Reversal of wave momentum in isotropic left-handed media

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The electromagnetic wave momentum is derived for a Lorentz medium and applied to study the momentum transfer to stationary, isotropic left-handed materials. The model includes material dispersion and losses, which are necessary for a causal medium with negative index of refraction. The results provide a rigorous proof of the force on free currents in a lossy medium and a validation of the theoretical separation of force based on the real and imaginary parts of the permittivity and permeability. The resulting electromagnetic wave momentum conservation theorem proves that the momentum flux of a monochromatic wave in an isotropic left-handed material is opposite to the power flow direction. However, the momentum density in a lossy medium with a negative index of refraction may be parallel or antiparallel to the power flow. The results are applied to predict the reversal of radiation pressure on free currents in a material with a negative index of refraction. Furthermore, conservation of momentum at a material boundary states that the tangential component of the wave momentum is conserved. Thus there is no electromagnetic shear force at the boundary between isotropic media, regardless of the sign of the refractive index.

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I. INTRODUCTION

Left-handed materials (LHMs) have received much attention since their realization in 2001 [1] due to interesting physics such as negative refraction [2–4], reversal of the Čerenkov effect [5–8], and the potential to create a perfect lens [9]. However, the reversal of radiation pressure in LHMs first predicted in 1968 by Veselago [10] has received much less attention. Veselago's results show that the time-average momentum density of an electromagnetic wave

$$\langle \bar{G} \rangle = \frac{1}{2} \text{Re}\left\{ \epsilon\mu \bar{E} \times \bar{H}^* + \frac{\bar{k}}{2}\left(\frac{\partial \epsilon}{\partial \omega} |\bar{E}|^2 + \frac{\partial \mu}{\partial \omega} |\bar{H}|^2 \right) \right\} \quad (1)$$

is antiparallel to the average Poynting power $\langle \bar{S} \rangle = \frac{1}{2}\text{Re}\{\bar{E} \times \bar{H}^*\}$ in materials with simultaneously negative permeability μ and permittivity ϵ, giving rise to light attraction instead of light pressure [10]. In Eq. (1), $\bar{E}$ and $\bar{H}$ are the complex electric field and magnetic field, respectively, $\bar{k}$ is the wave vector, ω is the angular frequency, Re is the real part, and the asterisk denotes complex conjugate. Researchers still have not observed experimentally the predicted reversal of electromagnetic wave momentum. Experiments have revealed optical momentum transfer to dielectric media in direct proportion to the macroscopic index of refraction [11–14], but observations have not revealed dependence on the slope of the dielectric function as in Eq. (1). Furthermore, identification of electromagnetic momentum in media continues to be a controversial subject; various definitions and interpretations have been proposed [15–22].

The recent application of electromagnetic momentum conservation at the interface separating free-space and an isotropic LHM led to the conclusion that the change in momentum of the electromagnetic wave due to refraction must produce a force with a nonzero component directed parallel to the boundary [23]. This sheering force is claimed to be unique to LHMs, thus supporting the notion that the tangential component of the momentum is conserved at the boundary separating two right-handed materials (RHMs) [24]. However, the aforementioned sheering force results since the momentum flux in LHMs was assumed to be in the same direction as the power flow, which is in contrast to Veselago's prediction for the momentum density [10]. Thus electromagnetic wave momentum and radiation pressure in LHMs remains topical.

In this paper, we rigourously treat the momentum transfer to stationary, isotropic LHMs by applying the classical electromagnetic wave theory. It is argued that previous attempts to describe the momentum of the electromagnetic wave in LHMs failed to include material losses and/or dispersion, which cannot be ignored in a causal system with a negative index of refraction. We apply the concept of wave momentum, which includes contributions from the material response as proposed by the seminal work of Gordon [15]. The derived expressions define the wave momentum density and wave momentum flux similar to previous derivations for dispersive dielectrics [17,18]. The results presented here are analogous to the wave energy density and wave energy flux previously derived using the standard Lorentz model for the polarization and magnetization [25]. The Lorentz force is also applied directly to derive the wave momentum flux density in an LHM half-space resulting from oblique incidence of a monochromatic wave. It is shown that the force tangential to the interface results solely from the momentum transfer as the wave attenuates in the medium. Thus the tangential component of wave momentum is conserved due to reflection and refraction at the interface of any isotropic medium, and the tangential force on a hypothetical lossless medium is zero regardless of the direction of phase propagation. In the process, we provide a rigorous derivation for the cycle-averaged force on free currents, which, along with the force on bound currents and charges, gives the total force and describes the details of momentum transfer in lossy media

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[26,27]. Furthermore, the force on free currents due to the attenuation of the electromagnetic wave in a material with negative index of refraction is opposite to the direction of power flow.

II. ENERGY AND MOMENTUM IN UNBOUNDED MEDIA

In order to accurately describe wave propagation in an LHM, it is necessary to include material dispersion and losses. That is, the coupled electromagnetic field and material response must be considered together in determining the form of the energy and momentum of the electromagnetic wave. The equations governing the energy and momentum of an arbitrary subsystem generally take the form [28]

$$\bar{\nabla} \cdot \bar{S} + \frac{\partial W}{\partial t} = -\varphi, \tag{2a}$$

$$\bar{\nabla} \cdot \bar{\bar{T}} + \frac{\partial \bar{G}}{\partial t} = -\bar{f}, \tag{2b}$$

where the energy flow $\bar{S}$, energy density $\bar{W}$, momentum flow $\bar{\bar{T}}$, and momentum density $\bar{G}$ interact with other subsystems via φ and $\bar{f}$. In this section, we derive the equations that govern the energy and momentum of the wave from the classical electromagnetic theory and the equations of motion for the dielectric and magnetic response of a Lorentz medium.

A. Energy and momentum of the electromagnetic fields

We begin with the Maxwell-Minkowski fields $\bar{\mathcal{E}}(\bar{r},t)$, $\bar{\mathcal{H}}(\bar{r},t)$, $\bar{\mathcal{D}}(\bar{r},t)$, and $\bar{\mathcal{B}}(\bar{r},t)$ in a source-free region [29]. The electromagnetic fields and the material response fields are separated by defining the polarization $\bar{\mathcal{P}} \equiv \bar{\mathcal{D}} - \epsilon_0 \bar{\mathcal{E}}$ and magnetization $\mu_0 \bar{\mathcal{M}} \equiv \bar{\mathcal{B}} - \mu_0 \bar{\mathcal{H}}$ for a stationary medium, where the dependence upon space and time is now implied in the notation. The resulting Maxwell-Chu equations [28,29]

$$\bar{\nabla} \times \bar{\mathcal{H}} - \epsilon_0 \frac{\partial \bar{\mathcal{E}}}{\partial t} = \frac{\partial \bar{\mathcal{P}}}{\partial t} \equiv \bar{\mathcal{J}}_e, \tag{3a}$$

$$\bar{\nabla} \times \bar{\mathcal{E}} + \mu_0 \frac{\partial \bar{\mathcal{H}}}{\partial t} = -\mu_0 \frac{\partial \bar{\mathcal{M}}}{\partial t} \equiv -\bar{\mathcal{J}}_h, \tag{3b}$$

$$\mu_0 \bar{\nabla} \cdot \bar{\mathcal{H}} = -\bar{\nabla} \cdot \mu_0 \bar{\mathcal{M}} \equiv \rho_h, \tag{3c}$$

$$\epsilon_0 \bar{\nabla} \cdot \bar{\mathcal{E}} = -\bar{\nabla} \cdot \bar{\mathcal{P}} \equiv \rho_e \tag{3d}$$

give the electromagnetic fields $\bar{\mathcal{E}}$ and $\bar{\mathcal{H}}$ in the presence of the material represented by the source terms $\bar{\mathcal{J}}_e$, $\bar{\mathcal{J}}_h$, ρ_e, and ρ_h. The energy and momentum quantities for the electromagnetic subsystem can be derived without specifying models for $\bar{\mathcal{P}}$ and $\bar{\mathcal{M}}$.

The energy equation is derived in the usual way by subtracting Eq. (3a) dot multiplied by $\bar{\mathcal{E}}$ from Eq. (3b) dot multiplied by $\bar{\mathcal{H}}$ and applying the vector identity [29] $\bar{\mathcal{H}} \cdot (\bar{\nabla} \times \bar{\mathcal{E}}) - \bar{\mathcal{E}} \cdot (\bar{\nabla} \times \bar{\mathcal{H}}) = \bar{\nabla} \cdot (\bar{\mathcal{E}} \times \bar{\mathcal{H}})$. The quantities corresponding to the electromagnetic subsystem in Eq. (2a) are identified as [28]

$$\bar{S}_{eh} = \bar{\mathcal{E}} \times \bar{\mathcal{H}}, \tag{4a}$$

$$W_{eh} = \frac{\epsilon_0}{2} \bar{\mathcal{E}} \cdot \bar{\mathcal{E}} + \frac{\mu_0}{2} \bar{\mathcal{H}} \cdot \bar{\mathcal{H}}, \tag{4b}$$

$$\varphi_{eh} = \bar{\mathcal{J}}_e \cdot \bar{\mathcal{E}} + \bar{\mathcal{J}}_h \cdot \bar{\mathcal{H}}, \tag{4c}$$

where the subscript *eh* denotes quantities relating to the electromagnetic subsystem. Likewise, the equation describing the transfer of momentum to and from the electromagnetic subsystem is derived by adding the cross product of the vector equation (3a) and $\mu_0 \bar{\mathcal{H}}$, the cross product of the vector equation (3b) and $\epsilon_0 \bar{\mathcal{E}}$, the product of the scalar equation (3c) and $\bar{\mathcal{H}}$, and the product of the scalar equation (3d) and $\bar{\mathcal{E}}$. After some manipulation of the resulting vector equation, the momentum conservation equation for the electromagnetic subsystem can be written in the form of Eq. (2b) with the quantities given by [28]

$$\bar{\bar{T}}_{eh} = \frac{1}{2}(\epsilon_0 \bar{\mathcal{E}} \cdot \bar{\mathcal{E}} + \mu_0 \bar{\mathcal{H}} \cdot \bar{\mathcal{H}})\bar{\bar{I}} - \epsilon_0 \bar{\mathcal{E}}\bar{\mathcal{E}} - \mu_0 \bar{\mathcal{H}}\bar{\mathcal{H}}, \tag{5a}$$

$$\bar{G}_{eh} = \epsilon_0 \mu_0 \bar{\mathcal{E}} \times \bar{\mathcal{H}}, \tag{5b}$$

$$\bar{f}_{eh} = \rho_e \bar{\mathcal{E}} + \rho_h \bar{\mathcal{H}} + \bar{\mathcal{J}}_e \times \mu_0 \bar{\mathcal{H}} - \bar{\mathcal{J}}_h \times \epsilon_0 \bar{\mathcal{E}}, \tag{5c}$$

where $\bar{\bar{I}}$ is the 3×3 identity dyad and $\bar{\mathcal{E}}\bar{\mathcal{E}}$ represents a dyadic product. The quantities in Eqs. (4) and (5) represent the electromagnetic subsystem and may be regarded as the electromagnetic contributions to energy and momentum [28]. The quantities W_{eh} and $\bar{S}_{eh}$ are identified as the energy density of the electromagnetic fields and the Poynting power, respectively. The momentum density $\bar{G}_{eh}$ is often referred to as the Abraham momentum [30], and the momentum flux $\bar{\bar{T}}_{eh}$ takes the form of the free-space Maxwell stress tensor [31,32]. It is well-known that a material contribution to the energy density accompanies the propagation of electromagnetic energy in dielectrics [33–35]. In the section that follows, we derive the corresponding material contribution to the wave momentum.

B. Energy and momentum contribution from dispersive media

To model many experimental observations, it is necessary to include the dispersive characteristics of the material in describing the observed behavior of the electromagnetic wave. The inclusion of losses requires that a specific model for $\bar{\mathcal{P}}$ and $\bar{\mathcal{M}}$ be applied. The material response to the electromagnetic fields is described by the differential equations for a Lorentz medium,

$$\left(\frac{\partial^2}{\partial t^2} + \gamma_e \frac{\partial}{\partial t} + \omega_{e0}^2\right)\bar{\mathcal{P}} = \epsilon_0 \omega_{ep}^2 \bar{\mathcal{E}}, \tag{6a}$$

$$\left(\frac{\partial^2}{\partial t^2} + \gamma_m \frac{\partial}{\partial t} + \omega_{m0}^2\right)\bar{\mathcal{M}} = F \omega_{mp}^2 \bar{\mathcal{H}}, \tag{6b}$$

where the parameters of the equations have their usual meanings [25]. To derive the energy of the electromagnetic wave, the material equations (6a) and (6b) are dot multiplied by $\bar{\mathcal{J}}_e$ and $\bar{\mathcal{J}}_h$, respectively. The resulting equations

$$\bar{\mathcal{J}}_e \cdot \bar{E} = \frac{1}{2\epsilon_0 \omega_{ep}^2}\frac{\partial}{\partial t}\left\{\frac{\partial \bar{\mathcal{P}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t} + \omega_{e0}^2 \bar{\mathcal{P}}\cdot\bar{\mathcal{P}}\right\} + \frac{\gamma_e}{\epsilon_0 \omega_{ep}^2}\frac{\partial \bar{\mathcal{P}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t}, \tag{7a}$$

$$\bar{\mathcal{J}}_h \cdot \bar{\mathcal{H}} = \frac{\mu_0}{2F \omega_{mp}^2}\frac{\partial}{\partial t}\left\{\frac{\partial \bar{\mathcal{M}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t} + \omega_{m0}^2 \bar{\mathcal{M}}\cdot\bar{\mathcal{M}}\right\} + \frac{\gamma_m \mu_0}{F \omega_{mp}^2}\frac{\partial \bar{\mathcal{M}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t}, \tag{7b}$$

are then added to the energy conservation equation of the electromagnetic subsystem given by Eq. (2a) with the quantities defined by Eq. (4). The resulting energy conservation equation for the electromagnetic wave is in the form of Eq. (2a) with the energy flow $\bar{S}$, energy density W, and energy dissipation φ given by [25]

$$\bar{S} = \bar{S}_{eh} = \bar{\mathcal{E}} \times \bar{\mathcal{H}}, \tag{8a}$$

$$W = W_{eh} + \frac{1}{2\epsilon_0 \omega_{ep}^2}\left[\frac{\partial \bar{\mathcal{P}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t} + \omega_{e0}^2 \bar{\mathcal{P}}\cdot\bar{\mathcal{P}}\right] + \frac{\mu_0}{2F \omega_{mp}^2}\left[\frac{\partial \bar{M}}{\partial t}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t} + \omega_{m0}^2 \bar{\mathcal{M}}\cdot\bar{\mathcal{M}}\right], \tag{8b}$$

$$\varphi = \frac{\gamma_e}{\epsilon_0 \omega_{ep}^2}\frac{\partial \bar{\mathcal{P}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t} + \frac{\gamma_m \mu_0}{F \omega_{mp}^2}\frac{\partial \bar{\mathcal{M}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t}. \tag{8c}$$

A few remarks are in order regarding the quantities in Eq. (8), which are written without subscript to indicate values corresponding to the electromagnetic wave. First, the energy flow $\bar{S}$ is generally regarded as the Poynting power and retains its free-space form even in the presence of a lossy, dispersive material. Second, the energy density W contains contributions from the potential energy and kinetic energy of the electric and magnetic dipoles. Furthermore, the form of Eq. (8b) has been regarded as significant since the energy density remains positive in left-handed media [25]. Third, the energy dissipation term φ depends upon the damping factors γ_e and γ_m in Eq. (6). Thus φ=0 in the limiting case of a lossless material (i.e., the energy of the electromagnetic wave is conserved).

A similar mathematical derivation exists for the wave momentum. To determine the contribution of the material response fields to the wave momentum, the material dispersion equations (6) are dot multiplied by the dyads $-\nabla\bar{\mathcal{P}}$ and $-\mu_0\nabla\bar{\mathcal{M}}$, respectively. The resulting vector equations are then added to the electromagnetic conservation equation given by Eqs. (2b) and (5) to yield

$$\bar{\nabla}\cdot\bar{\bar{T}}_{eh} + \frac{\partial \bar{G}_{eh}}{\partial t} + \bar{f}_{eh} + \bar{\mathcal{E}}\cdot\bar{\nabla}\bar{\mathcal{P}} + \mu_0 \bar{\mathcal{H}}\cdot\bar{\mathcal{M}} - \nabla\bar{\mathcal{P}}\cdot\left(\frac{\partial^2 \bar{\mathcal{P}}}{\partial t^2} + \omega_{e0}^2 \bar{\mathcal{P}}\right)\frac{1}{\epsilon_0 \omega_{ep}^2} - \mu_0 \nabla\bar{\mathcal{M}}\cdot\left(\frac{\partial^2 \bar{\mathcal{M}}}{\partial t^2} + \omega_{m0}^2 \bar{\mathcal{M}}\right)\frac{\mu_0}{F\omega_{mp}^2} = \nabla\bar{\mathcal{P}}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t}\frac{\gamma_e}{\epsilon_0 \omega_{ep}^2} + \nabla\bar{\mathcal{M}}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t}\frac{\gamma_m \mu_0}{F\omega_{mp}^2}. \tag{9}$$

Application of identities from vector calculus allows us to write

$$\nabla\bar{\mathcal{P}}\cdot\left(\frac{\partial^2 \bar{\mathcal{P}}}{\partial t^2} + \omega_{e0}^2 \bar{\mathcal{P}}\right)\frac{1}{\epsilon_0 \omega_{ep}^2} = \frac{\partial}{\partial t}\left[\frac{1}{\epsilon_0 \omega_{ep}^2}\left(\nabla\bar{\mathcal{P}}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t}\right)\right] - \bar{\nabla}\cdot\left[\frac{1}{2\epsilon_0 \omega_{ep}^2}\left(\frac{\partial \bar{\mathcal{P}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{P}}}{\partial t} - \omega_{e0}^2 \bar{\mathcal{P}}\cdot\bar{\mathcal{P}}\right)\right], \tag{10a}$$

$$\nabla\bar{\mathcal{M}}\cdot\left(\frac{\partial^2 \bar{\mathcal{M}}}{\partial t^2} + \omega_{m0}^2 \bar{\mathcal{M}}\right)\frac{\mu_0}{F\omega_{mp}^2} = \frac{\partial}{\partial t}\left[\frac{\mu_0}{F\omega_{mp}^2}\left(\nabla\bar{\mathcal{M}}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t}\right)\right] - \bar{\nabla}\cdot\left[\frac{\mu_0}{2F\omega_{mp}^2}\left(\frac{\partial \bar{\mathcal{M}}}{\partial t}\cdot\frac{\partial \bar{\mathcal{M}}}{\partial t} - \omega_{m0}^2 \bar{\mathcal{M}}\cdot\bar{\mathcal{M}}\right)\right], \tag{10b}$$

$$\bar{f}_{eh} + \bar{\mathcal{E}}\cdot\bar{\nabla}\bar{\mathcal{P}} + \mu_0 \bar{\mathcal{H}}\cdot\bar{\mathcal{M}} = \frac{\partial}{\partial t}[\bar{\mathcal{D}}\times\bar{\mathcal{B}} - \epsilon_0\mu_0\bar{\mathcal{E}}\times\bar{\mathcal{H}}] + \bar{\nabla}\cdot[(\bar{\mathcal{P}}\cdot\bar{\mathcal{E}} + \mu_0\bar{\mathcal{M}}\cdot\bar{\mathcal{H}})\bar{\bar{I}} - \bar{\mathcal{P}}\bar{\mathcal{E}} - \mu_0\bar{\mathcal{M}}\bar{\mathcal{H}}]. \tag{10c}$$

By combining Eq. (9) and (10), the momentum conservation equation for the electromagnetic wave can be written in the form of Eq. (2b) with the momentum flow, momentum density, and force density

$$\bar{\bar{T}} = \frac{1}{2}(\bar{\mathcal{D}}\cdot\bar{\mathcal{E}} + \bar{\mathcal{B}}\cdot\bar{\mathcal{H}})\bar{\bar{I}} - \bar{\mathcal{D}}\bar{\mathcal{E}} - \bar{\mathcal{B}}\bar{\mathcal{H}} + \frac{1}{2}\Bigg[(\bar{\mathcal{P}}\cdot\bar{\mathcal{E}} + \mu_0\bar{\mathcal{M}}\cdot\bar{\mathcal{H}}) + \frac{1}{\epsilon_0\omega_{ep}^2}\left(\frac{\partial\bar{\mathcal{P}}}{\partial t}\cdot\frac{\partial\bar{\mathcal{P}}}{\partial t} - \omega_{e0}^2\bar{\mathcal{P}}\cdot\bar{\mathcal{P}}\right) + \frac{\mu_0}{F\omega_{mp}^2}\left(\frac{\partial\bar{\mathcal{M}}}{\partial t}\cdot\frac{\partial\bar{\mathcal{M}}}{\partial t} - \omega_{m0}^2\bar{\mathcal{M}}\cdot\bar{\mathcal{M}}\right)\Bigg]\bar{\bar{I}}, \tag{11a}$$

$$\bar{G} = \bar{\mathcal{D}}\times\bar{\mathcal{B}} - \frac{1}{\epsilon_0\omega_{ep}^2}\nabla\bar{\mathcal{P}}\cdot\frac{\partial\bar{\mathcal{P}}}{\partial t} - \frac{\mu_0}{F\omega_{mp}^2}\nabla\bar{\mathcal{M}}\cdot\frac{\partial\bar{\mathcal{M}}}{\partial t}, \tag{11b}$$

$$\bar{f} = -\frac{\gamma_e}{\epsilon_0\omega_{ep}^2}\nabla\bar{\mathcal{P}}\cdot\frac{\partial\bar{\mathcal{P}}}{\partial t} - \frac{\gamma_m\mu_0}{F\omega_{mp}^2}\nabla\bar{\mathcal{M}}\cdot\frac{\partial\bar{\mathcal{M}}}{\partial t}. \tag{11c}$$

The quantities in Eq. (11) define the quantities for the wave momentum conservation equation analogous to the wave energy quantities given by Eq. (8). The momentum density $\bar{G}$ contains the Minkowski [36] momentum $\bar{\mathcal{D}}\times\bar{\mathcal{B}}$ plus material dispersion terms. Likewise, the momentum flow $\bar{\bar{T}}$ is the Maxwell stress tensor $\frac{1}{2}(\bar{\mathcal{D}}\cdot\bar{\mathcal{E}}+\bar{\mathcal{B}}\cdot\bar{\mathcal{H}})\bar{\bar{I}}-\bar{\mathcal{D}}\bar{\mathcal{E}}-\bar{\mathcal{B}}\bar{\mathcal{H}}$ in nondispersive media [29,32] plus dispersive terms. We note that the momentum dissipation term $\bar{f}$ depends upon the damping factors γ_e and γ_m in Eq. (6). Thus $\bar{f}=0$ in the limiting case of a lossless, unbounded material (i.e., the momentum of the electromagnetic wave is conserved).

C. Average energy and momentum of a monochromatic wave

For the remainder of the paper, we consider the propagation of time-harmonic electromagnetic waves and employ complex notation such that the complex field $\bar{E}$ is related to the time-domain field by $\bar{\mathcal{E}}=\mathrm{Re}\{\bar{E}e^{-i\omega t}\}$. To arrive at various quantities of interest, the substitutions $\partial/\partial t\to -i\omega$ and $\nabla\to i\bar{k}$ are made, which are valid for plane wave solutions to the wave equation [29]. The constitutive parameters,

$$\epsilon(\omega) = \epsilon_0\left(1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 + i\omega\gamma_e}\right), \tag{12a}$$

$$\mu(\omega) = \mu_0\left(1 - \frac{F\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 + i\omega\gamma_m}\right), \tag{12b}$$

are functions of the frequency ω and consist of real and imaginary parts denoted by $\epsilon=\epsilon_R+i\epsilon_I$ and $\mu=\mu_R+i\mu_I$. Likewise, the time-average of the squared polarization and magnetization are

$$|\bar{P}|^2 = \frac{\epsilon_0^2\omega_{ep}^4}{(\omega^2-\omega_{e0}^2)^2+\gamma_e^2\omega^2}|\bar{E}|^2, \tag{13a}$$

$$|\bar{M}|^2 = \frac{F^2\omega_{mp}^4}{(\omega^2-\omega_{m0}^2)^2+\gamma_m^2\omega^2}|\bar{H}|^2. \tag{13b}$$

Similarly, $|\partial\bar{P}/\partial t|^2=\omega^2|\bar{P}|^2$ and $|\partial\bar{M}/\partial t|^2=\omega^2|\bar{M}|^2$. These quantities can now be applied to determine the time-average values relating to energy and momentum conservation given in the previous section.

The time average energy density found from Eq. (8b) is

$$\langle W\rangle = \frac{\epsilon_0}{2}\left[1 + \frac{\omega_{ep}^2(\omega^2+\omega_{e0}^2)}{(\omega^2-\omega_{e0}^2)^2+\gamma_e^2\omega^2}\right]|\bar{E}|^2 + \frac{\mu_0}{2}\left[1 + \frac{F\omega_{mp}^2(\omega^2+\omega_{m0}^2)}{(\omega^2-\omega_{m0}^2)^2+\gamma_m^2\omega^2}\right]|\bar{H}|^2. \tag{14}$$

We note that in the lossless case, $\gamma_e=0$ and $\gamma_m=0$ implies that both $\epsilon_I=0$ and $\mu_I=0$, and the energy density satisfies the well-known relation [29,32–34]

$$\langle W\rangle = \frac{1}{4}\frac{\partial(\epsilon\omega)}{\partial\omega}|\bar{E}|^2 + \frac{1}{4}\frac{\partial(\mu\omega)}{\partial\omega}|\bar{H}|^2. \tag{15}$$

Eq. (15) is valid only for lossless media, and its application to lossy media produces unphysical phenomena such as a negative energy in LHM. In contrast, the average energy density in Eq. (14) remains positive for all ω. However, it is the rate of change in energy that appears in the conservation equation (2a), which tends to zero upon cycle averaging. That is, $\langle\partial W/\partial t\rangle=0$, and the resulting conservation equation

$$-\langle\bar{\nabla}\cdot\bar{S}\rangle = \frac{1}{2}[\omega\epsilon_I|\bar{E}|^2 + \omega\mu_I|\bar{H}|^2] \tag{16}$$

is generally regarded as the complex Poynting's theorem, where $\langle\bar{S}\rangle=\frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\}$ is the time average Poynting power [29].

A similar analysis is applied to the wave momentum conservation equation. The average momentum density,

$$\langle\bar{G}\rangle = \frac{1}{2}\mathrm{Re}\left\{\bar{D}\times\bar{B}^* + \bar{k}\frac{\epsilon_0\omega\omega_{ep}^2}{(\omega^2-\omega_{e0}^2)^2+\gamma_e^2\omega^2}|\bar{E}|^2 + \bar{k}\frac{\mu_0\omega F\omega_{mp}^2}{(\omega^2-\omega_{m0}^2)^2+\gamma_m^2\omega^2}|\bar{H}|^2\right\}, \tag{17}$$

is obtained from Eq. (11b). It is simple to show using Eq. (12) that the average momentum given in Eq. (17) satisfies Eq. (1) when the medium is lossless. Thus the expression for the momentum given by Vesselago is valid only when absorption of electromagnetic energy can be ignored. The momentum flow reduces to

$$\langle\bar{\bar{T}}\rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}(\bar{D}\cdot\bar{E}^* + \bar{B}\cdot\bar{H}^*)\bar{\bar{I}} - \bar{D}\bar{E}^* - \bar{B}\bar{H}^*\right\} \tag{18}$$

since the dispersive terms in Eq. (11a) tend to zero upon cycle averaging. The momentum flow in Eq. (18) is also referred to as the Maxwell stress tensor in matter [29,32]. Since the average rate of change in momentum density is zero [37] (i.e., $\langle\partial\bar{G}/\partial t\rangle=0$), the momentum conservation theorem for a monochromatic wave reduces to

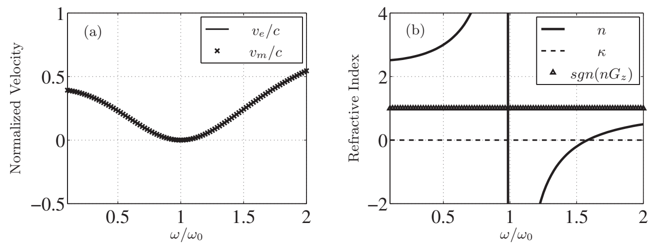

FIG. 1. (a) Normalized energy velocity and momentum velocity for an electromagnetic wave in a lossless medium. (b) The complex index of refraction is given by $n+i\kappa=c\sqrt{\mu\epsilon}$, where μ and ϵ are given in Eq. (12). The triangles clearly show that n and G_z have the same signs over the entire frequency range. The parameters of the material are $\omega_{e0}=\omega_{m0}\equiv\omega_0$, $\omega_{ep}^2=F\omega_{mp}^2=1.5\omega_0^2$, and $\gamma_e=\gamma_m=0$.

$$-\langle\bar{\nabla}\cdot\bar{\bar{T}}\rangle=\frac{1}{2}\,\text{Re}\{\omega\epsilon_I\bar{E}\times\bar{B}^*-\omega\mu_I\bar{H}\times\bar{D}^*\}. \qquad (19)$$

The right-hand side of Eq. (19) is recognized as the force density on free currents [27], which will be discussed later in this paper.

It is now possible to study the propagation of electromagnetic energy and momentum in a dispersive LHM. Consider an electromagnetic wave $\bar{E}=\hat{x}E_x=\hat{x}e^{ikz}$ propagating in an unbounded medium with complex index of refraction $n+i\kappa=kc/\omega=c\sqrt{\epsilon\mu}$, where c is the speed of light in vacuum. The magnetic field satisfies the relation $\sqrt{\mu}H_y=\sqrt{\epsilon}E_x$. The analysis is simplified by taking the ratio of the energy flow and energy density. This ratio $\langle S_z\rangle/\langle W\rangle\equiv v_e$ is generally referred to as the energy velocity of the wave [33,35]. The $\hat{z}$-directed time-average Poynting power is simply $\langle S_z\rangle=\frac{1}{2}\text{Re}\{E_xH_y^*\}$. Similarly, a momentum velocity $\langle T_{zz}\rangle/\langle G_z\rangle\equiv v_m$ may also be defined [17]. The average momentum flux is

$$\langle T_{zz}\rangle=\frac{1}{4}\,\text{Re}\{\epsilon|E_x|^2+\mu|H_y|^2\}=\frac{n}{c}\langle S_z\rangle. \qquad (20)$$

It is obvious from Eq. (20) that the momentum flow is antiparallel to the energy flow when the index of refraction is negative. Furthermore, it has been previously argued that the momentum density $\langle G_z\rangle$ is also antiparallel to $\langle S_z\rangle$ when both $n<0$ and when absorption is negligible [10]. As an illustration, the energy velocity and momentum velocity have been plotted along with the index of refraction for a lossless medium in Fig. 1. Since there is no loss, v_e and v_m are both equivalent to the group velocity [29]. In the negative index region, both $\langle W\rangle$ and $\langle S_z\rangle$ are positive while $\langle G_z\rangle$ and $\langle T_{zz}\rangle$ are negative. This latter point is evident by the fact that $v_m>0$, which implies that $\langle G_z\rangle$ and $\langle T_{zz}\rangle$ have the same sign for all ω. A lossy medium is considered for a second illustration as shown in Fig. 2. In this example, the energy velocity and momentum velocity are quite different in the region where $n<0$. The energy velocity remains positive since both $\langle W\rangle$ and $\langle S_z\rangle$ are positive. However, the momentum velocity becomes negative for part of this region. While the sign of $\langle T_{zz}\rangle$ follows exactly the sign of n via Eq. (20), the momentum density $\langle G_z\rangle$ may be positive or negative in a frequency band with a negative index of refraction.

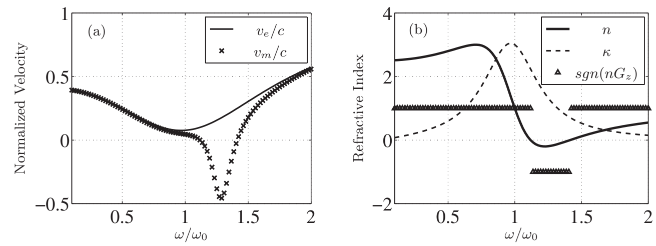

FIG. 2. (a) Normalized energy velocity and momentum velocity for an electromagnetic wave in a lossy medium. (b) The complex index of refraction is given by $n+i\kappa=c\sqrt{\mu\epsilon}$, where μ and ϵ are given in Eq. (12). The triangles clearly show the frequency range where n and G_z have different signs [i.e., $\text{sgn}(nG_z)=-1$]. The parameters of the material are $\omega_{e0}=\omega_{m0}\equiv\omega_0$, $\omega_{ep}^2=F\omega_{mp}^2=1.5\omega_0^2$, and $\gamma_e=\gamma_m=0.5\omega_0$.

The results of this section prove that the momentum density $\langle\bar{G}\rangle$ may be parallel or antiparallel to the power flow $\langle\bar{S}\rangle$ in a lossy LHM. This is in contrast to the results for a lossless LHM, where the momentum density is always antiparallel to the energy flow [10]. However, the momentum transfer from a monochromatic wave is independent of the momentum density. Instead, the momentum conservation equation reduces to Eq. (19). Thus it is expected that observed forces due to a continuous wave depend upon the average momentum flow, which, due to the direct dependence upon n shown in Eq. (20), is opposite the flow of energy in an LHM. In the following section, this conservation equation for a monochromatic wave will be applied to predict the reversal of radiation pressure in LHMs.

III. ELECTROMAGNETIC FORCE

The average force exerted by a monochromatic wave upon matter is given by the Lorentz force density applied directly to matter

$$\langle\bar{f}\rangle=\frac{1}{2}\,\text{Re}\{\epsilon_0(\bar{\nabla}\cdot\bar{E})\bar{E}^*+\mu_0(\bar{\nabla}\cdot\bar{H})\bar{H}^* - i\omega(\epsilon-\epsilon_0)\bar{E}\times\bar{B}^*+i\omega(\mu-\mu_0)\bar{H}\times\bar{D}^*\}, \qquad (21)$$

which has been shown to be consistent with the conservation of free space momentum via numerous examples of lossless [24,37] and lossy [27] media. Furthermore, the Lorentz force density can be decomposed into the force on free currents and the force on bound currents and charges [27]. The total force density $\langle\bar{f}\rangle=\langle\bar{f}_b\rangle+\langle\bar{f}_c\rangle$ is separated based on the real and imaginary contributions to the complex permittivity $\epsilon=\epsilon_R+i\epsilon_I$ and $\mu=\mu_R+i\mu_i$. According to Eq. (19), the force density

$$\langle\bar{f}_c\rangle=\frac{1}{2}\,\text{Re}\{\omega\epsilon_I\bar{E}\times\bar{B}^*-\omega\mu_I\bar{H}\times\bar{D}^*\} \qquad (22)$$

relates the force density on free currents to the momentum transfer as the wave attenuates in the medium, and the force density on bound charges and currents,

$$\langle \bar{f}_b \rangle = \frac{1}{2}\,\mathrm{Re}\{\epsilon_0(\bar{\nabla}\cdot\bar{E})\bar{E}^* + \mu_0(\bar{\nabla}\cdot\bar{H})\bar{H}^* - i\omega(\epsilon_R-\epsilon_0)\bar{E}\times\bar{B}^* + i\omega(\mu_R-\mu_0)\bar{H}\times\bar{D}^*\}, \quad (23)$$

gives the remaining momentum transfer to the host material. Therefore the total forces $\langle \bar{F}\rangle$, $\langle \bar{F}_c\rangle$, and $\langle \bar{F}_b\rangle$ resulting from the total volume integration of the corresponding force densities can be found equivalently, after application of the divergence theorem, by appropriate integration of the Maxwell stress tensor [27]. In what follows, we apply the force on free currents and the force on bound currents and charges to determine the electromagnetic momentum transfer to LHMs.

A. Radiation pressure on an LHM interface

The radiation pressure on an LHM interface was previously calculated by applying momentum conservation at the boundary separating free space and the material [23]. However, the momentum transmitted into the LHM was assumed to be parallel to the Poynting power. In contrast, we derive the radiation pressure on the interface by applying the Lorentz force directly. That is, the force density (22) is applied to derive the electromagnetic wave momentum transmitted into a medium occupying the region $z>0$. The transverse electric (TE) and transverse magnetic (TM) polarized waves are treated identically by considering the fields $\bar{E}=\hat{e}E_0e^{i\bar{k}\cdot\bar{r}}$ and $\bar{H}=\hat{h}H_0e^{i\bar{k}\cdot\bar{r}}$ transmitted into the medium, where E_0 and H_0 are the magnitudes of the electric field and magnetic field at $z=0^+$, and $\bar{k}=k\hat{s}$ is the complex wave vector. The fields satisfy the relations $\omega\mu\bar{H}=\bar{k}\times\bar{E}$ and $\omega\epsilon\bar{E}=-\bar{k}\times\bar{H}$ and the field solutions are omitted since the details of the reflection and transmission phenomenon (RT) of an electromagnetic wave incident onto an LHM interface have been extensively treated by analytical methods [3,38–40]. Since $\epsilon=|\epsilon|\exp(i\phi_\epsilon)$ and $\mu=|\mu|\exp(i\phi_\mu)$ are complex, the phase propagation direction is determined by the sign of k_R, which is defined by $k=k_R+ik_I=\omega\sqrt{|\epsilon||\mu|}\,\exp[i(\phi_\epsilon+\phi_\mu)/2]$. Inserting the fields into Eq. (22) and using the relationship $\hat{s}=\hat{k}/k=(\bar{k}/k)^*$ yields

$$\langle \bar{f}_c \rangle = \frac{1}{2}\,\mathrm{Re}\{\bar{k}^*[\epsilon_I|\bar{E}|^2 + \mu_I|\bar{H}|^2]\}. \quad (24)$$

By applying Poynting's theorem (16), the force density on free currents (24) is written as

$$\langle \bar{f}_c \rangle = -\frac{1}{2}\frac{\bar{k}_R}{\omega}\,\mathrm{Re}\{\bar{\nabla}\cdot\bar{S}\} \equiv -\hat{s}\frac{1}{2}\,\mathrm{Re}\{\bar{\nabla}\cdot\bar{p}\}, \quad (25)$$

where $\bar{S}=\bar{E}\times\bar{H}^*$ is the complex Poynting power and $\bar{p}\equiv n\bar{S}/c$ is the momentum flux density of the wave. Thus the direction of the force on free currents depends upon the sign of the index of refraction $n=ck_R/\omega$.

Transmission of the momentum flux density $\bar{p}$ ensures that the tangential force due to RT at the boundary is zero. To demonstrate this fact, we treat a TE polarized wave $\bar{E}=\hat{y}E_0e^{-k_{zI}z}e^{ik_{zR}z}e^{ik_xx}$ transmitted into the material and integrate the force density on free currents over the region $z\in[0,\infty)$. The analysis is simplified by applying the divergence theorem to Eq. (25). Thus the total pressure on free currents is $\bar{p}$ evaluated at $z=0+$, where it is assumed that the fields attenuate to zero as $z\to\infty$ due to losses. The tangential component of the resulting force is

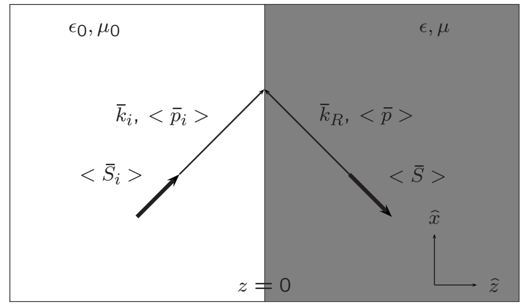

FIG. 3. Average power and momentum flux of a monochromatic wave refracted at the boundary of free space (μ_0,ϵ_0) and a matched LHM $(\mu=-\mu_0,\ \epsilon=-\epsilon_0)$ occupying the region $z>0$. The incident power $\langle \bar{S}_i\rangle$ and momentum $\langle \bar{p}\rangle$ are parallel, while the transmitted power $\langle \bar{S}\rangle$ and momentum flux $\langle \bar{p}\rangle$ are antiparallel.

$$\hat{x}\cdot\langle \bar{F}_c\rangle = \frac{\epsilon_0}{2}|E_i|^2(1-|R_{hs}|^2)\cos\theta_i\sin\theta_i, \quad (26)$$

where $|E_i|^2$ is the intensity of the incident wave and θ_i is the incident angle. In deriving Eq. (26), we have applied $E_0=E_iT_{hs}$ and the fundamental relationship between the half-space reflection coefficient R_{hs} and transmission coefficient T_{hs} resulting from the boundary conditions [29]. The fact that Eq. (26) gives the total tangential momentum transfer to the half-space [20,27,41] leads to the conclusion that the tangential component of $\langle \bar{F}_b\rangle$ is identically zero. In fact, it is straightforward to verify that this is true by integrating Eq. (23) over $z\in[0,\infty)$. Thus the force on a half-space void of free currents is normal to the surface. Just as $\langle \bar{F}_c\rangle$ gives the momentum transfer to free carriers inside the medium, $\langle \bar{F}_b\rangle$ can be interpreted as the momentum transfer due to RT at the boundary since it can be computed by the application of the Maxwell stress tensor along a surface that just encloses the boundary [24,27]. This view of momentum conservation is shown in Fig. 3 for the case of $\epsilon=-\epsilon_0$ and $\mu=-\mu_0$ previously considered [23]. The momentum flux is seen to be in the opposite direction of the Poynting power and the tangential projection of the wave momentum is conserved at the interface with an LHM resulting in zero tangential force.

B. Radiation pressure on a slab

A more physically realistic situation of an electromagnetic wave incident from free space onto an absorbing slab is now considered. The expression for the radiation pressure [24]

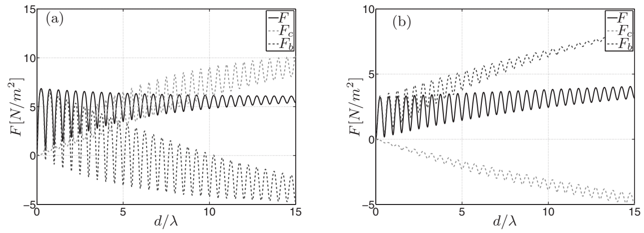

FIG. 4. Radiation pressure of a normally incident wave on an absorbing slab as a function of thickness d. The radiation pressure is decomposed into the force on free currents and the force on bound currents for (a) dielectric slab ($\mu=\mu_0$) with index of refraction $n+i\kappa=4+i0.04$ and (b) LHMs with $\mu=(-1+0.01)\mu_0$ and $\epsilon=(-4+i0.04)\epsilon_0$. The thickness is normalized to the wavelength inside the material.

$$\langle \bar{F} \rangle = \frac{\epsilon_0}{2}|E_i|^2[1+|R_{slab}|^2-|T_{slab}|^2], \tag{27}$$

due to a plane wave of intensity $|E_i|^2$ incident normal to the surface of a slab occupying the region $0<z<d$ satisfies the conservation of free space electromagnetic momentum. The reflection coefficient R_{slab} and transmission coefficient T_{slab} have well-known closed form solutions [29]. Also, the total force can be decomposed into the force on bound currents and the force on free currents.

Figure 4(a) gives an example of momentum transfer to an absorbing dielectric. In most situations, the observed force is expected to be the total force $\langle F \rangle$, which is always positive. However, several experiments have been devised to confirm the dependence of $\langle F_c \rangle$ upon upon n. Indeed, the photon drag effect [11,13], photon recoil in a dilute gas of atoms [14], and the deflection of mirrors immersed in dielectric fluids [12,42] provide experimental evidence that the wave momentum is directly proportional to the index of refraction in dielectrics. This is evident in Fig. 4(a) since the force on free currents is greater than the total force on the slab when the slab thickness is comparable to or greater than the penetration depth of the wave. By virtue of linear momentum conservation, a negative force on bound currents is required as shown in Fig. 4(a). This recoil force is unobserved in most experiments. However, we believe that one of the earliest observations of this force was made by Poynting and Barlow. In an experiment to measure the recoil of light against its source, Poynting and Barlow measured the deflection of disks under steady illumination [43]. One of these experiments involving light incident upon the absorbing side of a disk produced an initial suction. It was explained that the effect was due to heating of occluded gas from the silver film on the unilluminated side of the disk causing back pressure on the film. From the theory presented here, the heated gas obtained a momentum greater than the incident radiation pressure due to the absorption of electromagnetic energy in the disk. However, Poynting and Barlow observed the recoil force directly, which we explain as the force on bound currents $\langle F_b \rangle$.

A similar situation may be envisioned for an LHM slab as shown in Fig. 4(b). As with the dielectric slab in Fig. 4(a), the total pressure is always positive. However, the momentum transfer to free currents in LHMs is negative since $\langle \bar{f}_c \rangle$ is proportional to n. Therefore as the wave attenuates in an LHM, the free currents are pulled toward the incident wave, thus proving that radiation pressure in LHMs is negative. As required by momentum conservation, the force on bound currents is positive. Thus while the total force remains positive, the force on free currents is negative, and the force on bound currents is positive for an absorbing LHM slab, which is in contrast to the situation for a RHM slab.

IV. DISCUSSION

We have rigorously treated the momentum transfer to stationary, isotropic LHMs by applying the classical electromagnetic wave theory. Contrary to previous attempts to describe the momentum of the electromagnetic wave in LHMs, we have included material dispersion and losses, which are necessary for a causal medium with negative index of refraction. In this regard, the standard Lorentz model was employed for the polarization and magnetization, which is consistent with a previous derivation of the electromagnetic wave energy [25]. Thus the derived expressions for the momentum given by Eq. (11) is analogous to the energy of the wave in Eq. (8). We recognize that the results in Eqs. (8) and (11) are open to some interpretation. It is known that while the mathematical validity of Poynting's theorem is unquestionable, its interpretation is subject to some criticism [31]. For example, it is certainly possible in many cases to algebraically rearrange terms in Eq. (8) so that the energy flow and energy density take different forms, while the prediction of measurable quantities such as time-average Poynting power and energy dissipation remain unaltered. Likewise, we may assume that the momentum conservation theorem given by Eq. (11) may also be subject to similar manipulations. However, the measurable results predicted by the application of the cycle-average theorems (16) and (19) are unambiguous. Also, the results presented here depend upon the model used for the material response in the presence of the electromagnetic fields. For example, the two time derivative Lorentz material model

$$\left(\frac{\partial^2}{\partial t^2}+\gamma_m\frac{\partial}{\partial t}+\omega_{m0}^2\right)\bar{\mathcal{M}}=\left(\omega_{mp}^2\chi_\alpha^m+\omega_{mp}\chi_\beta^m\frac{\partial}{\partial t}+\chi_\gamma^m\frac{\partial^2}{\partial t^2}\right)\bar{\mathcal{H}} \tag{28}$$

was previously employed to describe the magnetic response of an LHM [44]. This model reduces to the standard Lorentz model for $\xi_\beta^m=\xi_\gamma^m=0$. Another variation in which $\xi_\alpha^m=\xi_\beta^m=0$ was recently applied to derive an alternate form of the electromagnetic energy density in LHMs, and it was acknowledged that this alternate form maps very closely to the result derived from the standard Lorentz media model [45]. In this regard, the various models produce equations for the electromagnetic wave energy that differ in form, but give similar quantitative results in the negative index frequency ranges where the models have overlapping validity. Thus it is expected that the results for momentum, like the energy, should remain both qualitatively and quantitatively similar in frequency bands where multiple models are valid. Furthermore, we note that the results derived here for momentum and energy reduce to the known expressions for a single resonance Lorentz dielectric [17,35].

We have also shown that the average momentum density vector of a monochromatic wave may be either parallel or antiparallel to the average Poynting vector in a material with negative index of refraction. However, the momentum conservation equation given by Eq. (19) depends only upon the average momentum flow in Eq. (18). Thus we expect that any observation of momentum transfer in RHMs or LHMs due to a monochromatic wave is independent of the details of dispersion. Furthermore, these results provide a rigorous derivation of the force on free currents and further validation of the theoretical separation of force based on the real and imaginary parts of the permittivity and permeability [27]. The theory was applied to calculate the radiation pressure on an infinite half-space and a lossy slab. For the hypothetical problem of a lossless half-space (i.e., $\mu_I = \epsilon_I = 0$), the material is pulled toward the incident wave when $n > 1$ [24,46] and pushed when $n < 1$, which includes the negative index regime. We also determined that a slab in vacuum is pushed by a monochromatic wave regardless of the values for μ and ϵ. This differs from the results previously reported for a finite pulse [44], where it was concluded that the pulse attracts a slab when $0 < \mu, \epsilon < 1$. It is possible, however, to consider a slab embedded in a left-handed vacuum [47], where we make the replacements $\mu_0 \rightarrow -\mu_0$ and $\epsilon_0 \rightarrow -\epsilon_0$ for the free space background. In this case, the slab is pulled toward the incident wave consistent with the prediction of radiation attraction by Veselago [10]. Although the results are not shown, we have also confirmed that the force of a plane wave exerted upon a cylinder [48–52] and upon a sphere [27] is qualitatively the same in that radiation pressure exists when $n > 0$ for the background and radiation attraction occurs when $n < 0$ for the background.

Finally, we may conclude that the theory presented here attaches fundamental physical meaning to Snell's law; the reflected and transmitted wave vectors ensure conservation of the momentum component which is parallel to the boundary. Likewise, the magnitudes of the reflected and transmitted waves ensure conservation of wave energy at the interface. This assertion holds for LHMs and ensures that no sheering force exists due to RT at the interface.

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[1] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[2] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[3] J. Pacheco, T. M. Grzegorczyk, B. I. Wu, Y. Zhang, and J. A. Kong, Phys. Rev. Lett. **89**, 257401 (2002).
[4] C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbah, and M. Tanielian, Phys. Rev. Lett. **90**, 107401 (2003).
[5] J. Lu, T. M. Grzegorczyk, Y. Zhang, J. Pacheco, B. I. Wu, and J. A. Kong, Opt. Express **11**, 723 (2003).
[6] R. Matloob and A. Ghaffari, Phys. Rev. A **70**, 052116 (2004).
[7] Y. O. Averkov and V. M. Yakovenko, Phys. Rev. B **72**, 205110 (2005).
[8] S. D. Korovin, A. A. Eltchaninov, V. V. Rostov, V. G. Shpak, M. I. Yalandin, N. S. Ginzburg, A. S. Sergeev, and I. V. Zotova, Phys. Rev. E **74**, 016501 (2005).
[9] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[10] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[11] A. F. Gibson, M. F. Kimmitt, and A. C. Walker, Appl. Phys. Lett. **17**, 75 (1970).
[12] R. V. Jones and B. Leslie, Proc. R. Soc. London, Ser. A **360**, 347 (1978).
[13] A. F. Gibson, M. F. Kimmitt, A. O. Koohian, D. E. Evans, and G. F. D. Levy, Proc. R. Soc. London, Ser. A **370**, 303 (1980).
[14] G. K. Campbell, A. E. Leanhardt, J. Mun, M. Boyd, E. W. Streed, W. Ketterle, and D. E. Pritchard, Phys. Rev. Lett. **94**, 170403 (2005).
[15] J. P. Gordon, Phys. Rev. A **8**, 14 (1973).
[16] D. F. Nelson, Phys. Rev. A **44**, 3985 (1991).
[17] R. Loudon, L. Allen, and D. F. Nelson, Phys. Rev. E **55**, 1071 (1997).
[18] S. Stallinga, Phys. Rev. E **73**, 026606 (2006).
[19] Y. N. Obukhov and F. W. Hehl, Phys. Lett. A **311**, 277 (2003).
[20] M. Mansuripur, Opt. Express **12**, 5375 (2004).
[21] R. Loudon, Fortschr. Phys. **52**, 1134 (2004).
[22] M. Scalora, G. D'Aguanno, N. Mattiucci, M. J. Bloemer, M. Centini, C. Sibilia, and J. W. Haus, Phys. Rev. E **73**, 056604 (2006).
[23] S. Riyopoulos, Opt. Lett. **31**, 2480 (2006).
[24] B. A. Kemp, T. M. Grzegorczyk, and J. A. Kong, Opt. Express **13**, 9280 (2005).
[25] T. J. Cui and J. A. Kong, Phys. Rev. B **70**, 205106 (2004).
[26] R. Loudon, S. M. Barnett, and C. Baxter, Phys. Rev. A **71**, 063802 (2005).
[27] B. A. Kemp, T. M. Grzegorczyk, and J. A. Kong, Phys. Rev. Lett. **97**, 133902 (2006).
[28] P. Penfield and H. A. Haus, *Electrodynamics of Moving Media* (MIT Press, Cambridge, MA, 1967).
[29] J. A. Kong, *Electromagnetic Wave Theory* (EMW Publishing, Cambridge, MA, 2005).
[30] M. Abraham, Rend. Pal. **28**, 1 (1909).
[31] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).
[32] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York,

1999).
[33] L. Brillouin, *Wave Propagation and Group Velocity* (Academic Press, New York, 1960).
[34] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, New York, 1984).
[35] R. Loudon, J. Phys. A **3**, 233 (1970).
[36] H. Minkowski, Nachr. Ges. Wiss. Goettingen, Math.-Phys. Kl. **1**, 53 (1908).
[37] B. A. Kemp, T. M. Grzegorczyk, and J. A. Kong, J. Electromagn. Waves Appl. **20**, 827 (2006).
[38] T. M. Grzegorczyk, C. D. Moss, J. Lu, X. Chen, J. Pacheco, and J. A. Kong, IEEE Trans. Microwave Theory Tech. **53**, 1443 (2005).
[39] T. M. Grzegorczyk, X. Chen, J. Pacheco, J. Chen, B.-I. Wu, and J. A. Kong, Prog. Electromagn. Res. **PIER 51**, 83 (2005).
[40] J. J. Chen, T. M. Grzegorczyk, B. I. Wu, and J. A. Kong, J. Appl. Phys. **98**, 094905 (2005).
[41] J. H. Poynting, Philos. Mag. **24**, 156 (1905).
[42] R. V. Jones and J. C. S. Richards, Proc. R. Soc. London, Ser. A **221**, 480 (1954).
[43] J. H. Poynting and G. Barlow, Proc. R. Soc. London, Ser. A **83**, 534 (1910).
[44] R. W. Ziolkowski, Phys. Rev. E **63**, 046604 (2001).
[45] A. D. Boardman and K. Marinov, Phys. Rev. B **73**, 165110 (2006).
[46] P. Daly and H. Gruenberg, J. Appl. Phys. **38**, 4486 (1967).
[47] W. C. Chew, Prog. Electromagn. Res. **PIER 51**, 1 (2005).
[48] T. M. Grzegorczyk, B. A. Kemp, and J. A. Kong, J. Opt. Soc. Am. A **23**, 2324 (2006).
[49] T. M. Grzegorczyk, B. A. Kemp, and J. A. Kong, Phys. Rev. Lett. **96**, 113903 (2006).
[50] T. M. Grzegorczyk, B. A. Kemp, and J. A. Kong, Opt. Lett. **31**, 3378 (2006).
[51] T. M. Grzegorczyk and J. A. Kong, J. Opt. Soc. Am. B **24**, 644 (2007).
[52] D. Maystre and P. Vincent, J. Opt. A, Pure Appl. Opt. **8**, 1059 (2007).