Larkin-Ovchinnikov state in resonant Fermi gas

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We construct the phase diagram of a homogeneous two-component Fermi gas with population imbalance under a Feshbach resonance. In particular, we study the physics and stability of the Larkin-Ovchinnikov phase. We show that this phase is stable over a much larger parameter range than has been previously reported by other authors.

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Trapped Fermi gases offer us a wonderful opportunity to study strongly interacting fermion systems [1]. With Feshbach resonance, one is able to tune the effective interaction from weakly attractive for magnetic fields above the resonance to strongly attractive below. This problem is also closely related to studies of quark and nuclear matter [2].

Recent attention has shifted to systems with unequal populations [3–6]. The latter problem is analogous to the physics of a superconductor under the influence of an external Zeeman field, which provides a chemical potential difference $\mu_\uparrow - \mu_\downarrow \equiv 2h$ between the two species $\uparrow$ and $\downarrow$. In the weak-coupling limit, it was shown by Sarma [7] that the uniform state with population imbalance is unstable at zero temperature. By comparing the free energy of the normal state and the completely paired superconducting state, he concluded that, as the magnetic field is increased, there is a first-order phase transition from the equal-population superconducting state (with gap Δ_0) to the normal state at $h = \Delta_0/\sqrt{2}$. This implies that a system with given unequal numbers of the two spin species will either phase separate (with $h = \Delta_0/\sqrt{2}$), or be in the normal state (with $h > \Delta_0/\sqrt{2}$), depending on the imbalance. However, later, Fulde and Ferrell [8] and Larkin and Ovchinnikov [9] showed that there are better alternatives. Fulde and Ferrell (FF) considered a state with order parameter $\Delta(\vec{r}) = \Delta_0 e^{i\vec{q}\cdot\vec{r}}$, that is, pairs with finite momentum. The order parameter has a spatially varying phase with, however, the magnitude still constant in space. They showed that in certain parameter space this state has lower free energy than the states considered by Sarma. Larkin and Ovchinnikov (LO), however, demonstrated that, at least in the small-order-parameter limit, states with certain choices of sinusoidal variations of the order parameter [such as $\Delta(\vec{r}) = \Delta_0 \cos(qx)$, etc.] are more energetically favorable than the FF state. Notice that, in the LO state, the magnitude of the order parameter is no longer a constant in space. With decreasing magnetic field and hence decreasing population difference n_d, many authors showed that the LO state evolves into a state with a set of domain walls. This has been demonstrated for one [10], two [11], as well as three [12] dimensions and also for a d-wave superconductor [13]. In the small-population-imbalance limit, the order parameter has a constant magnitude almost everywhere in space (with value identical to that in the state with no population difference), except near the domain walls. The phase of the order parameter changes by π when these walls are crossed, and these are also the locations where the magnetization concentrates. This local magnetization arises from the occupation of bound states, available due to the presence of the domain walls. The physics of these domain walls is closely related to that of the π junctions in superconductor-ferromagnet-superconductor junctions [14], where for suitable parameters it is energetically favorable for the two superconductors to acquire a phase difference π.

Indeed, in three dimensions, in the zero-temperature and weak-coupling limit, Matsuo *et al.* [12] demonstrated that the energy of the domain walls becomes negative for h above $h_{dw} \equiv \frac{1}{2}(0.745\pi T_c) \approx 0.665\Delta_0$, where Δ_0, is the zero-temperature gap for the completely paired superfluid of equal populations. This h_{dw} is less than the phase separation field $h_{ps} = \Delta_0/\sqrt{2} \approx 0.707\Delta_0$ (where the free energies of the normal and the completely paired superfluid states are equal). Hence the LO state must be more stable than both the uniform superfluid and the normal state (at least) for $h_{dw} < h < h_{ps}$. For h slightly above h_{dw}, the domain walls are far apart. The system resembles the uniform paired state except for the occasional domain walls. This state thus allows for the possibility of arbitrarily small population difference between the two species. This problem is quite analogous to the lower critical field h_{c1} in type-II superconductors, where the vortex energy becomes negative and vortices begin to penetrate the superconductor for h slightly above h_{c1}. For increasing h, more and more domain walls are formed, the order parameter becomes more sinusoidal-like, and the state evolves smoothly to the picture given by LO [9].

For the resonant Fermi gas, it has been recognized that, at intermediate coupling strengths, the uniform state with population imbalance must be unstable [3,4]. Indeed, this has also been demonstrated also by experiments [6]. However, the investigation into the actual phase diagram, that is, the question as to which phase appears where instead of the unstable uniform state, even for the case without a trap, cannot be regarded as complete, especially regarding the stability of the FFLO states. Many papers [15,16] examined only phase separation, whereas some [4,5,17–19] considered in addition only the FF state with $\Delta \propto e^{i\vec{q}\cdot\vec{r}}$. However, none of these works actually investigated the LO states. They conclude that the FF state exists only in a very narrow region next to the normal state in the weak-coupling regime, and deduce then that phase separation occurs throughout the rest of the entire region where the uniform phase is unstable. In particular, they conclude that, for small population differences, phase separation occurs unless the dimensionless coupling constant $1/k_f a \to -\infty$. However, as seen already in the above paragraph, this is simply an artifact of the FF state, which cannot smoothly go into the completely paired equal-population state [20]. We expect that, at least in the weak-coupling limit, the state should be LO for any population imbalance below that of the normal state.

In this work, we shall investigate the stability of the LO state for arbitrary strength of the attractive interaction between the two species, thus generalizing previous works such as [11,12] beyond the BCS limit. We shall concentrate in particular on the small-n_d limit. More specifically, we compare the critical chemical potential difference h_{dw} at which the domain wall energy becomes negative to the critical field h_{ps} for phase separation, where the free energy of the normal phase becomes equal to that of the completely paired equal-population superfluid phase. We find that for $1/k_f a \lesssim (\gtrsim) -0.845$, $h_{dw} < (>) \, h_{ps}$. Therefore we conclude that, for small n_d, the LO state is more stable than the phase-separated state for $1/k_f a \lesssim -0.845$. By combining with previous results [3,5,23] (and with some reasonable extrapolations), we sketch the appropriate phase diagram for our system.

The mean-field Hamiltonian of our system can be written as

$$H = \int d^3\vec{r}\left[\sum_\sigma\left(\frac{\hbar^2\,\nabla\,\psi_\sigma^\dagger\,\nabla\,\psi_\sigma}{2m} - \mu_\sigma\psi_\sigma^\dagger\psi_\sigma\right) - [\Delta^*(\vec{r})\psi_\downarrow\psi_\uparrow + \text{c.c.}] - \frac{|\Delta(\vec{r})|^2}{g}\right] \quad (1)$$

where $\sigma = \uparrow, \downarrow$ for the two species, $\psi_\sigma(\vec{r})$ are their corresponding field operators, $\Delta(\vec{r})$ is a position-dependent order parameter, and g is the coupling constant. For convenience below we shall also write $\mu_\uparrow = \mu + h$ and $\mu_\downarrow = \mu - h$.

This Hamiltonian can be diagonalized by the Bogoliubov transformation for a general inhomogeneous system [21]

$$\begin{pmatrix}\psi_\uparrow(\vec{r})\\ \psi_\downarrow^\dagger(\vec{r})\end{pmatrix} = \sum_J \begin{pmatrix} u_J(\vec{r}) & v_J^*(\vec{r}) \\ -v_J(\vec{r}) & u_J^*(\vec{r})\end{pmatrix}\begin{pmatrix}\alpha_J\\ \beta_J^\dagger\end{pmatrix}, \quad (2)$$

where α_J and β_J are annihilation operators for quasiparticles with spin $\uparrow$ and $\downarrow$ of the state labeled by a set of quantum numbers J. $u_J(\vec{r})$ and $v_J(\vec{r})$ obey $\int d^3\vec{r}[|u_J(\vec{r})|^2 + |v_J(\vec{r})|^2] = 1$ and the Bogoliubov–de Gennes (BdG) equation

$$\begin{pmatrix} -\frac{\hbar^2\nabla^2}{2m} - \mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & \frac{\hbar^2\nabla^2}{2m} + \mu \end{pmatrix}\begin{pmatrix}u_J(\vec{r})\\ v_J(\vec{r})\end{pmatrix} = E_J\begin{pmatrix}u_J(\vec{r})\\ v_J(\vec{r})\end{pmatrix}. \quad (3)$$

Substituting Eq. (2) into Eq. (1) and using Eq. (3) to eliminate the derivatives ∇u_J and ∇v_J, we obtain the ground-state free energy

$$F = \int d^3\vec{r}\left[\sum_J\left(-2E_J|v_J(\vec{r})|^2 + \frac{1}{V}\frac{|\Delta(\vec{r})|^2}{2\epsilon_J}\right) - \frac{m}{4\pi\hbar^2 a}|\Delta(\vec{r})|^2\right] + \sum_J (E_J - h)f(E_J - h). \quad (4)$$

The last term arises from the occupation of states α_J with $E_J < h$. Here f is the Fermi function, V the volume, and we have eliminated the interaction constant g in favor of the scattering length a via the relation $1/g = m/(4\pi\hbar^2 a) - (1/V)\Sigma_J 1/2\epsilon_J$ where ϵ_J is the energy of the state J in the normal phase. (Strictly speaking we should label the quantum states in the normal phase by another set of quantum numbers J', but we shall not make such a distinction for simplicity in notations. See further below.) In Eq. (4), $\Delta(\vec{r})$ should be viewed as a variational parameter with respect to which F has to be minimized.

We are interested in the domain wall energy for given μ, h, and a. We evaluate this by calculating the energy difference between a state with a single planar domain wall and the uniform completely paired superfluid state. The latter is easy, since it is independent of h and the BdG equation (3) can be solved by Fourier transform. We obtained, for wave vector $\vec{k}$, the familiar quasiparticle energies $E_{\vec{k}} = [(\epsilon_k - \mu)^2 + |\Delta|^2]^{1/2}$ (where $\epsilon_k \equiv \hbar^2 k^2/2m$) and hence the bulk free energy $F_S(\mu) = \Sigma_{\vec{k}}[\epsilon_k - \mu - E_k + (m/\hbar^2 k^2)|\Delta|^2] - (m/4\pi\hbar^2 a)|\Delta|^2$. Minimizing with respect to Δ gives the usual BCS gap equation $-(m/4\pi\hbar^2 a)\Delta = \Delta(1/V)\Sigma_{\vec{k}}(1/2E_k - m/\hbar^2 k^2)$. We can also compute the corresponding density via $n = \int[d^3k/(2\pi)^3][1 - (\epsilon_k - \mu/E_k)]$ and the corresponding Fermi wave vector $k_f \equiv (3\pi^2 n)^{1/3}$ and express a in the dimensionless combination $k_f a$. These results are identical with those in [22]. We had also used $\Delta(z) = \text{const}$ instead of a domain wall in the procedure described below and verified numerically that we indeed obtained the same free energy density for the uniform state. (By equating this free energy to that of the normal state at the same μ and h, we obtained the phase separation field h_{ps} discussed.)

Next we evaluate the free energy of a planar domain wall. For this, we put our system in a box of dimensions L_x, $L_y = L_x$, $L_z \equiv L$. We assume that the order parameter varies only along the z direction and thus Fourier-transform the x and y coordinates. Calling the resulting wave vector $\vec{p}$, we rewrite (u_J, v_J) as

$$\begin{pmatrix}u_J(\vec{r})\\ v_J(\vec{r})\end{pmatrix} = \frac{1}{L_x L^{1/2}}\begin{pmatrix}u_{p,j}(z)\\ v_{p,j}(z)\end{pmatrix}e^{i\vec{p}\cdot\vec{r}_p} \quad (5)$$

where $\vec{r}_p$ is the component of $\vec{r}$ in the x-y plane, and j is now a quantum number for the z dependences. $u_{p,j}(z)$ and $v_{p,j}(z)$ are dimensionless quantities obeying $\int dz[|u_{p,j}(z)|^2 + |v_{p,j}(z)|^2]/L = 1$. Equation (3) becomes

$$\begin{pmatrix} -\dfrac{\hbar^2\partial_z^2}{2m}-\tilde{\mu} & \Delta(z) \\ \Delta^*(z) & \dfrac{\hbar^2\partial_z^2}{2m}+\tilde{\mu} \end{pmatrix}\begin{pmatrix} u_{p,j}(z) \\ v_{p,j}(z) \end{pmatrix} = E_{p,j}\begin{pmatrix} u_{p,j}(z) \\ v_{p,j}(z) \end{pmatrix}, \tag{6}$$

where $\tilde{\mu}\equiv\mu-p^2/2m$. The energies $E_{p,j}$ and the wave functions $u_{p,j}(z)$, $v_{p,j}(z)$ depend on $\vec{p}$ only through its magnitude p. The free energy can then be written as

$$F=\int dz\left[\frac{1}{L}\sum_{\vec{p}}\sum_{j}^{j_{max}}\left(-2E_{p,j}|v_{p,j}(z)|^2+\frac{|\Delta(z)|^2}{2\epsilon_{p,j}}\right)-L_x^2\frac{m}{4\pi\hbar^2 a}|\Delta(z)|^2\right]+\sum_{\vec{p},j}(E_{p,j}-h)f(E_{p,j}-h). \tag{7}$$

In this equation, we arrange the quasiparticle states for both the superfluid and the normal states as increasing functions of the counting number j. We solve Eq. (6) by discretizing the z coordinate. After the energies and the function $v_{p,j}(z)$ are calculated, we put them in Eq. (7) and calculate the free energy. For $\Delta(z)$, we limit ourselves to the one-parameter ansatz

$$\Delta(z)=\Delta_0\tanh\left(\frac{k_\mu z}{\beta}\right) \tag{8}$$

with β as the variational parameter and $k_\mu\equiv(2m\mu)^{1/2}$. (Thus the width of the domain wall is given by βk_μ^{-1}.) Here Δ_0 is the bulk order parameter for the given μ and a at $h=0$, as the order parameter should approach this value far away from the domain wall. Our Eq. (8) is motivated by earlier investigations [11,12], where their numerical results can be well fitted by a function of the form (8). [Since we are employing periodic boundary conditions in z, this ansatz actually introduces a sharp ($\beta=0$) domain wall also at $z=\pm L/2$. However, this can be taken care of easily by removing the contributions due to this extra domain wall.]

We shall then input the ansatz Eq. (8) into Eq. (6) to solve for $E_{p,j}$. Our analysis of the free energy Eq. (7) is simplified by the following observations. At $h=0$, since all quasiparticle energies are positive, $f(E_{p,j})=0$ and the free energy is given simply by the integral in Eq. (7). The free energy at finite h of a given β is related to that of $h=0$ at the same β by simply adding the negative term $\Sigma_{\vec{p},j}(E_{p,j}-h)f(E_{p,j}-h)$ due to the occupation of the quasiparticle states.

An example for our results for $1/k_\mu a=-1.0866$, $1/k_f a=-1.0676$, and $\Delta_0/\mu=0.2$ is discussed below. The bound state energies $E_b\equiv E_{p,j}$ are illustrated in Fig. 1. We see that in general we have states below the continuum. ($E_{p,j}<|\Delta_0|$ for $\tilde{\mu}>0$, $E_{p,j}<\sqrt{|\tilde{\mu}|^2+|\Delta_0|^2}$ for $\tilde{\mu}<0$. It is the bound states that are essential: as we shall see, the relevant values of h are below the gap edges.) The $\beta=0$ results were checked against the analytical ones in the Appendix. The bound-state energy for a given p decreases with increasing width of the domain walls, as one expects. The significance of this will be discussed again below.

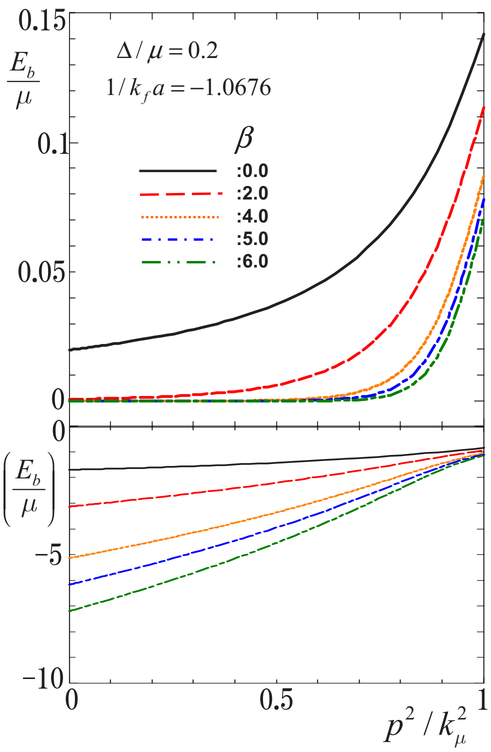

FIG. 1. (Color online) Upper panel: Bound-state energies E_b for $1/k_f a=-1.0676$, corresponding to $1/k_\mu a=-1.0866$, $\Delta_0/\mu=0.2$, for widths of the domain walls given by βk_μ^{-1}. Energies are normalized to μ and the momentum p in the x-y plane normalized to k_μ. Lower panel: Same as upper panel except with the vertical axis being $\ln(E_b/\mu)$.

The integrand of the first term in Eq. (7) is plotted in Fig. 2. [The double-hump structure for small β ($\neq 0$) is due to the fact that $|\Delta(z)|\to 0$ as $z\to 0$: see Eq. (7).] As expected, the integrand decreases to a constant corresponding to the free

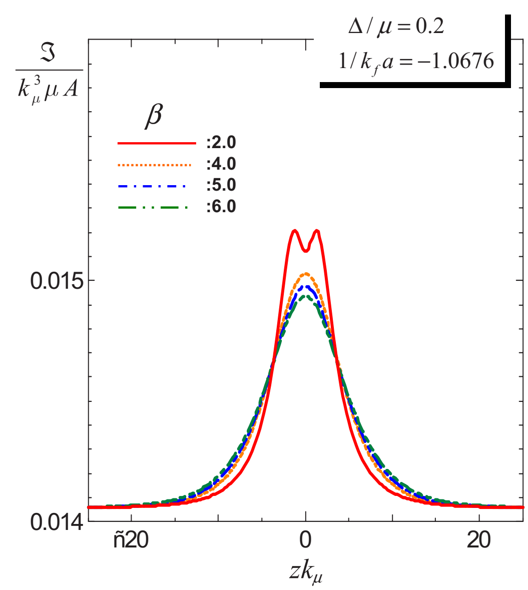

FIG. 2. (Color online) Integrand of Eq. (7), in units of $k_\mu^3\mu A$ where A is the area, as a function of position z (in units of k_μ^{-1}). Parameters are the same as in Fig. 1.

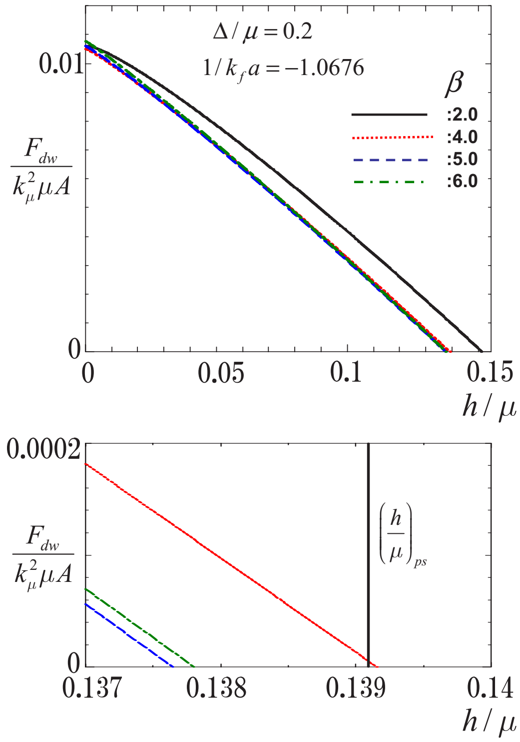

FIG. 3. (Color online) Upper panel: Domain wall free energy F_{dw} per unit area A (in units of $k_\mu^2\mu$) versus h (in units of μ). Parameters are the same as in Fig. 1. The lower panel magnifies the region near $F_{dw}=0$.

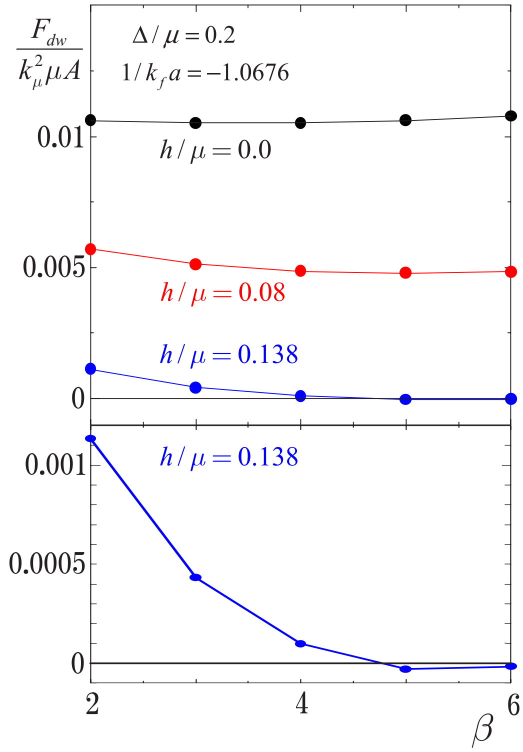

FIG. 4. (Color online) Upper panel: Domain wall free energy F_{dw} per unit area A (in units of $k_\mu^2\mu$) versus domain wall width β. Parameters are the same as in Fig. 1. The lower panel shows the detailed behavior for $h/\mu=0.138$.

energy density in the bulk for distances sufficiently far away from the domain wall. We can then evaluate the domain wall energy F_{dw} at $h=0$ by simply integrating this excess contribution over z. At $h=0$, F_{dw} is minimum at $\beta\approx 3$. F_{dw} is positive for all β's, as expected since the uniform state should have a lower energy than a domain wall. For finite h, the free energies are evaluated by adding the negative term from bound-state occupation as discussed above. The results are depicted in Fig. 3. The free energy decreases due to the occupation of the bound states. Since the bound-state energies are smaller for larger β, the wall energy decreases faster for larger β, shifting the β for minimum wall energy to larger values with increasing h (see Fig 4). The domain wall energy becomes negative at sufficiently large h for all β's. The important question is whether it will become negative for some β at a value of h which is less than that for phase separation. For the parameters under discussion, the domain wall energy first becomes negative for $\beta\approx 5$ at $h\equiv h_{dw}\approx 0.1376\mu$, which is less than the phase separation value $h_{ps}\approx 0.1391\mu$. We thus conclude that for $1/k_f a=-1.0676$, when n_d is infinitesimally small, the system is in the LO state but not the phase-separated state.

At this point, it is of interest to compare our results more quantitatively with those in [12], where the quasiclassical limit was assumed in the calculations at the outset. The authors of [12] obtained the critical value for the sign change of the domain wall energy at $h=h_{dw}=\frac{1}{2}(0.745\pi T_c)\approx 0.665\Delta_0$ (using $\Delta_0=1.76T_c$). Our h_{dw} is close to their value if we simply rewrite their result as $h_{dw}/\mu=0.665\Delta_0/\mu$ and substitute $\Delta_0/\mu=0.2$. Also, their Fig. 1 indicates the domain wall width near the critical field to be roughly given by $2\Delta_0 v_f/(\pi T_c)^2$ where $v_f=k_f/m$ is the Fermi velocity. Rewriting this expression as $\approx 1.25(k_f/k_\mu)(\mu/\Delta_0)k_\mu^{-1}$ and simply subsituting the values of Δ_0/μ, etc., we again find very good agreement. Thus, even though we did not assume the quasiclassical limit at the outset, for $1/k_f a=-1.067$ our results can be understood from the quasiclassical limit calculations of [12] with extrapolations.

For increasing $1/k_f a$, both h_{dw} and h_{ps} increase (and the β for the optimal domain wall decreases). However, h_{dw} increases faster than h_{ps}, and eventually $h_{dw}<h_{ps}$ no longer holds. The situation is shown in Fig. 5. If $h_{dw}>h_{ps}$, then the formation of a domain wall is no longer favorable since phase separation already occurs at h slightly above h_{ps}. We conclude that, for $n_d\to 0_+$, the transition from the LO state to the phase-separation state occurs at $1/k_f a\approx -0.845$. This point is indicated by the point A in Fig. 6.

After locating this transition point for $n_d\to 0_+$, we now attempt to construct a phase diagram for general n_d by combining the present results with previous ones in the literature. For $n_d/n\neq 0$ and sufficiently negative $1/k_f a$, the system is in the normal state [3,5]. We consider in turn the transition lines between this normal state and the LO and phase-separated states. Assuming that the transition to the LO state is second order [9] (cf. below), the transition line between the normal and LO states can be found by solving the Cooper problem at finite wave vector $\vec{q}$:

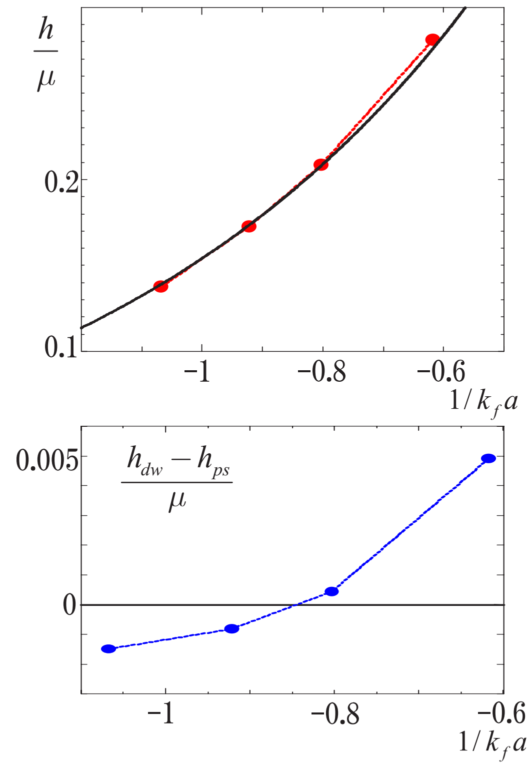

FIG. 5. (Color online) Upper panel: Comparison between h_{dw} (dotted line) and h_{ps} (full line) as a function of $1/k_f a$. Lower panel: $h_{dw}-h_{ps}$ as a function of $1/k_f a$.

$$-\frac{m}{4\pi\hbar^2 a}=\frac{1}{V}\sum_{\vec{k}}\left(\frac{1-f[\epsilon_{\vec{k}+\vec{q}}-(\mu+h)]-f[\epsilon_{-\vec{k}}-(\mu-h)]}{\epsilon_{\vec{k}+\vec{q}}+\epsilon_{\vec{k}}-2\mu}-\frac{m}{\hbar^2 k^2}\right). \quad (9)$$

The transition line is determined by finding the optimal q corresponding to the weakest attractive interaction, i.e., the most negative $1/k_f a$. We note here that, since Eq. (9) is obtained by linearizing in the order parameter, the same equation determines the second-order transition line into the FF state [9]. Our result can be turned into a line in our phase diagram (an equation of $1/k_f a$ versus n_d/n) by using $k_f=(3\pi^2 n)^{1/3}$, with $n=[(2m)^{3/2}/6\pi^2][(\mu+h)^{3/2}+(\mu-h)^{3/2}]$ and $n_d/n=[(\mu+h)^{3/2}-(\mu-h)^{3/2}]/[(\mu+h)^{3/2}+(\mu-h)^{3/2}]$. This gives the long-dashed line in Fig. 6. Our numerical results (see also [23]) agree well with [5]. It should, however, be remarked that the transition from the normal state to the LO state has also claimed to be first order [24] at very low temperatures in three dimensions. However, the difference between the actual transition line and the second-order line is very small [24], and we ignore this difference here.

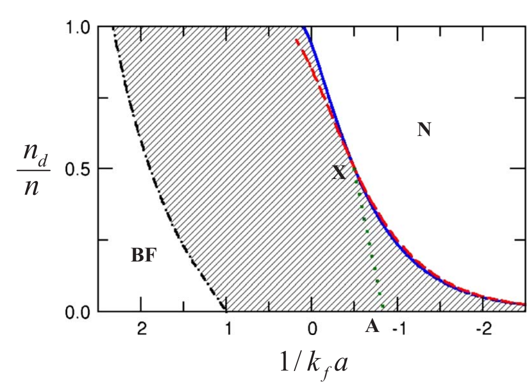

FIG. 6. (Color online) Phase diagram. N, normal state, BF, homogeneous superfluid (Bose-Fermi mixture) phase. Shaded regions to the right of XA, Larkin-Ovchinnikov state; shaded regions to the left of XA, phase separation.

For the transition line between the normal state and phase separation, we equate [7] the free energy of the completely paired superfluid state $F_S(\mu)$ to that of the normal state: $F_N(\mu,h)=-V[(2m)^{3/2}/15\pi^2][(\mu+h)^{5/2}+(\mu-h)^{5/2}]$. This again yields a line $1/k_f a$ as a function of h/μ and hence n_d/n, shown as the solid line in Fig. 6. Our numerical result (see also [23]) is also in good agreement with [5]. The two transition lines intersect at the point labeled by X at $1/k_f a \approx -0.50$. Therefore the transition is to the LO (phase-separation) state if $1/k_f a$ is less (larger) than this value. Interpolating between points A and X, we thus conclude that the system is in the LO state for the shaded region to the right of the line XA, whereas phase separation occurs for the shaded region to the left of this line. The transition lines between this phase-separated state and the homogeneous superfluid (Bose-Fermi mixture) phase for positive $1/k_f a$ have already been discussed elsewhere in the literature ([5,15,16]; see also other references in [23]), and we will not repeat the details here.

In conclusion, we studied the stability of the LO state in a homogeneous system, in particular in the limit of small population imbalance, by calculating the domain wall energies. Our investigation here is analogous to the determination of the lower critical field of the vortex phase of a type-II superconductor. The determination of the detailed structures of the LO state, such as the question of the lattice structure of the domain walls (cf., e.g., [24], analogous to the vortex lattice structure in type-II superconductors), as well as the phase diagram in the presence of a trap (studied already partially in [25]), are left for the future.

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APPENDIX: BOUND STATES

Due to the important role played by the bound states in the LO state, we give the analytic results for a sharp domain wall where $\Delta(z)=\Delta_0\,\mathrm{sgn}(z)$ (corresponding to $\beta\to 0$). It should be noted that, even though $|\Delta(z)|$ is a constant throughout space, bound states still exist due to the sudden change in phase factor of the order parameter from $-\pi$ to 0 at $z=0$. As already seen from Eq. (6), the bound-state energy is a function of μ and p only through the combination $\tilde{\mu}$. The BdG equation (6) can be solved easily in this case since $\Delta(z)$ is piecewise constant. We require that the functions $u_{p,j}(z)$, $v_{p,j}(z)$ as well as their derivatives be continuous at $z=0$. The calculation is similar to solving the Schrödinger equation in piecewise constant potentials. It is straightforward and we simply state the results. It is convenient to divide them into two regions.

(i) $\tilde{\mu}>0$. In this case states are bound when $E_b \equiv E_{p,j} < |\Delta_0|$. We find that there is only one bound state (thus correspondingly one bound state for each $\vec{p}$), with energy given by

$$\frac{E_b}{|\Delta_0|} = \frac{\sqrt{1+2\delta^2}-1}{2\delta}, \tag{A1}$$

where $\delta \equiv |\Delta_0| / \tilde{\mu}$. We note here that the quasiclassical limit corresponds to $|\Delta_0| \ll \tilde{\mu}$ and hence $\delta \ll 1$. In this limit, one can approximate the BdG equation by the Andreev equation [26] and the bound-state energy vanishes (irrespective of the detailed positional dependence of $\Delta(z)$ so long as there is a π phase change; see, e.g., [27] and also [28]). For small $|\Delta_0| / \tilde{\mu}$, we have $E_b \approx |\Delta_0|^2 / 2\tilde{\mu}$ (cf. the minigap for bound states near the vortex core [21]). With increasing $|\Delta_0|$ or equivalently decreasing $\tilde{\mu}, E_b/|\Delta_0|$ increases. For $\delta \to \infty$, $E_b \to |\Delta_0| / \sqrt{2}$.

(ii) $\tilde{\mu}<0$. In this case the continnuum starts at $\sqrt{\tilde{\mu}^2+|\Delta_0|^2}$. Again we find only one bound state, with

$$\frac{E_b}{|\Delta_0|} = \frac{\sqrt{1+2\delta^2}+1}{2\delta} \tag{A2}$$

where $\delta \equiv |\Delta_0| / |\tilde{\mu}|$. For $\tilde{\mu}=0_-$ ($\delta \to \infty$), $E_b \to |\Delta_0| / \sqrt{2}$ [cf. case (i) above]. For decreasing $\tilde{\mu}$ (decreasing δ), E_b increases. E_b reaches $|\Delta_0|$ at $\delta=2$. For $\tilde{\mu} \to -\infty$, E_b approaches the continuum from below. It is remarkable that a bound state exists even for $\tilde{\mu}<0$.We note that in this regime we have always $E_b > |\tilde{\mu}|$. A similar situation occurs also for the bound states at a vortex [29] for general coupling strength $1/k_f a$.

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