\langle\cdots\rangle_T$ is the thermal average, and $B^z=zJ(n+1)\cos\theta_0$. There are two solutions of Eq. (11), one of which is a trivial one, $B^+=0$, which corresponds to a Mott state. It is stable when the temperature exceeds the critical one, $T\gg T_c$. A nontrivial solution exists at $T<T_c$. Setting $B^+=0$ and $z=6$ in Eq. (11) one defines T_c in three-dimensional lattice:

$$T_c = 3J(n+1)\cos\theta_0/\tanh^{-1}(\cos\theta_0), \tag{12}$$

where $\cos\theta_0=2(\rho_0-n)-1$ is defined by the number of bosons ρ_0 per lattice site. As expected the critical temperature goes to zero as one approaches the Mott states at $|\cos\theta_0|\approx 1$. The dependence of the critical temperature on the boson density is illustrated in Fig. 2(a). The maximum value $T_{3D}=3J(n+1)$ defines the critical temperature for wide interlayers in the trapped case.

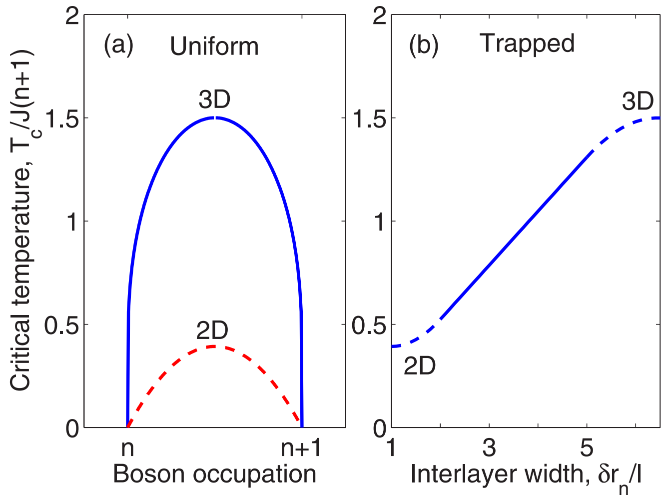

FIG. 2. (Color online) (a) The critical temperature of the superfluid transition as a function of the boson density in the strongly interacting ($U\gg J$) Bose gas in the two-dimensional (dashed line) and three-dimensional (solid line) uniform geometry. The critical temperature vanishes at the integer values of the density when the system becomes insulating. (b) The critical temperature of the superfluid transition in the trapped geometry as a function of the interlayer width, interpolating between two- and three dimensional limiting values. Dashed lines sketch smooth interpolation to the corresponding limits.

The thermal properties of narrow interlayers, $\delta r_n\simeq\ell$, are qualitatively different. In this case the angular excitations play the dominant role. In accordance with the Mermin-Wagner-Hohenberg theorem [22], the long-range order is destroyed but the system retains power-law correlations in the phase of the order parameter. In the limit that the interlayer width is comparable to the lattice spacing, $\delta r_n\simeq\ell$, a simple model capturing the properties of the two-dimensional system involves only phase variables and leads to the effective Hamiltonian $\mathcal{H}_\varphi=(K/2)\int d^2x(\nabla\varphi)^2$, where the integration is over the surface of the spherical layer and we have introduced the phase stiffness $K=J(n+1)/2$. The Kosterlitz-Thouless (KT) transition [23] between the high-temperature normal and low-temperature superfluid states occurs at temperature $T_{2D}=(\pi/2)K=(\pi/4)J(n+1)$. It is the maximum value of the KT critical temperature in the two-dimensional uniform geometry, $T_{KT}=(\pi/4)J(n+1)\sin^2\theta_0$, as a function of the boson density [see Fig. 2(a)]. At intermediate widths $\delta r_n\gtrsim\ell$, the phase stiffness is approximated by $K=[J(n+1)/2](\delta r_n/\ell)\overline{\sin^2\theta_0}$ with $\overline{\sin^2\theta_0}=(1/\delta r_n)\int dr\sin^2\theta_0$. For the interlayer in the trap, $\overline{\sin^2\theta_0}=2/3$, and the critical temperature of the KT transition is given by

$$T_c = (\pi/6)(\delta r_n/\ell)J(n+1), \tag{13}$$

which is a linear function of the interlayer width, interpolating between two-dimensional and three-dimensional limits, $T_{2D}\leq T_c\leq T_{3D}$, as shown in Fig. 2(b). In the range of current experiments, for $J\approx h\times 120$ Hz, one obtains an estimate of the critical temperature, $T_c\simeq 10$ nK.

VI. CONCLUSION

In conclusion, we have shown that the interlayer with fluctuating site occupation confined between two Mott states becomes superfluid at low but experimentally accessible temperatures. Employing the pseudospin model, we have identified the effective potential confining the superfluid and analyzed the low-energy excitations in the system. We have demonstrated that the effective dimensionality of the interlayer can be changed by tuning external parameters. As an example of the ensuing physics we have suggested that the critical temperature interpolates between two-dimensional and three-dimensional limits as one changes the width of the interlayer. A clear experimental signature of the interlayer condensate, either through time-of-flight and interference experiments, excitation of collective modes, or radio-frequency spectroscopy, is yet to be obtained.

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