Entangling two atoms in spatially separated cavities through both photon emission and absorption processes

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We consider a system consisting of a Λ-type atom and a V-type atom, which are individually trapped in two spatially separated cavities that are connected by an optical fiber. We show that an extremely entangled state of the two atoms can be deterministically generated through both photon emission of the Λ-type atom and photon absorption of the V-type atom in an ideal situation. The influence of various decoherence processes such as spontaneous emission and photon loss on the fidelity of the entangled state is also investigated. We find that the effect of photon leakage out of the fiber on the fidelity can be greatly diminished in some special cases. In regard to the effect of spontaneous emission and photon loss from the cavities, we find that the present scheme with high fidelity may be realized under current experiment conditions. We also show that either the Λ- or V-type level configuration can be realized on the same atom by initializing the atomic state, and we show the present scheme is scalable.

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I. INTRODUCTION

Entangled quantum state is one of the essential key ingredients in the implementation of quantum communication and quantum computation [1–4]. Among the kinds of schemes proposed to generate entangled states [5–9], cavity quantum electrodynamic systems (CQEDS), which marry atomic and photonic quantum bits together, are paid more attention because of both its low decoherent rate and promising feasibility to scale up. In recent years, several elegant works based on CQEDS have been done. In [10], a pair of momentum- and polarization-entangled photons is sent to two spatially separated cavities, each of which contains a V-type atom. After the atoms absorb the photons, the photon entanglement is transferred to the atoms. In [11–14], entangled states of two Λ-type atoms individually trapped in spatially separated cavities are generated through the interference of the polarized photons leaking out of the cavities on a beam splitter and co-instantaneous single-photon detections. In these schemes, in order to guarantee the interference effect, several conditions are required. First, the two atoms have to be simultaneously excited or driven. Second, spatial shapes of photon pulses leaking from the cavities should be similar. Third, the two photon paths from the cavities to the beam splitter must be symmetrical. In addition, the success of generating entangled atomic states is probabilistic since the photon-state-projection detection behind the beam splitter is performed. We also notice that the previous schemes are based on either photon emission or photon absorption process. In this paper, we consider a CQED system in which a Λ-type atom and a V-type atom are trapped individually in two spatially separated cavities that are connected by an optical fiber. This setup is closely related to two previous schemes. In [15], Pellizzari proposed a cavity-fiber-cavity system to realize reliable transfer of a quantum state. In that scheme, two three-level Λ atoms that are adiabatically and simultaneously driven by a laser beam are trapped individually in two single-mode cavities connected by an optical fiber. In [16], Serafini *et al.* employed the similar setup in which two-level atoms are trapped in fiber-connected cavities to realize highly reliable swap and entangling gates. We notice that quantum information processes realized in the fiber-connected cavity setup depends on atomic level configurations and laser driving ways. In the present scheme, without using any laser driving, only the Λ-type atom is initially to be pumped in the excited state. Instead of using either photon absorption or photon emission as done in the previous scheme, both the photon emission of the Λ-type atom and the photon absorption of the V-type atom are involved. In this way, an entangled state of the two atoms can deterministically be generated through the atom-field interaction without the photon interference and the photon-state projection detection.

II. MODEL

As shown in Fig. 1, a Λ-type atom with two degenerate ground states $|g_{-1}\rangle$ and $|g_{+1}\rangle$ and a V-type atom with two degenerate excited states $|e_{-1}\rangle$ and $|e_{+1}\rangle$ are trapped in two spatially separated cavities that are linked by an optical fiber. The two atoms interact with local cavity fields, respectively. In recent years, several theoretical schemes based on such

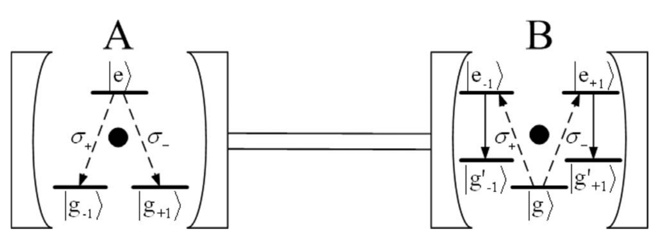

FIG. 1. A Λ-type atom and a V-type atom are trapped in two spatially separated cavities A and B, respectively. The two cavities are linked through an optical fiber.

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fiber-connected cavity systems for realizing quantum computation have been proposed [15–17].

The Hamiltonian of the whole system can be written as [15]

$$H = H_A + H_B + H_F, \tag{1}$$

where

$$H_A = \sum_{j=\pm1} [\omega^f_{A,j} a^\dagger_{A,j} a_{A,j} + \omega^a_A \sigma^z_{A,j} + \lambda_{A,j} a_{A,j} \sigma^\dagger_{A,j} + \text{H.c.}], \tag{2}$$

$$H_B = \sum_{j=\pm1} [\omega^f_{B,j} a^\dagger_{B,j} a_{B,j} + \omega^a_B \sigma^z_{B,j} + \lambda_{B,j} a_{B,j} \sigma^\dagger_{B,j} + \text{H.c.}], \tag{3}$$

$$H_F = \sum_{k=1}^{\infty} \sum_{j=\pm1} [\omega_{j,k} b^\dagger_{j,k} b_{j,k} + \nu_{j,k} b_{j,k} [a^\dagger_{A,j} + (-1)^k e^{i\phi} a^\dagger_{B,j}] + \text{H.c.}]. \tag{4}$$

In the above, $H_{A/B}$ is the energy of the system consisting of atom A/B and the corresponding local cavity fields, and H_F represents the free energy of fiber modes and the interaction between cavity and fiber modes. In Eqs. (2)–(4), $a_{A/B,j}$ and $a^\dagger_{A/B,j}$ are annihilation and creation operators for photons of frequency $\omega^f_{A/B,j}$ and polarization $j(=-1,+1$ corresponding to right and left circular polarizations, respectively) in cavity A/B, and $b_{j,k}$, $b^\dagger_{j,k}$ are annihilation and creation operators for photons of frequency $\omega_{j,k}$ and polarization j in mode k of the fiber field, $\sigma^z_{A,j}=(|e\rangle\langle e|-|g_j\rangle\langle g_j|)_A/2$, $\sigma^z_{B,j}=(|e_j\rangle\langle e_j|-|g\rangle\langle g|)_B/2$, $\sigma^\dagger_{A,j}=(|e\rangle\langle g_j|)_A$, and $\sigma^\dagger_{B,j}=(|e_j\rangle\langle g|)_B$, $\omega^a_{A/B}$ is the frequency of the transition $|e/e_j\rangle \to |g_j/g\rangle$, $\lambda_{A/B,j}$ is the coupling constant of atom A/B with cavity mode j, and $\nu_{j,k}$ the coupling constant of cavity mode j with fiber mode k. Here, we assume that the right and left circular polarization modes in both the cavities have the same frequency ω and the interaction between the atoms and the cavity fields is on resonance. That is, $\omega^f_{A,\pm1}=\omega^f_{B,\pm1}=\omega^a_A=\omega^a_B=\omega$.

In the short fiber limit $(2L\bar{\nu})/(2\pi C) \leqslant 1$, where L is the length of fiber and $\bar{\nu}$ is the decay rate of the cavity fields into a continuum of fiber modes, only resonant modes of the fiber interacts with the cavity modes [16]. For this case, the Hamiltonian H_F may be approximated to

$$H_F = \sum_{j=\pm1} [\omega b^\dagger_j b_j + \nu_j b_j (a^\dagger_{A,j} + a^\dagger_{B,j}) + \text{H.c.}], \tag{5}$$

where the phase $(-1)^k e^{i\phi}$ in Eq. (4) has been absorbed into the annihilation and creation operators of the modes of the second cavity field.

In the interaction picture, the total Hamiltonian now becomes

$$H_I = \sum_{j=\pm1} [\lambda_{A,j} a_{A,j} \sigma^\dagger_{A,j} + \lambda_{B,j} a_{B,j} \sigma^\dagger_{B,j} + \nu_j b_j (a^\dagger_{A,j} + a^\dagger_{B,j}) + \text{H.c.}]. \tag{6}$$

III. THE GENERATION OF ENTANGLED STATES

In this section, we show that an extremely entangled states of the atoms can be deterministically generated in an ideal situation.

To explain the scheme of the generation of entangled states, first, let us see how the entangling process works. Suppose that at the initial time the atom A is pumped in the excited state $|e\rangle_A$ and the atom B is in the ground state $|g\rangle_B$. Through interacting with local cavity fields, the atom A emits either a σ_+ or σ_- polarized photon and then the atom-field system is in the state $|\Psi_1\rangle = \frac{1}{2}(|g_{-1}\rangle|1_{\sigma_+}\rangle + |g_{+1}\rangle|1_{\sigma_-}\rangle)$, where $|1_{\sigma_+}\rangle/|1_{\sigma_-}\rangle$ is a cavity field state with one photon in mode $j=-1/+1$. Assume that the coupling of the cavity modes with the fiber modes is so strong that the emitted photon can be transferred into the fiber modes before being reabsorbed by the atom A. Through the fiber, the photon enters the cavity B. After absorbing the photon, the atom B is excited in either $|e_{-1}\rangle_B$ or $|e_{+1}\rangle_B$. In this way, the entangled state $|\Psi_t\rangle = \frac{1}{\sqrt{2}}(|g_{-1}\rangle_A|e_{-1}\rangle_B + |g_{+1}\rangle_A|e_{+1}\rangle_B)$ can be created. Here, we should stress that to retain this entangled state some local operations on the atom B have to be performed in time, which will be discussed in detail at the end of this section.

Now, let us go on to investigate the generation of the entangled state in detail. The time evolution of the whole system state is governed by the Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi_1(t)\rangle = H_I|\psi_1(t)\rangle. \tag{7}$$

Suppose that at the initial time the system is in the state $|\psi_1(0)\rangle = |e\rangle_A|0\rangle_A|0\rangle_f|0\rangle_B|g\rangle_B$. The state vector $|\psi_1(t)\rangle$ at time t can be expanded as

$$\begin{aligned}
|\psi_1(t)\rangle = {} & d_1|e\rangle_A|0\rangle_A|0\rangle_f|0\rangle_B|g\rangle_B + d_2|g_{-1}\rangle_A|1_{-1}\rangle_A|0\rangle_f|0\rangle_B|g\rangle_B \\
& + d_3|g_{-1}\rangle_A|0\rangle_A|1_{-1}\rangle_f|0\rangle_B|g\rangle_B \\
& + d_4|g_{-1}\rangle_A|0\rangle_A|0\rangle_f|1_{-1}\rangle_B|g\rangle_B \\
& + d_5|g_{-1}\rangle_A|0\rangle_A|0\rangle_f|0\rangle_B|e_{-1}\rangle_B \\
& + d_6|g_{+1}\rangle_A|1_{+1}\rangle_A|0\rangle_f|0\rangle_B|g\rangle_B \\
& + d_7|g_{+1}\rangle_A|0\rangle_A|1_{+1}\rangle_f|0\rangle_B|g\rangle_B \\
& + d_8|g_{+1}\rangle_A|0\rangle_A|0\rangle_f|1_{+1}\rangle_B|g\rangle_B \\
& + d_9|g_{+1}\rangle_A|0\rangle_A|0\rangle_f|0\rangle_B|e_{+1}\rangle_B,
\end{aligned} \tag{8}$$

where $|1_{\pm1}\rangle$ represents either a photon number state with one right (-1) and/or left $(+1)$ circular polarized photon. Upon substituting Eq. (8) in Eq. (7) with the conditions $\lambda_{A,-1}=\lambda_{A,1}=\lambda$, $\lambda_{B,-1}=\lambda_{B,1}=\sqrt{2}\lambda$, and $\nu_{+1}=\nu_{-1}=\nu$, one finds

$$\begin{aligned}
d_1 = {} & \frac{1}{2}[\cos(\sqrt{2}\lambda t) + 1] + \frac{\lambda^2}{2(\lambda^2+\nu^2)} \\
& \times \left[\cos\left(\sqrt{1+\frac{\nu^2}{\lambda^2}}\sqrt{2}\lambda t\right) - 1\right],
\end{aligned} \tag{9}$$

$$\begin{aligned}
d_2 = d_6 = {} & -\frac{i}{2\sqrt{2}}\left[\sin(\sqrt{2}\lambda t)\right. \\
& \left. + \frac{\lambda}{\sqrt{\lambda^2+\nu^2}}\sin\left(\sqrt{1+\frac{\nu^2}{\lambda^2}}\sqrt{2}\lambda t\right)\right],
\end{aligned} \tag{10}$$

$$d_3 = d_7 = \frac{\lambda\nu}{2(\lambda^2+\nu^2)}\left[\cos\left(\sqrt{1+\frac{\nu^2}{\lambda^2}}\sqrt{2}\lambda t\right) - 1\right], \quad (11)$$

$$d_4 = d_8 = \frac{i}{2\sqrt{2}}\left[\sin(\sqrt{2}\lambda t) - \frac{\lambda}{\sqrt{\lambda^2+\nu^2}}\sin\left(\sqrt{1+\frac{\nu^2}{\lambda^2}}\sqrt{2}\lambda t\right)\right], \quad (12)$$

$$d_5 = d_9 = \frac{1}{2\sqrt{2}(\lambda^2+\nu^2)}\left[\nu^2 + \lambda^2\cos\left(\sqrt{1+\frac{\nu^2}{\lambda^2}}\sqrt{2}\lambda t\right) - (\lambda^2+\nu^2)\cos(\sqrt{2}\lambda t)\right]. \quad (13)$$

We notice that $d_1=d_2=d_3=d_4=0$, $d_5=d_9=1/\sqrt{2}$, and the system is in the state $|\psi_1\rangle=\frac{1}{\sqrt{2}}(|g_{-1}\rangle_A|e_{-1}\rangle_A+|g_{+1}\rangle_A|e_{+1}\rangle_A)|0\rangle_A|0\rangle_f|0\rangle_B$ when $\sqrt{2}\lambda t=m\pi$ $(m=1,3,5,\ldots)$ and $\sqrt{1+\frac{\nu^2}{\lambda^2}}=n$ $(n=2,4,6,\ldots)$. In this state, the atoms are completely separated from the cavity fields and in the extremely entangled state $|\Psi_t\rangle$.

In Eqs. (9)–(13), one may also find that if $\nu\gg\lambda$ the conditions $d_1=d_2=d_3=d_4=0$ and $d_5=d_9=1/\sqrt{2}$ can also be satisfied at the time $\sqrt{2}\lambda t=m\pi$ $(m=1,3,5,\ldots)$. It means that the requirement $\sqrt{1+\frac{\nu^2}{\lambda^2}}=n$ $(n=2,4,6,\ldots)$ can be loosed in the limit $\nu\gg\lambda$. In order to make the thing more clear, let us introduce the normal modes [16]

$$c_{0,j} = \frac{a_{A,j}-a_{B,j}}{\sqrt{2}}, \quad (14)$$

$$c_{\pm,j} = \frac{a_{A,j}+a_{B,j}\pm 2b_j}{\sqrt{2}}. \quad (15)$$

In terms of these normal modes, the Hamiltonian (6) can be rewritten as

$$H_I = \sum_{j=\pm1}\left(\lambda_{A,j}\frac{e^{i\sqrt{2}\nu_j t}c_{-,j}+e^{-i\sqrt{2}\nu_j t}c_{+,j}+\sqrt{2}c_{0,j}}{2}\sigma_{A,j}^{\dagger} + \lambda_{B,j}\frac{e^{i\sqrt{2}\nu_j t}c_{-,j}+e^{-i\sqrt{2}\nu_j t}c_{+,j}-\sqrt{2}c_{0,j}}{2}\sigma_{B,j}^{\dagger} + \text{H.c.}\right). \quad (16)$$

As noted in Eq. (16), the normal mode $c_{0,j}$ is resonant with the atomic transitions from both $|e\rangle$ to $|g_j\rangle$ of the atom A and $|g\rangle$ to $|e_j\rangle$ of the atom B, but the normal modes $c_{\pm,j}$ are off resonant. In the limit $\nu_j\gg\lambda_{A,j},\lambda_{B,j}$, the off-resonant modes can be safely neglected [16]. In this case, the Hamiltonian (16) becomes

$$H_I = \frac{1}{\sqrt{2}}\sum_{j=\pm1}[\lambda_{A,j}c_{0,j}\sigma_{A,j}^{\dagger} - \lambda_{B,j}c_{0,j}\sigma_{B,j}^{\dagger} + \text{H.c.}]. \quad (17)$$

In the Hamiltonian (17), the fiber modes are completely depressed since the resonant modes c_0 do not contain the fiber modes and the original system is modeled by a system consisting of two atoms interacting with two resonant modes in one cavity. For the initial condition $|\psi_2(0)\rangle=|e\rangle_A|g\rangle_B|0\rangle_c$, where $|0\rangle_c$ is the vacuum state of the normal mode c_0, the solution of the Schrödinger equation (7) with the Hamiltonian (17) can be found to be

$$|\psi_2(t)\rangle = \tilde{d}_1|e\rangle_A|0\rangle_c|g\rangle_B + \tilde{d}_2|g_{-1}\rangle_A|1_{-1}\rangle_c|g\rangle_B + \tilde{d}_3|g_{+1}\rangle_A|1_{+1}\rangle_c|g\rangle_B + \tilde{d}_4|g_{-1}\rangle_A|0\rangle_c|e_{-1}\rangle_B + \tilde{d}_5|g_{+1}\rangle_A|0\rangle_c|e_{+1}\rangle_B, \quad (18)$$

where $|1_{\pm1}\rangle_c$ is a state in which there is either one right (-1) and/or left $(+1)$ circular polarized photon in the normal mode c_0. Under the condition $\lambda_{A,-1}=\lambda_{A,+1}=\lambda$ and $\lambda_{B,-1}=\lambda_{B,+1}=\sqrt{2}\lambda$, substituting Eq. (18) in Eq. (7) with the Hamiltonian (17), we find the expansion coefficients in Eq. (18),

$$\tilde{d}_1 = \frac{1+\cos(\sqrt{2}\lambda t)}{2}, \quad \tilde{d}_2 = \tilde{d}_3 = \frac{-i\sin(\sqrt{2}\lambda t)}{2},$$

$$\tilde{d}_4 = \tilde{d}_5 = \frac{1-\cos(\sqrt{2}\lambda t)}{2\sqrt{2}}. \quad (19)$$

From Eqs. (18) and (19), one can see that at the moments $t=m\pi/\sqrt{2}\lambda$ $(m=1,3,5,\ldots)$ the system is in the state $|\Psi_t\rangle|0\rangle_c$ in which the two atoms are separated from the normal modes and the extremely entangled state of the atoms $|\Psi_t\rangle$ is achieved. This result depends on the condition $\nu\gg\lambda$. In order to check the approximation validity, in Fig. 2, we show the fidelities $F_1=|\langle 0_A,0_F,0_B|\langle\Psi_t|\psi_1(t)\rangle|^2$, where $|0_A,0_F,0_B\rangle$ is the vacuum state for photons in both the cavities and the fiber, and $F_2=|_c\langle 0|\langle\Psi_t|\psi_2(t)\rangle|^2$ that corresponds to the limit $\nu\to\infty$. As shown in Fig. 2, the results related to the Hamiltonians (6) and (17) become nearly the same and the approximation condition $\nu\gg\lambda$ holds very well when $\nu\geqq 10\lambda$.

From the above results, we see that to generate the extremely entangled state of the atoms it is needed to choose the right interaction time and the right ratio of ν to λ when ν is comparable to λ, but it is needed only to control the interaction time when $\nu\gg\lambda$. At the time $\sqrt{2}\lambda t=m\pi$ $(m=1,3,5,\ldots)$, the atoms evolve into the extremely entangled state. To hold and protect the entangled state at that moment, we must interrupt the interaction of the atom B with the cavity fields and transfer the atoms to the stable states in time. For completing these local operations, as shown in Fig. 1, one may apply a π-polarized light to the transition between $|e_{\pm1}\rangle_B$ and $|g'_{\pm1}\rangle_B$, and transfer the entangled state to the stable one: $|\psi_s\rangle=\frac{1}{\sqrt{2}}(|g_{-1}\rangle_A|g'_{-1}\rangle_B+|g_{+1}\rangle_A|g'_{+1}\rangle_B)$.

Finally, let us briefly discuss how to realize either the Λ- or V-type level configuration on the same atom and make the scheme be scalable. In Fig. 3, the hyperfine structure of $^2S_{1/2}$ and $^2P_{1/2}$ of ^{87}Rb is shown. Suppose that the cavity supports two modes $\sigma^{\dagger}$ and σ^{-} of the electromagnetic field, corresponding the transition between the $^2P_{1/2}$, $F=1$ level and the $^2S_{1/2}$, $F=1$ level. If the atom is initially prepared in the $^2P_{1/2}$, $F=1$, $M_F=0$ hyperfine level, the atom interacting with the cavity modes has the Λ-type level configuration, as shown in Fig. 3(a). If the atom is initially prepared in the $^2S_{1/2}$, $F=1$, $M_F=0$ hyperfine level, the atom interacting with the cavity modes has the V-type level configuration, as shown in Fig.

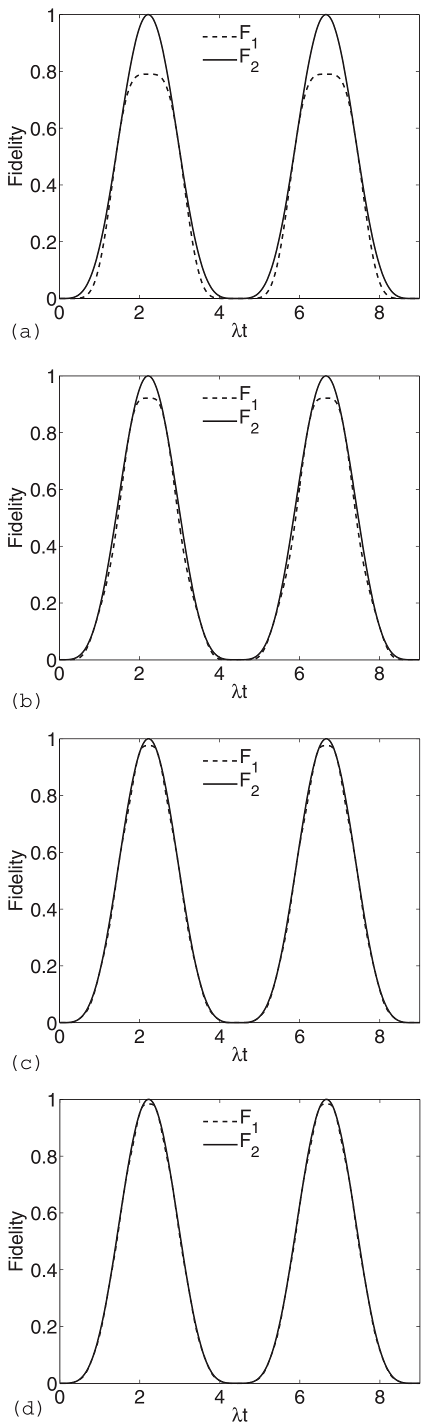

FIG. 2. The fidelity to obtain the extremely entangled state $|\Psi_t\rangle$ versus time. The solid lines represent the result of F_2 with the limit of $\nu \gg \lambda$. The dashed lines represent the results when ν is comparable to λ, and take the values (a) $\nu=\sqrt{8}\lambda$; (b) $\nu=\sqrt{24}\lambda$; (c) $\nu=\sqrt{80}\lambda$; (d) $\nu=\sqrt{120}\lambda$.

3(b). Therefore, either the Λ- or V-type level configuration can be realized on the same atom by initializing the atomic state. To scale up the scheme, we may arrange the identical two-mode cavities in a line, along which two neighbor cavities are connected through an optical fiber. Suppose that each of the cavities along the line contains one ^{87}Rb atom and

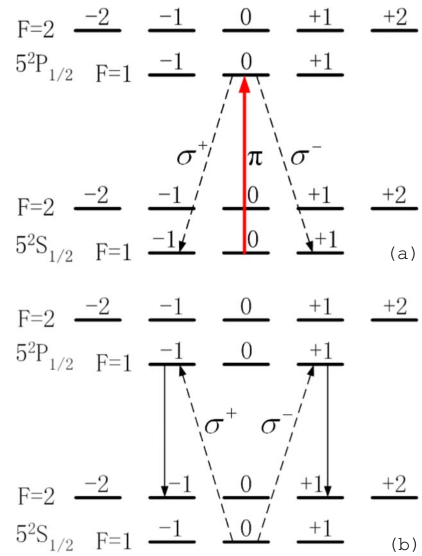

FIG. 3. (Color online) (a) Λ-type level scheme of the atom (b) V-type level scheme of the atom.

laser beams can respectively address the atoms trapped in different cavities to initialize the atomic states to form the combined Λ- and V-type level configuration and protect the resulting entangled state. This atom-cavity-fiber coupled system could be realized by use of the integrated fiber-coupled cavity technique with atom chips [20,21]. By use of the Stark shift technique [22], one may switch on or off the interaction between the atom and the cavity modes. In this way, addressing any two atoms trapped in two distant cavities by laser beams and turning off the interaction between atoms and cavities except the addressed atoms and the related cavities by the Stark shift, we can put the two addressed atoms in the extremely entangled state by employing the above-mentioned approach. Therefore, the present scheme is a scalable entanglement generation scheme in which the entangled state of any two qubits in the qubit line network can be generated.

IV. EFFECTS OF SPONTANEOUS EMISSION AND PHOTON LOSS

In this section, we investigate the influence of spontaneous emission and photon leakage on the generation of atomic entangled states. The master equation for the density matrix of the whole system is

$$\dot{\rho} = -i[H_I,\rho] - \sum_{j=-1,1} \frac{\gamma_{A,j}}{2}(a_{A,j}^\dagger a_{A,j}\rho - 2a_{A,j}\rho a_{A,j}^\dagger + \rho a_{A,j}^\dagger a_{A,j})$$

$$- \sum_{j=-1,1} \frac{\gamma_{B,j}}{2}(a_{B,j}^\dagger a_{B,j}\rho - 2a_{B,j}\rho a_{B,j}^\dagger + \rho a_{B,j}^\dagger a_{B,j})$$

$$- \sum_{j=-1,1} \frac{\gamma_{f,j}}{2}(b_j^\dagger b_j\rho - 2b_j\rho b_j^\dagger + \rho b_j^\dagger b_j)$$

$$- \sum_{j=-1,1} \frac{\kappa_{A,j}}{2}(\sigma_{A,j}^\dagger \sigma_{A,j}\rho - 2\sigma_{A,j}\rho\sigma_{A,j}^\dagger + \rho\sigma_{A,j}^\dagger \sigma_{A,j})$$

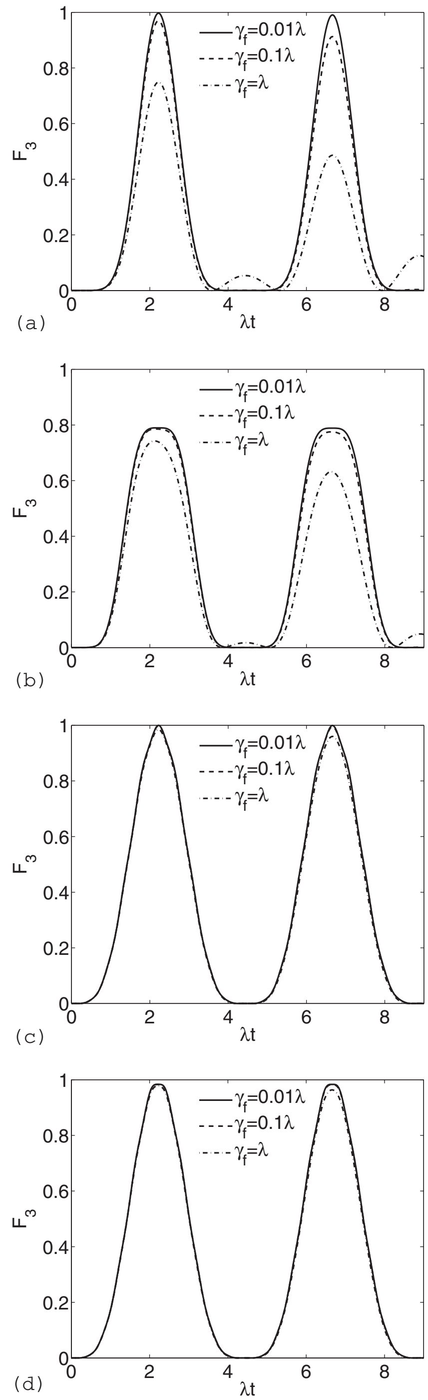

FIG. 4. The influence of fiber photon leakage on the fidelity to obtain the extremely entangled state with $\kappa=\gamma_c=0$. (a) $\nu=\sqrt{3}\lambda$; (b) $\nu=2\sqrt{2}\lambda$; (c) $\nu=\sqrt{99}\lambda$; (d) $\nu=\sqrt{120}\lambda$.

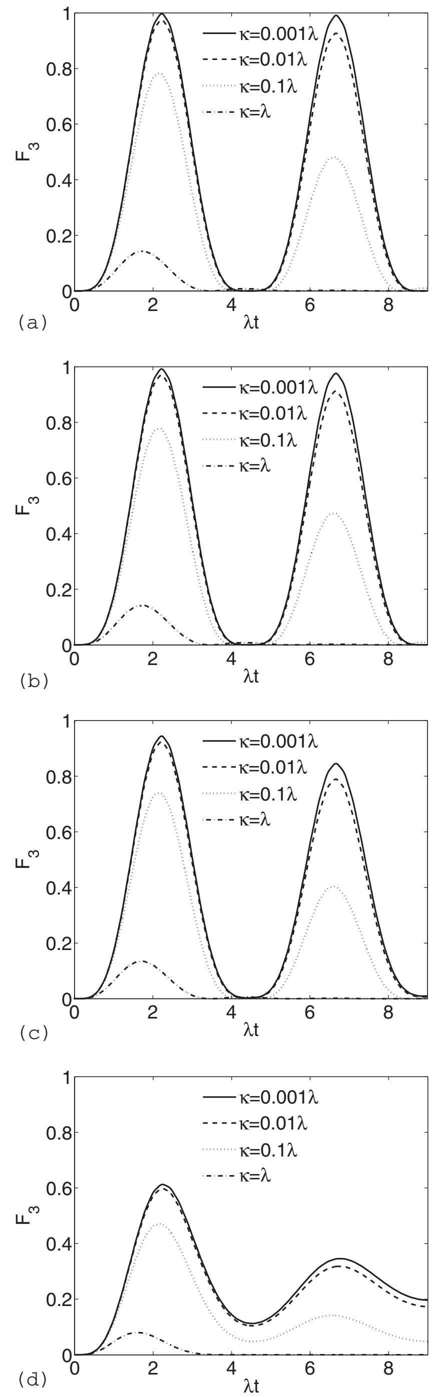

FIG. 5. The influence of spontaneous emission and cavity leakage on the fidelity to obtain the extremely entangled state with $\nu=\sqrt{399}\lambda$, $\gamma_f=0$, and different values of γ_c and κ. (a) $\gamma_c=0.001\lambda$; (b) $\gamma_c=0.01\lambda$; (c) $\gamma_c=0.1\lambda$; (d) $\gamma_c=\lambda$.

$$-\sum_{j=-1,1}\frac{\kappa_{B,j}}{2}(\sigma_{B,j}^{\dagger}\sigma_{B,j}\rho-2\sigma_{B,j}\rho\sigma_{B,j}^{\dagger}+\rho\sigma_{B,j}^{\dagger}\sigma_{B,j}), \qquad (20)$$

where H_I is given by Eq. (6), and $\gamma_{A,j}$, $\gamma_{B,j}$ and $\gamma_{f,j}$ are the decay rates for photons in mode j of cavities A and B, and fiber, respectively, and $\kappa_{A,j}$ and $\kappa_{B,j}$ are spontaneous emission rates that are related to the decay channel of atom A, $|e\rangle\to|g_j\rangle$, and the decay channel of atom B, $|e_j\rangle\to|g\rangle$, respectively. In solving the master equation, we always choose $\lambda_{B,\pm1}=\sqrt{2}\lambda_{A,\pm1}=\sqrt{2}\lambda$, and $\kappa_{A,\pm1}=\kappa_{B,\pm1}=\kappa$, $\gamma_{f,\pm1}=\gamma_f$, and $\gamma_{A,\pm1}=\gamma_{B,\pm1}=\gamma_c$. By use of the solution of Eq. (20), the fidelity $F_3=\langle 0_A,0_F,0_B|\langle\Psi_t|\rho(t)|\Psi_t\rangle|0_A,0_F,0_B\rangle$ can be calculated. In Fig. 4, the fidelity F_3 is plotted for the cases with $\kappa=\gamma_c=0$, but with different values of ν and γ_f.

As shown in Figs. 4(c) and 4(d), even for the case with $\gamma_f=\lambda$, the effect of photon leakage out of the fiber can be greatly depressed when the cavity-fiber coupling strength ν is much larger than the cavity-atom coupling strength λ. This result can be easily understood. From the discussion in the preceding section, there is only the resonant normal mode c_0 that plays the role when $\nu \gg \lambda$. According to Eq. (14), the resonant normal mode is not involved with the fiber mode. In this limit, therefore, the fiber mode is really not excited and is always kept in the vacuum state. On the other hand, the condition $\nu \gg \lambda$ means that the transmission time of photons through the fiber becomes much shorter than the interaction time of the atoms with the cavity fields and then the fiber photon is equivalently kept in the vacuum state in the whole process. This reason clearly explains why the influence of photon loss can be diminished in the limit $\nu \gg \lambda$.

From Eqs. (9)–(13), $d_3 = d_7 = 0$ at $t = m\pi/\sqrt{2}\lambda$ $(m=1, 3, 5, \ldots)$ if the condition $\sqrt{1+\frac{\nu^2}{\lambda^2}} = n$ $(n=2,4,6,\ldots)$ is satisfied. It means that all the terms associated with the basic vectors with one fiber photon in Eq. (8) disappear under this condition. In this case, therefore, the fiber field in the state vector (8) is in the vacuum state. Therefore, the appropriate choice of the ratio ν/λ may be useful to obtain a higher fidelity of the entangled state even if the condition $\nu \gg \lambda$ is not well satisfied. In Figs. 4(a) and 4(b), the results with $\sqrt{1+\frac{\nu^2}{\lambda^2}} = 2$ and $\sqrt{1+\frac{\nu^2}{\lambda^2}} = 3$ are shown. It is observed that the former result is obviously better than the latter one when $\gamma_f < 0.1\lambda$.

Finally, let us discuss the influence of spontaneous emission and cavity leakage. In Fig. 5, the fidelity F_3 is shown for the cases with $\nu = \sqrt{399}\lambda$ and $\gamma_f = 0$, and different values of γ_c and κ. Comparing these figures, we find that the atomic spontaneous emission has a stronger influence on the fidelity than the cavity photon leakage does. To obtain the fidelity higher than 0.97, one should keep the decay rates $\kappa \leqslant 0.01\lambda$ and $\gamma_c \leqslant 0.1\lambda$. In recent experiments [18,19], for an optical cavity with the wavelength region 630 nm–850 nm, the condition ($\lambda/2\pi = 750$ MHz, $\kappa/2\pi = 2.62$ MHz, $\gamma_c/2\pi = 3.5$ MHz) is achievable. With these parameters and neglecting the defect of the coupling between the cavity and the fiber, it seems that the present entanglement generation scheme with a high fidelity larger than 0.98 could be feasible in an experiment. However, we should emphasize that the fidelity will be certainly reduced in a real situation because the coupling between the cavity and the fiber is not ideal in the current experiment [20].

V. CONCLUSION

In this paper, we propose a scheme to generate an entangled state of two atoms trapped in two spatially separated cavities that are connected by an optical fiber. We show that an extremely entangled state of the atoms can be deterministically generated if the ratio of the coupling constant of the cavity field with the atoms to the coupling constant of the cavity field with the fiber mode satisfies a certain condition. The scheme has three features. First, it is based on both the photon emission of a Λ-type atom and the photon absorption of a V-type atom, either of which level configuration can be realized on the same atom by initializing the atomic state. Therefore, in contrast to the previous schemes [10–12], either a pair of entangled photons or the photon interference process is not required. Second, since the present scheme is based on the interaction dynamics of atoms with cavity fields and then the co-instantaneous single photon detection is not required, the generation of entangled states is deterministic. Third, in contrast to the previous schemes in which manipulations such as excitation, photon emission, or absorption have to be performed simultaneously on all of the atoms under consideration, such manipulations are performed, respectively, on each of the atoms in the present scheme. This may reduce difficulties resulting from manipulations performed at the same time on two atoms. The influence of various decoherence processes such as spontaneous emission and photon loss on the generation of entangled states is also investigated. We find that the effect of photon leakage out of the fiber can be greatly depressed and even ignored if the coupling of the fiber mode with the cavity fields is much stronger than the coupling of the atoms with the cavity fields. If the condition is not well satisfied, the appropriate ratio of the coupling constant of the cavity field with the atoms to the coupling constant of the cavity field with the fiber mode can also ensure a high fidelity of generating entangled states. As regards the effect of photon loss out of the cavities and spontaneous emission of the atoms, we find that the implementation of the present scheme may be within the scope of the current techniques. Moreover, by appropriately arranging the cavity-fiber-atom coupled systems and addressing the atoms, we show that the present scheme is scalable.

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