Optimal quantum-chain communication by end gates

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The scalability of solid-state quantum computation relies on the ability of connecting the qubits to the macroscopic world. Quantum chains can be used as quantum wires to keep regions of external control at a distance. However, even in the absence of external noise their transfer fidelity is too low to assure reliable connections. We propose a method of optimizing the fidelity by minimal usage of the available resources, consisting of applying a suitable sequence of two-qubit gates at the end of the chain. Our scheme also allows the preparation of states in the first excitation sector as well as cooling.

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I. INTRODUCTION

It is often noted that the advantage of solid-state computation is its *scalability.* This is because a typical chip can contain a large amount of qubits and because the fabrication of many qubits is in principle no more difficult than the fabrication of a single one. In the last couple of years, remarkable progress was made in experiments with quantum dots [1] and superconducting qubits [2]. It should, however, be emphasized that for initialization, gating, and readout, those qubits have to be connected to the macroscopic world. For example, in a typical flux qubit gate, microwave pulses are applied onto specific qubits of the sample. This requires many (classical) wires on the chip, which is thus a compound of quantum and classical components. Unfortunately, any extra classical control wire is potentially an independent source of noise as it adds extra coupling between the quantum computing device and the external world. Consequently the number of wires is likely to be the bottleneck of the scalability as a whole: too few will make the device not powerful enough, too many will make it noisy.

In this situation, quantum chains may turn out to be extremely useful in the development of solid-state-based quantum computer technology. They consist of lines of coupled single qubits *without external classical control*. In many cases, such permanent couplings are easy to build in solid-state devices. Indeed the really difficult part usually is to *modulate* or to *suppress* them, as has been clearly pointed out for fabricated hard-wired couplings between superconducting qubits [3] or tunnel coupled quantum dots [4]. Naturally then the question arises as to whether one can use such quantum chains as nearly perfect channels for quantum communication despite the lack of classical controllability. If successful, it will also be the application of a quantum many-body system for a useful quantum information processing task. The setup we have in mind is sketched in Fig. 1. It is a distributed quantum computing architecture [5] built out of blocks of qubits, some of which are dedicated to communication and therefore connected to another block through a quantum chain. The block size is essentially determined by the minimum number of controlling wires necessary to perform reliable arbitrary unitary operations on the block spins: ultimately it depends the ability of implementing fault-tolerant gates [6] with the available current technology. The distance between the blocks is instead determined by the length of the quantum chains between them. It should be large enough to allow for classical control wiring of each block, but short enough such that the time scale of the quantum chain communication is well below the time scale of decoherence in the system.

Many interesting aspects of quantum chain communication were investigated in the last years [8–21], both from a physics point of view and from a quantum information point of view. Here, we would like to concentrate on those schemes [9–12] which require no further resources than those outlined in Fig. 1. The chain couplings may be engineered [10,11] to improve the theoretical communication fidelity, but coupling fluctuations and energy mismatches will lower the fidelity in practice [13–17]. Hence even without the contribution of external noise [17,18] the quality of transfer may well be too low to yield a scalable system.

In this paper we will show that the fidelity can be improved easily using the gates available in the regions of

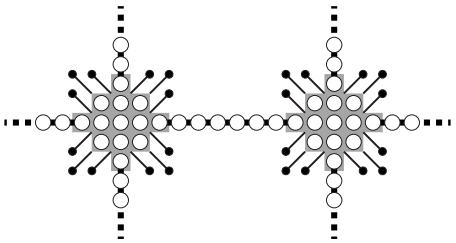

FIG. 1. Small blocks (gray) of qubits (white circles) connected by quantum chains. Each block consists of (say) 13 qubits, four of which are connected to outgoing quantum chains (the thick black lines denote their nearest-neighbor couplings). The blocks are connected to the macroscopic world through classical wires (thin black lines with black circles at their ends) through which arbitrary unitary operations can be triggered on the block qubits. The quantum chains require no external control. This architecture is an example of distributed quantum computation [5] where the computational and the communication qubits are the same objects (i.e., the spins): in this respect no interfacing among different qubits species is required (compare this with the implementations of Ref. [7]), whose extreme difficulty in the context of solid-state qubits is discussed, for example, in Ref. [15].

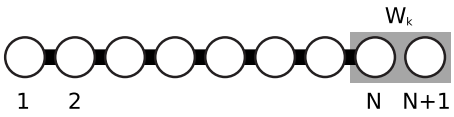

FIG. 2. A quantum chain (qubits $1,2,\ldots,N$) and a target qubit ($N+1$). By applying a sequence of two-qubit unitary gates W_k on the last qubit of the chain and the target qubit, arbitrarily high fidelity can be achieved for the transmission of quantum information from the left-hand side to the right-hand side of the chain.

quantum control. The main idea is to apply in certain time intervals two-qubit gates at the receiving end of the chain. The resulting sequence is determined *a priori* by the Hamiltonian of the system. As we shall see, the maximal fidelity that can be reached this way is limited only by external noise, and not by the spatial fluctuations of the couplings (cf. [17]). This is similar in spirit to the dual-rail [17] and memory protocols [19], but here we give a protocol that is *optimal* in the resources used: a single spin chain and a two-qubit gate at the each end. It is optimal because two-qubit gates at the sending and receiving end are required in order to connect the chain to the blocks in *all* above protocols (though often not mentioned explicitly). Our scheme has some similarities with [12], but the gates used here are much simpler, and arbitrarily high fidelity is guaranteed by a convergence theorem for arbitrary coupling strengths and all non-Ising coupling types that conserve the number of excitations. Furthermore, we show numerically that our protocol could also be realized by a simple switchable interaction. This means that quantum state transfer experiments with our protocol could be performed well before the realization of the blocks from Fig. 1.

The paper is organized as follows. In Sec. II we introduce the protocol and we present an analytical proof of the asymptotical convergence of the associated transfer fidelity. To do so we restrict ourselves to a regime in which the two-qubit gates applied at the end of the chain act instantaneously, i.e., they are activated over time intervals which are much shorter than the typical time scale of the free spin evolution. This hypothesis is not fundamental but it allows us to simplify the math: we will drop it in Sec. III where, by using numerical techniques, we generalize the convergence analysis to cases in which the timing of the end gates are comparable with those of the free dynamical evolution of the chain. The manuscript ends with the conclusions in Sec. IV.

II. ARBITRARILY PERFECT STATE TRANSFER

Here we present an analytical proof of the convergence of our transferring protocol. For the sake of simplicity we will focus on a single chain from the setup of Fig. 1. In this case, as in Ref. [9], the left end side of the chain plays effectively the role of a sender of quantum information while the right end side plays the role of a receiver. Within this framework we will show that the receiving block (gray area of Fig. 2) can improve the transmission fidelity to an arbitrarily high value by applying suitable two-qubit gates W_k (see below) between the end of the chain and a "target qubit" of the block. As mentioned in the Introduction, in order to get analytical results, we will restrict the analysis to the case in which the gates W_k act instantaneously on the system.

A. Notation

Before entering into the details of the derivation let us fix some notation and define the property of the system. We label the qubits of the chain by $1,2,\ldots,N$ and the target qubit by $N+1$. We also define the states

$$|\mathbf{0}\rangle \equiv |00\cdots0\rangle,$$

$$|\mathbf{n}\rangle \equiv \sigma_n^+|\mathbf{0}\rangle, \quad n=1,2,\ldots,N+1,$$

where σ_n^+ is the Pauli σ^+ operator acting on the nth qubit. With these definitions the typical initial configuration of our communication protocol will be described by vectors of the form

$$|\psi_{\text{initial}}\rangle = \alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle, \tag{1}$$

where all the qubits from 2 to $N+1$ are in the reference state $|0\rangle$ while the first qubit has been prepared into the logical state $\alpha|0\rangle+\beta|1\rangle$. This the quantum bits that one would like to propagate along the chain.

The free evolution of the system is described by a Hamiltonian H which couples all the qubits but the target. Our main assumption on H is that it has $|\mathbf{0}\rangle$ as eigenstate with eigenvalue 0, i.e., $H|\mathbf{0}\rangle=0$, and a N-dimensional invariant subspace spanned by the vectors $\{|\mathbf{n}\rangle; n=1,2,\ldots,N\}$. Under this condition H corresponds to a Hamiltonian that conserves the number of excitations along the chain, which would be the case, for example, of the Heisenberg or XY chains considered in most of the protocol proposed so far [8,9,11,12,17,19]. Thanks to this property the analysis of the protocol can be restricted to the $N+2$-dimensional Hilbert $\mathcal{H}=\text{span}\{|\mathbf{n}\rangle; n=0,1,2,\ldots,N+1\}$. Our final assumption about H is that there exists a time t such that $\langle\mathbf{N}|\exp\{-itH\}|\mathbf{1}\rangle\neq 0$. Physically this means that the Hamiltonian has the capability of transporting excitations (and hence information) from the first to the last qubit of the chain. As mentioned in the Introduction, the fidelity of this transport may be very bad in practice.

We denote the unitary evolution operator for a given time t_k as $U_k\equiv\exp\{-it_kH\}$ and introduce the projector

$$P = \mathbb{1}_{\mathcal{H}} - |\mathbf{0}\rangle\langle\mathbf{0}| - |\mathbf{N}\rangle\langle\mathbf{N}| - |\mathbf{N+1}\rangle\langle\mathbf{N+1}|.$$

A crucial ingredient to our protocol is the unitary transformation

$$W(c,d) \equiv P + |\mathbf{0}\rangle\langle\mathbf{0}| + d|\mathbf{N}\rangle\langle\mathbf{N}| + d^*|\mathbf{N+1}\rangle\langle\mathbf{N+1}| + c^*|\mathbf{N+1}\rangle\langle\mathbf{N}| - c|\mathbf{N}\rangle\langle\mathbf{N+1}|, \tag{2}$$

where c and d are complex normalized amplitudes. One can easily verify that W acts as the identity on all but the last two qubits, and can hence be realized by *a local two-qubit gate on the qubits N and N+1*. Furthermore, we have $WP=P$ and

$$W(c,d)[\{c|\mathbf{N}\rangle + d|\mathbf{N+1}\rangle\}] = |\mathbf{N+1}\rangle. \tag{3}$$

The operator $W(c,d)$ has the role of moving probability amplitude c from the Nth qubit to the target qubit. It can be applied locally by the receiving block.

B. The protocol

Using the time-evolution operator U_k and two-qubit unitary gates on the qubits N and $N+1$ defined in Eq. (2) we will now develop a protocol that transforms the state $|\mathbf{1}\rangle$ into $|\mathbf{N+1}\rangle$. Let us first look at the action of U_1 on $|\mathbf{1}\rangle$. Using the projector P we can decompose this time-evolved state as

$$U_1|\mathbf{1}\rangle = PU_1|\mathbf{1}\rangle + |\mathbf{N}\rangle\langle\mathbf{N}|U_1|\mathbf{1}\rangle \equiv PU_1|\mathbf{1}\rangle + \sqrt{p_1}\{c_1|\mathbf{N}\rangle + d_1|\mathbf{N+1}\rangle\}, \quad (4)$$

where $d_1=0$ and

$$p_1 = |\langle\mathbf{N}|U_1|\mathbf{1}\rangle|^2, \quad c_1 = \langle\mathbf{N}|U_1|\mathbf{1}\rangle/\sqrt{p_1}.$$

Let us now consider the instantaneous application of the unitary transformation $W_1 \equiv W(c_1,d_1)$ on the time-evolved state of Eq. (4). According to Eq. (3) this yields

$$W_1U_1|\mathbf{1}\rangle = PU_1|\mathbf{1}\rangle + \sqrt{p_1}|\mathbf{N}+1\rangle. \quad (5)$$

Hence with a probability of p_1, the excitation is now in the position $N+1$, where it is "frozen" (that qubit is not coupled to the chain). We will now show that at the next step, this probability is increased. Applying U_2 to Eq. (5) we get

$$U_2W_1U_1|\mathbf{1}\rangle = PU_2PU_1|\mathbf{1}\rangle + \langle\mathbf{N}|U_2PU_1|\mathbf{1}\rangle|\mathbf{N}\rangle + \sqrt{p_1}|\mathbf{N}+1\rangle = PU_2PU_1|\mathbf{1}\rangle + \sqrt{p_2}\{c_2|\mathbf{N}\rangle + d_2|\mathbf{N}+1\rangle\}$$

with

$$c_2 = \langle\mathbf{N}|U_2PU_1|\mathbf{1}\rangle/\sqrt{p_2}, \quad d_2 = \sqrt{p_1}/\sqrt{p_2}, \quad (6)$$

$$p_2 = p_1 + |\langle\mathbf{N}|U_2PU_1|\mathbf{1}\rangle|^2 \geqslant p_1. \quad (7)$$

Applying $W_2 \equiv W(c_2,d_2)$ we get

$$W_2U_2W_1U_1|\mathbf{1}\rangle = PU_2PU_1|\mathbf{1}\rangle + \sqrt{p_2}|\mathbf{N}+1\rangle.$$

Repeating this strategy ℓ times we get

$$\left(\prod_{k=1}^{\ell} W_kU_k\right)|\mathbf{1}\rangle = \left(\prod_{k=1}^{\ell} PU_k\right)|\mathbf{1}\rangle + \sqrt{p_\ell}|\mathbf{N}+1\rangle, \quad (8)$$

where the products are arranged in the time-ordered way. Using the normalization on the right-hand side of Eq. (8) we get

$$p_\ell = 1 - \left\|\left(\prod_{k=1}^{\ell} PU_k\right)|\mathbf{1}\rangle\right\|^2.$$

From Ref. [17] we know that there exists a $\tau>0$ such that for equal time intervals $t_1=t_2=\cdots=t_k=\tau$ we have $\lim_{\ell\to\infty}p_\ell=1$. Therefore the limit of infinite gate operations for Eq. (8) is given by

$$\lim_{\ell\to\infty}\left(\prod_{k=1}^{\ell} W_kU_k\right)|\mathbf{1}\rangle = |\mathbf{N}+\mathbf{1}\rangle. \quad (9)$$

It is also easy to see that

$$\lim_{k\to\infty} d_\ell = 1, \quad \lim_{k\to\infty} c_\ell = 0, \quad (10)$$

which shows that for large k the gates W_k converge to the identity operator, i.e., $\lim_{k\to\infty}W_k=\mathbb{1}_{\mathcal{H}}$.

Equation (9) is the main result of the paper. Together with the fact that W_kU_k leaves the vector $|\mathbf{0}\rangle$ invariant (i.e., $W_kU_k|\mathbf{0}\rangle=|\mathbf{0}\rangle$), this expression can be used to show that an arbitrary and unknown qubit at the first site (1) is transferred to the last site, i.e.,

$$|\psi_{\text{initial}}\rangle \to |\psi_{\text{final}}\rangle = \alpha|\mathbf{0}\rangle + \beta|\mathbf{N}+\mathbf{1}\rangle. \quad (11)$$

This corresponds to an arbitrarily perfect state transfer. As discussed in [21], the convergence of Eq. (9) is asymptotically exponentially fast in the number of gates applied (a detailed analysis of the relevant scaling can be found in [17]). Equation (9) shows that *any imperfect transfer can be made arbitrarily perfect* by only applying two-qubit gates on one end of the quantum chain. On one hand, it avoids restricting the gate times to specific times (as opposed to the scheme in [17]) while requiring no additional memory qubit (as opposed to the scheme in [19]). On the other hand, given the similarities with the convergence proof of the protocols of Ref. [17], the speed of the convergence of the present scheme is expected to be similar to that associated with such protocols. It is worth noticing that the sequence of unitary transformations $W_k \equiv W(c_k,d_k)$ that needs to be applied to the end of the chain to perform the state transfer is only depending on the Hamiltonian H of the quantum chain. The relevant properties can in principle be determined *a priori* by preceding measurements and tomography on the quantum chain (as discussed in Ref. [17]). Furthermore, by performing projective measurements and conditional spin flips on the memory qubit instead, the chain can also be *cooled* (this follows from the convergence theorem in higher excitation sectors given in [19,22]). Even state preparation of arbitrary known states in the first excitation sector is possible by using a time-inversed protocol [23].

Of course, even though perfect quantum state transfer is achieved only in the limit of infinitely many steps, nothing prohibits one to stop the protocol after a finite number of applications of W_k. In this case the resulting communication fidelity will not be optimal but will be still higher than that obtained in those schemes which only exploit direct propagation of the excitations along the chain [9] (see Fig. 3). In realistic scenarios the choice of the maximum number of steps one can use will depend upon the presence of external noise sources that determine the coherence time scales of the system. Since our scheme applies to all Hamiltonians that conserve the number of excitations, it can also be applied to improve the schemes that use engineered couplings [10,11] in the presence of disorder. In this situation the initial fidelity is already quite high, and the required number of operations is even lower.

III. GENERALIZATION

Motivated by the result of the previous sections we now investigate how the protocol may be implemented in practice, well before the realization of the quantum computing blocks from Fig. 1.

The two-qubit gates W_k are essentially rotations in the $\{|01\rangle, |10\rangle\}$ space of the qubits N and $N+1$. It is therefore to be expected that they can be realized (up to a irrelevant

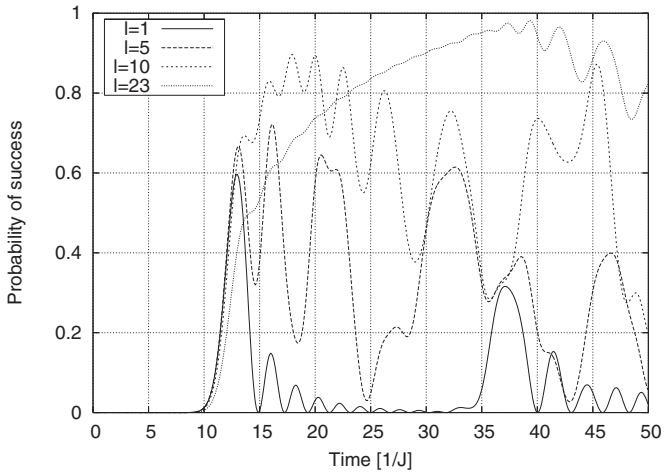

FIG. 3. Even with a finite number ℓ of two-qubit operations, the success probability of the transfer can be improved significantly. We give a numerical example of a Heisenberg chain of length $N=23$, where the gate times are equidistant. In particular, the plots show the transfer fidelity achievable after a time t has elapsed from the initial condition assuming that in the time interval $[0,t]$, ℓ two-qubit operations W_k have being performed at times $t_1=t/\ell$; $t_2=2t/\ell,\ldots,t_{\ell-1}=(\ell-1)t/\ell$, and $t_\ell=t$. For $\ell=1$ only a single two-qubit gate is performed to transfer the information and our result coincides with the original protocol [9]. Already for $\ell=10$ we find an improvement of approximately 50% within the same time scale. For $\ell=N$ we obtain a near-perfect transfer. Notice that starting to extract information from the chain too early causes a small quantum Zeno effect (e.g., see the case $\ell=23$ which for $t\sim 10/J$ is outperformed by the original protocol [9]).

phase) by a switchable Heisenberg or XY type coupling between the Nth and the target qubit. However, in the above, we have assumed that the gates W_k can be applied instantaneously, i.e., in a time scale much smaller than the time scale of the dynamics of the chain. This corresponds to a switchable coupling that is much stronger than the coupling strength of the chain. Here, we numerically investigate if a convergence similar to the above results is still possible when this assumption is not valid. We *do*, however, assume that the switching of the interaction is still describable by an instantaneous switching (i.e., the sudden approximation is valid). This assumption is mainly made to keep the numerics simple. We do not expect qualitative differences when the switching times become finite as long as the time-dependent Hamiltonian is still conserving the number of excitations in the chain. In fact it has recently been shown that the finite switching time can even *improve* the fidelity [16].

We have investigated two types of switching. For the first type, the coupling itself is switchable, i.e.,

$$H(t)=J\sum_{n=1}^{N-1}\sigma_n^-\sigma_{n+1}^+ + \Delta(t)\sigma_N^-\sigma_{N+1}^+ + \text{H.c.}, \tag{12}$$

where $\Delta(t)$ can be 0 or 1. For the second type, the target qubit is *permanently* coupled to the remainder of the chain, but a strong magnetic field on the last qubit can be switched,

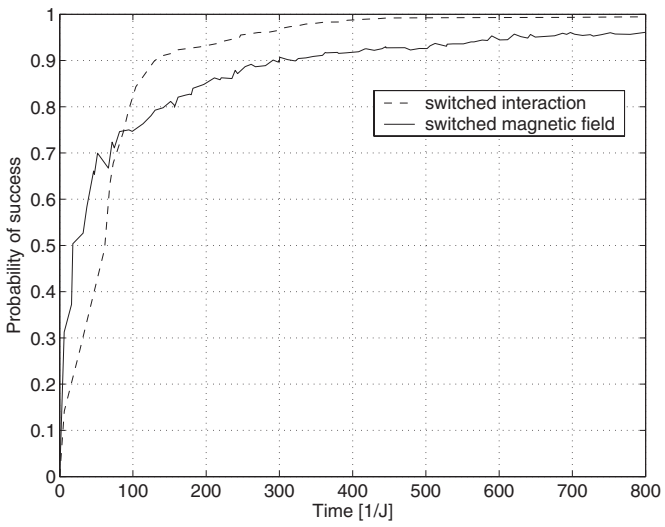

FIG. 4. Numerical example for the convergence of the success probability. Simulated is a quantum chain of length $N=20$ with the Hamiltonian from Eq. (12) (dashed line) and Eq. (13) with $B/J=20$ (solid line). Using the original protocol [9], the same chain would only reach a success probability of 0.63 in the above time interval.

$$H(t)=J\sum_{n=1}^{N}\sigma_n^-\sigma_{n+1}^+ + \text{H.c.} + B\Delta(t)\sigma_{N+1}^z, \tag{13}$$

where again $\Delta(t)$ can be 0 or 1 and $B\gg 1$. This suppresses the coupling between the Nth and $N+1$th qubit due to an energy mismatch.

For the purposes of the present discussion it is sufficient to focus on a specific choice of control pulses $\Delta(t)$: this will not give us the best achievable performances but it will prove our point. Therefore in both cases, we first numerically optimize the times for unitary evolution t_k over a fixed time interval such that the probability amplitude at the Nth qubit is maximal. The algorithm then finds the optimal time interval during which $\Delta(t)=1$ such that the probability amplitude at the target qubit is increased. In some cases the phases are not correct, and switching on the interaction would result in probability amplitude floating back into the chain. In this situation, the target qubit is left decoupled and the chain is evolved to the next amplitude maximum at the Nth qubit. Surprisingly, even when the time scale of the gates is comparable to the dynamics, near-perfect transfer remains possible (Fig. 4). In the case of the switched magnetic field, the achievable fidelity depends on the strength of the applied field. This is because the magnetic field does not fully suppress the coupling between the two last qubits. A small amount of probability amplitude is lost during each time evolution U_k, and when the gain by the gate is compensated by this loss, the total success probability no longer increases.

IV. CONCLUSIONS

We found an optimal strategy for achieving arbitrarily perfect state transfer and state preparation (including cooling) by applying a sequence of two-qubit operations at the

receiving end of a quantum chain. Surprisingly, the gates can be realized by a switchable interaction of the same strength as the chain coupling. By pointing out the rather counterintuitive fact that minimal control at one end enables a large class of quantum many-body systems to be used as a perfect quantum wire, we open up the field of whether a similar result holds for other many-body systems.

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