Irreversible decay of nonlocal entanglement via a reservoir of a single degree of freedom

A. Vaglica* and G. Vetri

Dipartimento di Scienze Fisiche ed Astronomiche and CNISM, Università di Palermo via Archirafi 36, 90123 Palermo, Italy

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Recently, it has been realized that nonlocal disentanglement may take a finite time as opposite to the asymptotic decay of local coherences. We find in this paper that a sudden irreversible death of entanglement takes place in a two atom optical Stern-Gerlach model. In particular, the one degree nondissipative environment here considered suddenly destroys the initial entanglement of any Bell's states $|\phi^{\pm}\rangle$ superposition.

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I. INTRODUCTION

Superposition principle is a basic feature of quantum mechanics (QM). When applied to composite systems superposition leads to nonlocal coherences and quantum correlations (entanglement) which have been extensively studied in the last years, and are even expected of central interest for physics to come [1].

As it is known, quantum coherences and entanglement are potential resources in various quantum information processes, even if they suffer any sort of environmental action which represents a serious obstacle towards these applications. On the other hand, it is just this fragility with respect to the environment, that is, the propensity to disperse information throughout inaccessible degrees of freedom that explicates the quantum-classical transition [2].

An interesting point raised in [3–7] concerns the possible finite time disentanglement for bipartite systems, as opposite to the usual local decoherence asymptotic time. An experimental evidence of this peculiar trait of entanglement has been reported recently by Almeida *et al.* [8]. This issue has been analyzed [4] in a simple and realistic model where two initially entangled two-level atoms separately interact with the multimode vacuum noise of two distinct cavities. The authors find out that the nonlocal decoherence may take place suddenly or at least as fast as the sum of the normal single atom decay rates. This sudden death of entanglement has been analyzed also for two Jaynes-Cummings (JC) atoms [9]. Also in this case the dynamics of the entanglement between the atomic internal variables shows different peculiarities for different initial states, with possible sudden decays that are however followed by periodic revivals, due to recovery of information by the system, being the cavities lossless.

In this paper we wish to investigate if the separate nondissipative interaction of each two-level atom with one only degree of freedom, but of continuous variables, may play the role of the interaction with a reservoir leading to an irreversible disentanglement of the bipartite system.

As is well known, the optical Stern-Gerlach (OSG) model [10] gives the opportunity of modifying the JC model by including the interaction between the internal and the external atomic dynamics via the electromagnetic mode of the cavity field, so actually providing the coupling of each qubit to only one degree of freedom. It has already been shown that this nondissipative coupling leads to a decoherence in the dynamics of a single atom [11–13] and also affects the entanglement in the internal dynamics of two atoms that successively interact with the same cavity [14]. As we will show in what follows, the same interaction gives rise to a disentanglement of the bipartite system which exhibits strong analogies with the disentanglement generated by the vacuum noise [4,8].

II. OPTICAL STERN-GERLACH MODEL FOR TWO ATOMS

Let us consider two identical isolated two-level atoms, A and B, respectively, crossing two distinct ideal cavities, a and b (in a direction orthogonal to the cavity axis), as in the OSG model. These two uncoupled subsystems are described by the total Hamiltonian

$$\hat{H} = \hat{H}_{Aa} + \hat{H}_{Bb}, \tag{1}$$

where, in resonant conditions and in the rotating wave approximation for both atoms,

$$\hat{H}_{Aa} = \frac{\hat{p}_A^2}{2m} + \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \hat{S}_z^A + \frac{1}{2}\right) + \hbar\varepsilon \sin(k\hat{x}_A)(\hat{a}^{\dagger}\hat{S}_-^A + \hat{S}_+^A\hat{a}), \tag{2a}$$

$$\hat{H}_{Bb} = \frac{\hat{p}_B^2}{2m} + \hbar\omega\left(\hat{b}^{\dagger}\hat{b} + \hat{S}_z^B + \frac{1}{2}\right) + \hbar\varepsilon \sin(k\hat{x}_B)(\hat{b}^{\dagger}\hat{S}_-^B + \hat{S}_+^B\hat{b}). \tag{2b}$$

To simplify the notation we will sometimes use the indices A and B to indicate the two subsystems Aa and Bb, respectively. The cavity fields are described, as usually, by the bosonic operators $\hat{a}$, $\hat{a}^{\dagger}$ and $\hat{b}$, $\hat{b}^{\dagger}$, and the two cavity frequencies $\omega = ck$, as well as the coupling constants ε for the two subsystems, are assumed to be the same. The $1/2$ spin operators $\hat{S}_{\pm}^j$, $\hat{S}_z^j$ account for the internal dynamics of jth atom, while the conjugate variables $\hat{x}_j$ and $\hat{p}_j$ describe the external motion of the same atom along the axis direction of the corresponding cavity $(j = A, B)$.

When the initial uncertainties Δx_j of the atomic wave packets are sufficiently small with respect to the wavelength $\lambda = 2\pi/k$ of both modes a and b, the Hamiltonian (1) as-

*vaglica@fisica.unipa.it

sumes the form (see, for example, Ref. [15] for one atom case)

$$\hat{H} = \frac{\hat{p}_A^2}{2m} + \frac{\hat{p}_B^2}{2m} + \hbar\omega(\hat{N}_A + \hat{N}_B) + \hbar\hat{\Omega}_x^A\hat{\mu}_x^A + \hbar\hat{\Omega}_x^B\hat{\mu}_x^B, \quad (3)$$

where

$$\hat{\mu}_x^A = \frac{\hat{a}^\dagger\hat{S}_-^A + \hat{S}_+^A\hat{a}}{2\sqrt{\hat{N}_A}}, \quad \hat{N}_A = \hat{a}^\dagger\hat{a} + \hat{S}_z^A + \frac{1}{2} \quad (4)$$

are constant of motion. Similarly for the subsystem B. $\hat{\Omega}_x^j \equiv 2\varepsilon k\sqrt{\hat{N}_j}\hat{x}_j$ is the (operatorial) Rabi frequency depending on the atomic position (with respect to a nodal point of the related cavity mode function) and on the excitation number of the jth subsystem. Using previous results [15] for the one atom case, it is easy to show that the evolution operator may be written as

$$\begin{aligned}\hat{U}(t,0) = e^{-i\hat{H}t/\hbar} = \exp\left\{-\frac{2it}{\hbar}m\hat{a}_N^A\hat{\mu}_x^A\hat{x}_A\right\} \\ \times\exp\left\{-\frac{it}{2m\hbar}\hat{p}_A^2\right\}\exp\left\{\frac{it^2}{\hbar}\hat{a}_N^A\hat{\mu}_x^A\hat{p}_A\right\} \\ \times\exp\left\{-\frac{2it}{\hbar}m\hat{a}_N^B\hat{\mu}_x^B\hat{x}_B\right\}\exp\left\{-\frac{it}{2m\hbar}\hat{p}_B^2\right\} \\ \times\exp\left\{\frac{it^2}{\hbar}\hat{a}_N^B\hat{\mu}_x^B\hat{p}_B\right\}e^{-i\vartheta_0(t)(\hat{N}_A+\hat{N}_B)}, \quad (5)\end{aligned}$$

where

$$\hat{a}_N^j = a_0\sqrt{\hat{N}_j}, \quad a_0 = \frac{\varepsilon\hbar k}{m}, \quad \vartheta_0(t) = \omega t + ma_0^2t^3/(6\hbar), \quad (6)$$

and it has been assumed that each atom enters their own cavity at the same time.

III. REDUCED DENSITY OPERATOR FOR THE QUBITS

In this section we will analyze for the two different configurations considered in Ref. [9], the time evolution of the initial entanglement between the two qubits under the effects of the interaction of each qubit with its own cavity and, consequently, of the coupling with its own translational motion. We will see that, differently from the usual JC model, in this case the elements of the reduced density matrices that describe the atomic internal dynamics show damped oscillations as a consequence of the OSG effect which causes a splitting of the atomic packets. As we will see later, this damping affects the entanglement decay whose rate is actually related to the phase space distance of the scattered packets. The section ends with a brief account of the results for the one atom OSG model, which are useful to explicate relations and differences between the decay rates of nonlocal entanglement and local coherences.

A. Two atoms

Let us first suppose that the configuration of the entire system at time $t=0$ may be written as

$$|\psi(0)\rangle = |\varphi_A(0)\rangle|\varphi_B(0)\rangle|\chi(0)\rangle, \quad (7)$$

where $|\varphi_A\rangle$, $|\varphi_B\rangle$ describe the translational dynamics along the cavity axes of atoms A and B, while

$$|\chi(0)\rangle = [\cos\gamma|+-\rangle + \sin\gamma|-+\rangle]|00\rangle, \quad 0 \leqslant \gamma \leqslant \frac{\pi}{2} \quad (8)$$

shows that the two qubits are initially in a superposition of the Bell's states usually denoted $|\psi^\pm\rangle$, and the two cavities are in the vacuum state. The ket $|+-\rangle$ indicates that the qubit A is in the upper state and the qubit B is in the lover state, and so on. Using the dressed states

$$|\chi_j^\pm\rangle = \frac{1}{\sqrt{2}}[|+0\rangle_j \pm |-1\rangle_j], \quad j = A,B, \quad (9)$$

state (8) assumes the form

$$\begin{aligned}|\chi(0)\rangle = \frac{1}{\sqrt{2}}\{\cos\gamma[|\chi_A^+\rangle + |\chi_A^-\rangle]|-0\rangle_B \\ + \sin\gamma[|\chi_B^+\rangle + |\chi_B^-\rangle]|-0\rangle_A\}. \quad (10)\end{aligned}$$

Applying the evolution operator (5) to the initial state (7) and using Eq. (10), one obtains the following expression for the state of the entire system at time t:

$$\begin{aligned}|\psi(t)\rangle = \frac{\cos\gamma}{\sqrt{2}}e^{-i\vartheta_0(t)}|\varphi_B(t)\rangle[|\phi_A^+(t)\rangle|\chi_A^+\rangle + |\phi_A^-(t)\rangle|\chi_A^-\rangle]|-0\rangle_B \\ + \frac{\sin\gamma}{\sqrt{2}}e^{-i\vartheta_0(t)}|\varphi_A(t)\rangle[|\phi_B^+(t)\rangle|\chi_B^+\rangle + |\phi_B^-(t)\rangle|\chi_B^-\rangle]|-0\rangle_A, \quad (11)\end{aligned}$$

where we have set

$$\begin{aligned}|\phi_j^\pm(t)\rangle = \exp\left\{\mp\frac{it}{\hbar}ma_0\hat{x}_j\right\}\exp\left\{-\frac{it}{2m\hbar}\hat{p}_j^2\right\} \\ \times\exp\left\{\pm\frac{it^2}{2\hbar}a_0\hat{p}_j\right\}|\varphi_j(0)\rangle, \quad (12)\end{aligned}$$

$$|\varphi_j(t)\rangle = \exp\left\{-i\frac{\hat{p}_j^2t}{2m\hbar}\right\}|\varphi_j(0)\rangle, \quad (13)$$

and the following relations have been used:

$$\begin{aligned}\hat{N}_j|\chi_j^\pm\rangle = |\chi_j^\pm\rangle, \quad \hat{N}_j|-0\rangle_j = 0, \\ \hat{\mu}_x^j|\chi_j^\pm\rangle = \pm\frac{1}{2}|\chi_j^\pm\rangle, \quad \hat{\mu}_x^j|-0\rangle_j = 0. \quad (14)\end{aligned}$$

If the initial spatial distributions of the two atoms are Gaussian functions of minimum uncertainty,

$$\varphi_j(x_j,0) = \left(\frac{1}{\sqrt{2\pi}\Delta x_0}\right)^{1/2}\exp\left\{-\frac{(x_j - x_{j,0})^2}{4\Delta x_0^2}\right\}, \quad (15)$$

centered in $x_{j,0}$ and with the same spread Δx_0, the x representation of Eq. (12) gives

$$\phi_j^{\pm}(x_j,t) = \left[\frac{\Delta x_0}{\sqrt{2\pi}\beta(t)}\right]^{1/2} \exp\left\{\mp \frac{it}{\hbar} m a_0 x_j\right\} \times \exp\left\{-\frac{[x_j - x_j^{\pm}(t)]^2}{4\beta(t)}\right\}, \qquad (16)$$

where

$$x_j^{\pm}(t) = x_{j,0} \mp a_0 t^2/2, \quad \beta(t) = \Delta x_0^2 + i\hbar t/(2m). \qquad (17)$$

As Eq. (13) shows, $|\varphi_j(t)\rangle$ describes the translational free motion of the jth atom, while the x representation of $|\phi_j^{\pm}(t)\rangle$ given by Eq. (16) accounts for the well-known splitting of the incoming wave packet in the OSG effect.

Since we are interested in the entanglement of the two qubits, we will identify the internal atomic degrees as the system of interest, while the cavities and the external dynamics will serve as environment. As a consequence, the density operator $|\psi(t)\rangle\langle\psi(t)|$ must be traced on the cavity fields and on the atomic translation variables. Taking into account the normalization of kets (12) and (13), and using relation (9) we obtain

$$\begin{aligned}\rho^{AB}(t) &= \mathrm{Tr}_{transl,field}(|\psi(t)\rangle\langle\psi(t)|) \\ &= a_1|+-\rangle\langle+-| + a_2|-+\rangle\langle-+| + a_3|+-\rangle\langle-+| \\ &\quad + a_3^*|-+\rangle\langle+-| + a_4|--\rangle\langle--|, \end{aligned} \qquad (18)$$

where

$$a_1 = \frac{1}{2}\cos^2\gamma[1 + \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle],$$

$$a_2 = \frac{1}{2}\sin^2\gamma[1 + \mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle],$$

$$a_3 = \frac{1}{4}\cos\gamma\sin\gamma[\langle\varphi_A(t)|\phi_A^+(t)\rangle + \langle\varphi_A(t)|\phi_A^-(t)\rangle] \times [\langle\phi_B^-(t)|\varphi_B(t)\rangle + \langle\phi_B^+(t)|\varphi_B(t)\rangle],$$

$$a_4 = \frac{1}{2} - \frac{1}{2}\cos^2\gamma\,\mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle - \frac{1}{2}\sin^2\gamma\,\mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle. \qquad (19)$$

By using Eqs. (13) and (16), it is possible to show that the following scalar products hold [12,15]:

$$\langle\phi_j^-(t)|\phi_j^+(t)\rangle = \exp\{-i\Omega_{j,0}t\}e^{-\Gamma(t)}, \qquad (20a)$$

$$\langle\varphi_j(t)|\phi_j^{\pm}(t)\rangle = \exp\left\{i\frac{m a_0^2 t^3}{4\hbar} \mp i\frac{\Omega_{j,0}t}{2}\right\}e^{-\Gamma(t)/4}, \qquad (20b)$$

where

$$\Gamma(t) = \frac{\delta x(t)^2}{8\Delta x_0^2} + \frac{\delta p(t)^2}{8\Delta p_0^2}, \qquad (21)$$

Δp_0 is the initial uncertainty of the momentum distributions for both atoms, and

$$\delta x(t) = -a_0 t^2, \quad \delta p(t) = -2m a_0 t, \quad \Omega_{j,0} = 2m a_0 x_{j,0}/\hbar. \qquad (22)$$

In the following we will assume that $x_{A,0}=x_{B,0}\equiv x_0$, which implies

$$\Omega_{A,0} = \Omega_{B,0} \equiv \Omega_0 = \frac{2m}{\hbar}a_0 x_0. \qquad (23)$$

By inserting this equation and Eqs. (20) in Eq. (19), we obtain

$$a_1 = \frac{1}{2}\cos^2\gamma\{1 + \cos(\Omega_0 t)e^{-\Gamma(t)}\}, \qquad (24a)$$

$$a_2 = \frac{1}{2}\sin^2\gamma\{1 + \cos(\Omega_0 t)e^{-\Gamma(t)}\}, \qquad (24b)$$

$$a_3 = \sin\gamma\cos\gamma\cos^2(\Omega_0 t/2)e^{-\Gamma(t)/2}, \qquad (24c)$$

$$a_4 = \frac{1}{2}\{1 - \cos(\Omega_0 t)e^{-\Gamma(t)}\}. \qquad (24d)$$

It is useful to emphasize that $\Gamma(t)$ is proportional to the square of the adimensional distance in phase space between the average positions of scattered packets in the OSG effect, as defined in Ref. [13] for the one atom case. The same $\Gamma(t)$ can also be interpreted as the *visibility* of the Rabi oscillations [12] in the ambit of the complementarity relation as introduced by Englert [16].

Considering the ordered basis $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$, the reduced density operator (18) takes on the form

$$\rho^{AB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_3^* & a_2 & 0 \\ 0 & 0 & 0 & a_4 \end{pmatrix}, \qquad (25)$$

where $(a_1+a_2+a_4)=1$. a_3 describes the coherence between the two atoms which assumes its maximum value at $t=0$ and, contrary to what happens in the JC case, goes irreversibly to zero because of the coupling of each qubit with its own external dynamics.

Now we consider the case in which the two qubits are initially in a superposition of the Bell's states denoted $|\phi^{\pm}\rangle$, and the two cavities are in the vacuum state,

$$|\chi(0)\rangle = [\cos\gamma|++\rangle + \sin\gamma|--\rangle]|00\rangle. \qquad (26)$$

With respect to the more convenient ordered basis $|+-\rangle$, $|++\rangle$, $|--\rangle$, $|-+\rangle$, the reduced density matrix of the two qubits assumes the form

$$\rho^{AB}(t) = \begin{pmatrix} b_3 & 0 & 0 & 0 \\ 0 & b_2 & b_1 & 0 \\ 0 & b_1^* & b_5 & 0 \\ 0 & 0 & 0 & b_3 \end{pmatrix}, \qquad (27)$$

where

$$b_1 = \frac{1}{4}\sin\gamma\cos\gamma e^{-i\vartheta_0(t)}[\langle\varphi_A(t)|\phi_A^-(t)\rangle + \langle\varphi_A(t)|\phi_A^+(t)\rangle] \times[\langle\phi_B^-(t)|\varphi_B(t)\rangle + \langle\phi_B^+(t)|\varphi_B(t)\rangle],$$

$$b_2 = \frac{\cos^2\gamma}{4}[1 + \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle + \mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle + \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle\mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle],$$

$$b_3 = \frac{\cos^2\gamma}{4}[1 + \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle - \mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle - \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle\mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle],$$

$$b_4 = \frac{\cos^2\gamma}{4}[1 - \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle + \mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle - \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle\mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle],$$

$$b_5 = \frac{\cos^2\gamma}{4}[1 - \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle - \mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle + \mathrm{Re}\langle\phi_A^-(t)|\phi_A^+(t)\rangle\mathrm{Re}\langle\phi_B^-(t)|\phi_B^+(t)\rangle] + \sin^2\gamma. \quad (28)$$

Using the results (20) we easily obtain

$$b_1 = \sin\gamma\cos\gamma\cos^2(\Omega_0 t/2)e^{-\Gamma(t)/2} \times\exp\{-i[\omega t - ma_0^2t^3/(3\hbar)]\}, \quad (29a)$$

$$b_2 = \frac{1}{4}\cos^2\gamma\{1 + \cos(\Omega_0 t)e^{-\Gamma(t)}\}^2, \quad (29b)$$

$$b_3 = \frac{1}{4}\cos^2\gamma\{1 - \cos^2(\Omega_0 t)e^{-2\Gamma(t)}\}, \quad (29c)$$

$$b_5 = \sin^2\gamma + \frac{1}{4}\cos^2\gamma\{1 - \cos(\Omega_0 t)e^{-\Gamma(t)}\}^2, \quad (29d)$$

where we have used $b_4 = b_3$ because of the condition (23).

B. One atom

To analyze analogies and differences with the disentanglement produced by different environments, it is useful to report in short on the dynamics of a single two-level atom in the OSG model. We assume that the initial state of the whole system is

$$|\psi(0)\rangle = |\varphi(0)\rangle|\chi(0)\rangle, \quad (30)$$

where

$$|\chi(0)\rangle = [\cos\gamma|+\rangle + \sin\gamma|-\rangle]|0\rangle \quad (31)$$

and $|\varphi\rangle$ describes the atomic translational dynamics along the cavity axis. Using some results of Ref. [15] (see Sec. IV) one obtains the time evolution of Eq. (30),

$$|\psi(t)\rangle = \sin\gamma|\varphi(t)\rangle|-0\rangle + \frac{\cos\gamma}{\sqrt{2}}e^{-i\vartheta_0(t)}[|\phi^+(t)\rangle|\chi^+\rangle + |\phi^-(t)\rangle|\chi^-\rangle]. \quad (32)$$

Removing the environment by tracing on the field and on the atomic translation variables, one obtains the reduced one qubit density matrix

$$\rho(t) = \begin{pmatrix} q_1 & q_2 \\ q_2^* & q_3 \end{pmatrix}, \quad (33)$$

where

$$q_1 = \frac{1}{2}\cos^2\gamma[1 + \mathrm{Re}\langle\phi^-(t)|\phi^+(t)\rangle], \quad (34a)$$

$$q_2 = \frac{1}{2}e^{-i\vartheta_0(t)}\cos\gamma\sin\gamma[\langle\varphi(t)|\phi^+(t)\rangle + \langle\varphi(t)|\phi^-(t)\rangle], \quad (34b)$$

$$q_3 = \frac{1}{2}\cos^2\gamma[1 - \mathrm{Re}\langle\phi^-(t)|\phi^+(t)\rangle] + \sin^2\gamma. \quad (34c)$$

Both the irreversible evolution of the populations [Eqs. (34a) and (34c)] and the decoherence (34b) are to be ascribed to the increasing distance in the phase space between the scattered atomic packets. However, while the population decay depends on the square distance $\Gamma(t)$ between the packets moving in opposite directions (through the scalar product $\langle\phi^{\mp}|\phi^{\pm}\rangle$), the coherence experiences a four time slower decay since it depends on the square distance $\Gamma(t)/4$ between one of the moving packet and the standing packets related to the ground state (scalar product $\langle\varphi|\phi^{\pm}\rangle$). Moreover, we stress that, because of the nondissipative character of the decay, the qubit upper population does not go to zero, as in the spontaneous emission, but tends to half of the initial value.

IV. ENTANGLEMENT EVOLUTION FOR THE TWO QUBITS

A. Concurrence

As it is generally convenient for these bipartite mixed states we will analyze the time evolution of the entanglement between the internal variables of the two atoms, looking at the concurrence [17]. It is not difficult to show that the concurrence of the state (25) is

$$C(\rho^{AB}) = \max\{0,(||a_3| + \sqrt{a_1a_2}| - ||a_3| - \sqrt{a_1a_2}|)\} = 2a_3. \quad (35)$$

The entanglement with the nonrelevant systems causes a degradation of the nonlocal correlation between the qubits. According to previous results (see, for example, Refs. [4,6]), and as shown in Fig. 1, state (8) gives rise to asymptotic disentanglement with the same rate of the decoherence. Because of the particular dynamics of the OSG model, in this case the disentanglement rate is one half time the population decay rate. Finally, we note that, for $\Gamma(t)=0$, i.e., when trans-

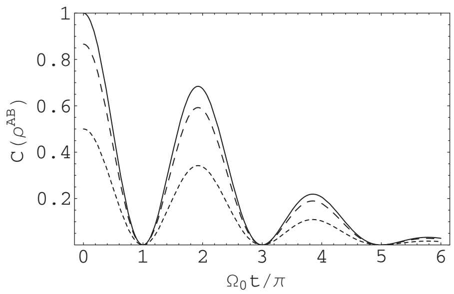

FIG. 1. Decay of concurrence, as given by Eq. (35), when the two qubits are initially in the superposition (8) of Bell's states $|\psi^{\pm}\rangle$. The superposition parameter is $\gamma=\pi/4$ (continuous line), $\pi/6$ (dashed line), $\pi/12$ (dotted line), with $x_{A,0}=x_{B,0}\equiv x_0=\lambda/10$, $\Delta x_0=\lambda/50$. The other parameters are $\lambda=10^{-2}$ m, $\epsilon=10^4$ sec^{-1}, $m=10^{-26}$ kg.

lation dynamics of the center of mass of the two atoms is not taken into account, the concurrence (35) reduces to that derived in [9] for the two JC atoms' case.

More interesting results are expected for the initial state (26), for which the concurrence is

$$C(\rho^{AB}) = \max\{0,d(t)\}, \tag{36}$$

where

$$d(t) = |b_1| + \sqrt{b_2 b_5} - ||b_1| - \sqrt{b_2 b_5}| - 2b_3 = 2(|b_1| - b_3). \tag{37}$$

The behavior of concurrence (36) is shown in Fig. 2. Also in this case, the coherence b_1 between the two atoms goes to zero as a_3 in the previous case does. However, the concurrence in this case experiences a sudden death because of the probabilities b_3 of finding the atoms in the two states $|+-\rangle$ and $|-+\rangle$, which are not correlated to the others and which tend to $\cos^2(\gamma)/4$ for long time. Figure 2 makes evident the strong dependence of the entanglement lifetime on the parameter of the initial two qubit state (26), similarly to the theoretical [4] and experimental [8] results. However, in Refs. [4,8] it is outlined that only some initial entanglements may undergo to sudden death. On the contrary, due to the particular environmental action of the OSG model, there is no threshold in our case. In fact, the function $d(t)$ of Eq. (37) is negative if $b_3/|b_1|>1$, and it turns out that

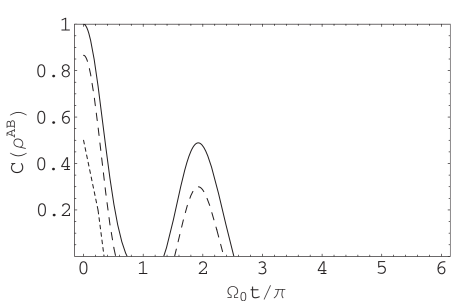

FIG. 2. Decay of concurrence, as given by Eq. (36), when the two qubits are initially in the superposition (26) of Bell's states $|\phi^{\pm}\rangle$. The values of the parameters are as in Fig. 1.

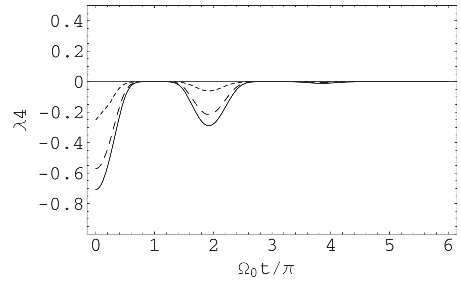

FIG. 3. Time behavior of the eigenvalue (41) of the matrix (39) for the one excitation initial state (8). An enlarged form of this figure should show that λ_4 takes on the zero value only for isolated points. The values of the parameters are as in Fig. 1.

$$b_3/|b_1| \geqslant \frac{\cot\gamma}{2}\sinh[\Gamma(t)/2]. \tag{38}$$

From this equation one can realize that for any value of γ there exists a finite time for which the concurrence goes to negative values and the two qubits become disentangled. In this respect, our model behaves similarly to the continuous variable two-atom model of Ref. [5].

B. Separability

We will now inquire on the separability of the reduced density matrices (25) and (27) adopting the Peres-Horodecki test [18,19]. As it is known, for two qubits the non-negativity of the eigenvalues of the partial transposed matrix works as a necessary and sufficient condition for the separability. Consider first the partial transposed of Eq. (25),

$$\sigma^{AB}(t) = \begin{pmatrix} 0 & 0 & 0 & a_3^* \\ 0 & a_1 & 0 & 0 \\ 0 & 0 & a_2 & 0 \\ a_3 & 0 & 0 & a_4 \end{pmatrix}, \tag{39}$$

whose elements are given by Eqs. (24). We easily find that three eigenvalues of Eq. (39),

$$\lambda_1 = a_1, \quad \lambda_2 = a_2, \quad \lambda_3 = \frac{a_4}{2} + \frac{1}{2}[a_4^2 + 4a_3^2]^{1/2}, \tag{40}$$

are non-negative, while the fourth eigenvalue

$$\lambda_4 = \frac{a_4}{2} - \frac{1}{2}[a_4^2 + 4a_3^2]^{1/2} \tag{41}$$

assumes only nonpositive values, as Fig. 3 shows.

We wish to point out that for the initial state (8) the two qubits display Einstein-Podolsky-Rosen (EPR) correlations [20] for all times, except for isolated points (see Fig. 1). Accordingly, one can see from the analytical expression of λ_4 (more accurately than from Fig. 3) that the bipartite system is not separable except for isolated points, and tends to the separability for long times.

Finally, for the density matrix (27) relative to the initial state (26) the partial transposition gives

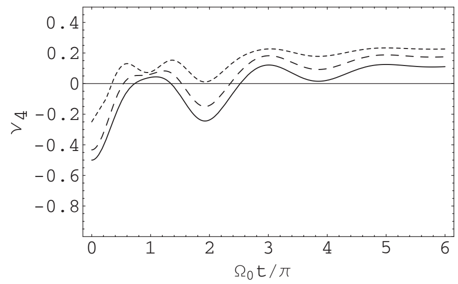

FIG. 4. Time behavior of the fourth eigenvalue (44) of the matrix (42) for the two excitations initial state (26). The values of the parameters are as in Fig. 1.

$$\sigma^{AB}(t) = \begin{pmatrix} b_3 & 0 & 0 & b_1^* \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_5 & 0 \\ b_1 & 0 & 0 & b_3 \end{pmatrix}, \tag{42}$$

whose elements are given in this case by Eqs. (29). As in the previous case, three eigenvalues

$$\nu_1 = b_2, \quad \nu_2 = b_5, \quad \nu_3 = b_3 + |b_1| \tag{43}$$

are non-negative, while a fourth eigenvalue

$$\nu_4 = b_3 - |b_1| \tag{44}$$

takes on negative values for some time intervals (see Fig. 4), in exact correspondence with the existence of a concurrence (see Fig. 2).

V. CONCLUSIONS

In this paper it has been analytically derived the time evolution of the initial entanglement of two atoms which separately interact with two distinct cavities. We have used the optical Stern-Gerlach model to include the external translational dynamics of each atom. The presence of entanglement has been ascertained through the concurrence and the separability and we have found, as in Ref. [4], that the two atom internal dynamics follows different patterns and becomes irreversibly separable in different times, depending on the different initial configurations. As found in other contests [4,6,8] which use different environments and where the disentanglement is due essentially to spontaneous emission, also in the OSG model the entanglement may undergo to an irreversible sudden death. In addition, the particular nondissipative environment here considered suddenly destroys the initial entanglement of Bell's states $|\phi^{\pm}\rangle$ superposition [Eq. (26)] for any initial value of γ. This can in part be ascribed to the peculiar dynamics of the qubits in the OSG model in which the population b_3 [matrix (27)] of the noncorrelates states $|+-\rangle$ and $|-+\rangle$ does not go to zero, but tend to $\cos^2(\gamma)/4$ for long time. We can consequently conclude that nonlocal coherences manifest an amazing fragility even toward a nondissipative environment made of a single degree of freedom.

[1] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, e-print arXiv:quant-ph/0702225v2.
[2] W. Zurek, e-print arXiv:quant-ph/0306072.
[3] L. Diósi, in *Irreversible Quantum Dynamics*, edited by F. Benatti and R. Floreanini (Springer, Berlin, 2003).
[4] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).
[5] P. J. Dodd and J. J. Halliwell, Phys. Rev. A **69**, 052105 (2004).
[6] M. F. Santos, P. Milman, L. Davidovich, and N. Zagury, Phys. Rev. A **73**, 040305(R) (2006).
[7] A. R. R. Carvalho, F. Mintert, S. Palzer, and A. Buchleitner, Eur. Phys. J. D **41**, 425 (2007).
[8] M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, Science **316**, 579 (2007); e-print arXiv:quant-ph/0701184.
[9] M. Yönaç, T. Yu, and J. H. Eberly, J. Phys. B **39**, S621 (2006).
[10] T. Sleator, T. Pfau, V. Balykin, O. Carnal, and J. Mlynek, Phys. Rev. Lett. **68**, 1996 (1992), and related references.
[11] A. Vaglica, Phys. Rev. A **58**, 3856 (1998).
[12] M. Tumminello, A. Vaglica, and G. Vetri, Europhys. Lett. **65**, 785 (2004).
[13] M. Chianello, M. Tumminello, A. Vaglica, and G. Vetri, Phys. Rev. A **69**, 053403 (2004).
[14] M. Tumminello, A. Vaglica, and G. Vetri, Europhys. Lett. **66**, 792 (2004); Eur. Phys. J. D **36**, 235 (2005).
[15] I. Cusumano, A. Vaglica, and G. Vetri, Phys. Rev. A **66**, 043408 (2002).
[16] B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).
[17] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[18] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
[19] R. Horodecki, P. Horodecki, and M. Horodecki, Phys. Lett. A **223**, 1 (1996).
[20] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).