Trapping of light pulses in ensembles of stationary Λ atoms

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We present a detailed theoretical description of the generation of stationary light pulses by standing-wave electromagnetically induced transparency in media comprised of stationary atoms such as ultracold gasses and solids. We show that, contrary to thermal gas media, the achievable storage times are limited only by the ground state dephasing rate of the atoms, making such media ideally suited for nonlinear optical interactions between stored pulses. Furthermore, we find significant quantitative and qualitative differences between the two types of media, which are important for quantum-information processing schemes involving stationary light pulses.

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I. INTRODUCTION

The coherent transfer of quantum states between light and atoms has been the subject of much research, both experimentally and theoretically, motivated by potential applications in quantum computing, quantum cryptography, and teleportation. While the transfer of quantum states from light to a single atom can in principle be achieved by cavity QED techniques [1], the required strong-coupling regime is experimentally very difficult to reach. To overcome this difficulty the use of atomic ensembles, rather than single atoms, has been proposed [2–5] and implemented for storage of classical light pulses [6,7], and recently storage of nonclassical pulses has been demonstrated [8,9]. One such light storage scheme is based on electromagnetically induced transparency (EIT) [10] in ensembles of Λ-type atoms [3]. While the storage and retrieval of light pulses with this scheme has been demonstrated utilizing many different types of atomic ensembles, including Bose-Einstein condensates [6] and thermal gasses [7], as well as solid-state media [11,12], the nontrivial manipulation of, and interaction between, stored pulses is hampered by the inherent tradeoff between storage time and field amplitude. A step towards overcoming this problem was taken with the suggestion of using standing-wave fields to create a periodic modulation of either the dispersive [13] or the absorptive [14] properties of the medium, inducing a photonic band gap [15] and creating a stationary light pulse. Schemes to implement a controlled phase gate using these techniques have been proposed using either the dispersive [16] or the absorptive [17] grating technique, but both are still hampered by a tradeoff between storage time and field amplitude. For the latter case, only thermal gas media have been considered, and a detailed theoretical treatment of this case is given in [18]. This theory, however, does not apply to media comprised of stationary atoms such as ultracold gasses or solid-state media [19]. In this article we present a detailed theoretical treatment of the creation of stationary light pulses by the absorptive grating technique for media comprised of stationary atoms. We find that the loss of excitations inherent to the thermal gas case is absent for stationary atoms, making such media ideally suited for the kind of nonlinear optical interactions envisaged in [17]. Furthermore, we find interesting quantitative and qualitative differences between the thermal gas and ultracold gas cases when quasi-standing-wave coupling fields are considered. These differences are important for the proposed controlled phase gate scheme [17] in stationary atom media.

In Sec. II we present a detailed account of our theory of stationary light pulses in media comprised of stationary atoms and compare the results to the thermal gas case. The theory is complemented in Sec. III by a calculation of nonadiabatic corrections. A summary of our results is provided in Sec. IV. The Appendix contains a brief review of the theory of stationary light pulses in thermal gas media [18], reformulated in terms of polariton fields, used for comparison with the stationary atom case.

II. STANDING-WAVE POLARITONS IN ENSEMBLES OF STATIONARY ATOMS

We consider an ensemble of N nonmoving Λ atoms interacting with probe and coupling lasers propagating parallel to the z axis. The two lower states $|b\rangle$ and $|c\rangle$ of the atoms (see Fig. 1) are assumed to be nearly degenerate, such that the magnitude of the wave vectors of the probe and coupling lasers can be considered identical ($k_p \simeq k_c = k$).

The Hamiltonian for the N atom problem is

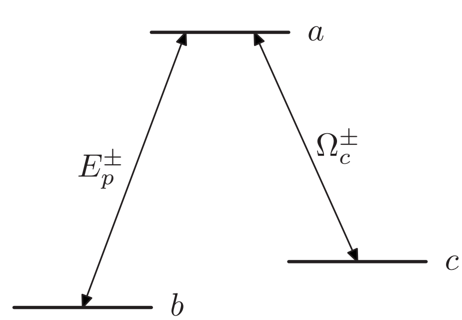


FIG. 1. The three-level Λ atom. The quantized probe field couples the ground state $|b\rangle$ to the excited state $|a\rangle$, while the classical coupling field couples the metastable state $|c\rangle$ to $|a\rangle$.

$$\hat{H} = \hat{H}_F + \sum_{j=1}^{N} (\hat{H}_A^j + \hat{H}_L^j + \hat{H}_V^j), \tag{1}$$

where $\hat{H}_F$ and $\hat{H}_A$ describe the free electromagnetic field and the atoms, $\hat{H}_L$ describes the interaction of the atoms with the probe and coupling fields, and $\hat{H}_V$ describes the interaction with the vacuum field modes. The individual terms are given by

$$\hat{H}_F = \sum_m \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m, \tag{2a}$$

$$\hat{H}_A^j = \hbar\omega_{cb}\hat{\sigma}_{cc}^j + \hbar\omega_{ab}\hat{\sigma}_{aa}^j, \tag{2b}$$

$$\hat{H}_L^j = -(\hat{\mathbf{E}}_p + \hat{\mathbf{E}}_c)\cdot(\mathbf{d}_{ba}\hat{\sigma}_{ba}^j + \mathbf{d}_{ca}\hat{\sigma}_{ca}^j + \text{H.a.}), \tag{2c}$$

$$\hat{H}_V^j = -\hat{\mathbf{E}}_V\cdot(\mathbf{d}_{ba}\hat{\sigma}_{ba}^j + \mathbf{d}_{ca}\hat{\sigma}_{ca}^j + \text{H.a.}). \tag{2d}$$

We introduce slowly varying field operators for the electromagnetic field. Since we are allowing for standing-wave fields, we write the field operator as a superposition of two traveling wave fields propagating in opposite directions,

$$\hat{\mathbf{E}}_{p,c}(z,t) = \sqrt{\frac{\hbar\omega_{p,c}}{2\varepsilon_0 V}}\mathbf{e}_{p,c}E_{p,c}(z,t)e^{-i\omega_{p,c}t} + \text{H.c.}, \tag{3}$$

where the operators $E_{p,c}$ are given by

$$E_{p,c}(z,t) = E_{p,c}^{+}(z,t)e^{ikz} + E_{p,c}^{-}(z,t)e^{-ikz}. \tag{4}$$

The field operators $E_{p,c}^{\pm}$ are the slowly varying field operators for the forward and backward propagating components of the probe and coupling fields with carrier frequencies $\omega_{p,c}$, and $\mathbf{e}_{p,c}$ are the respective polarization vectors.

We define continuum atomic operators $\hat{\sigma}_{\mu\nu}$ by summing over the individual atoms in a small volume V, and introduce slowly varying atomic operators $\sigma_{\mu\nu}$ defined by

$$\hat{\sigma}_{ba} = \sigma_{ba}e^{-i\omega_p t}, \tag{5a}$$

$$\hat{\sigma}_{ca} = \sigma_{ca}e^{-i\omega_c t}, \tag{5b}$$

$$\hat{\sigma}_{bc} = \sigma_{bc}e^{-i(\omega_p-\omega_c)t}. \tag{5c}$$

Notice that the operators defined by Eq. (5) are slowly varying in *time*, but not in *space*.

The Heisenberg-Langevin equations for these operators in the rotating-wave approximation are

$$\dot{\sigma}_{aa} = -i(g_pE_p^\dagger\sigma_{ba} + \Omega_c^*\sigma_{ca} - \text{H.a.}) - \gamma\sigma_{aa} + F_{aa}, \tag{6a}$$

$$\dot{\sigma}_{bb} = i(g_pE_p^\dagger\sigma_{ba} - \text{H.a.}) + \gamma_b\sigma_{aa} + F_{bb}, \tag{6b}$$

$$\dot{\sigma}_{cc} = i(\Omega_c^*\sigma_{ca} - \text{H.a.}) + \gamma_c\sigma_{aa} + F_{cc}, \tag{6c}$$

$$\dot{\sigma}_{ba} = i[g_pE_p(\sigma_{bb}-\sigma_{aa}) + \Omega_c\sigma_{bc}] - \Gamma_{ba}\sigma_{ba} + F_{ba}, \tag{6d}$$

$$\dot{\sigma}_{ca} = i[\Omega_c(\sigma_{cc}-\sigma_{aa}) + g_pE_p\sigma_{bc}^\dagger] - \Gamma_{ca}\sigma_{ca} + F_{ca}, \tag{6e}$$

$$\dot{\sigma}_{bc} = i(\Omega_c^*\sigma_{ba} - g_pE_p\sigma_{ca}^\dagger) - \Gamma_{bc}\sigma_{bc} + F_{bc}, \tag{6f}$$

where $\gamma = \gamma_b + \gamma_c$ is the decay rate of the excited state $|a\rangle$ into the two lower states. The complex decay rates $\Gamma_{\mu\nu}$ are given by

$$\Gamma_{ba} = \gamma_{ba} - i\delta_p, \tag{7}$$

$$\Gamma_{ca} = \gamma_{ca} - i\delta_c, \tag{8}$$

$$\Gamma_{bc} = \gamma_{bc} - i\Delta, \tag{9}$$

where $\gamma_{\mu\nu}$ are the dephasing rates of the respective coherences, $\delta_{p,c}$ are the one-photon detunings of the probe and coupling lasers, respectively, and Δ is the two-photon detuning. We have also assumed that the coupling field can be treated as a classical field with Rabi frequency Ω_c given by

$$\Omega_c(z,t) = \Omega_c^{+}(z,t)e^{ikz} + \Omega_c^{-}(z,t)e^{-ikz}. \tag{10}$$

In the following we shall disregard the noise operators $F_{\mu\nu}$ since we will be considering the adiabatic limit.

A. Weak probe approximation

In order to solve the propagation problem, we assume that the probe field is weak compared to the coupling field and that the probe photon density is small compared to the atomic density. In this case the Heisenberg-Langevin equations can be solved perturbatively. To first order in the probe field amplitude, the relevant Heisenberg-Langevin equations are

$$\sigma_{ba} = \frac{1}{i\Omega_c^*}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)\sigma_{bc}, \tag{11a}$$

$$\sigma_{bc} = -\frac{g_pE_p}{\Omega_c} - \frac{i}{\Omega_c}\left(\Gamma_{ba} + \frac{\partial}{\partial t}\right)\sigma_{ba}. \tag{11b}$$

Combining Eqs. (11) we can obtain a differential equation for σ_{bc},

$$\sigma_{bc} = -\frac{g_pE_p}{\Omega_c} - \frac{1}{\Omega_c}\left(\Gamma_{ba} + \frac{\partial}{\partial t}\right)\left[\frac{1}{\Omega_c^*}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)\sigma_{bc}\right]. \tag{12}$$

B. Adiabatic limit

In order to solve Eq. (12), we consider the adiabatic limit in which the fields vary slowly in time. Introducing a characteristic time scale T of the slowly varying operators, we expand σ_{bc} in powers of $(\gamma_{ba}T)^{-1}$. To zero order we find

$$\sigma_{bc} = -\frac{g_pE_p}{\Omega_c}. \tag{13}$$

Inserting this expression into Eq. (11a), we find an expression for σ_{ba} valid in the adiabatic limit

$$\sigma_{ba} = -\frac{1}{i\Omega_c^*}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)\left(\frac{g_pE_p}{\Omega_c}\right). \tag{14}$$

By inserting the field decomposition (4) into the adiabatic expression for σ_{ba} (14) we obtain

$$\sigma_{ba} = \frac{-g_p\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)}{i\Omega[1 + 2|\kappa^+||\kappa^-|\cos(2kz + \phi)]}\left(\frac{E_p^+ e^{ikz} + E_p^- e^{-ikz}}{\Omega}\right), \tag{15}$$

where we have also introduced the time-dependent total Rabi frequency $\Omega(t) = \sqrt{|\Omega_c^+|^2 + |\Omega_c^-|^2}$ and the ratios $\kappa^\pm = \frac{\Omega_c^\pm}{\Omega}$, which are assumed to be constant. The phase angle ϕ is defined by the relation

$$\kappa^+\kappa^{-*} = |\kappa^+||\kappa^-|e^{i\phi}. \tag{16}$$

C. Polariton field

We now introduce a dark-state polariton (DSP) field analogous to the DSP field defined in [3],

$$E_p^\pm(z,t) = \cos\,\theta(t)\Psi^\pm(z,t), \tag{17}$$

where the angle θ is given by the total coupling laser Rabi frequency through

$$\tan\,\theta(t) = \frac{g_p\sqrt{N_{\mathbf{r}}}}{\Omega(t)}. \tag{18}$$

By inserting the definition (17) of the DSP field into Eq. (15) we obtain

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba} = \frac{-\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)}{i\Omega[1 + 2|\kappa^+||\kappa^-|\cos(2kz + \phi)]} \times [\sin\,\theta(\Psi^+ e^{ikz} + \Psi^- e^{-ikz})]. \tag{19}$$

To derive wave equations for the components of the DSP field, we need to expand the optical coherence σ_{ba} in spatial Fourier components. We do this by inserting the Fourier series

$$\frac{1}{1 + y\cos x} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx), \tag{20}$$

where $y = 2|\kappa^+||\kappa^-|$ and $x = 2kz + \phi$, into Eq. (19). Note that $y \leq 1$, which guarantees the existence of the Fourier series except in the case of a standing-wave coupling field ($y=1$). Fortunately, we can treat this case successfully by considering the limit $y \to 1$ at the end of our calculation.

Inserting the Fourier series into Eq. (19) we find

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba} = \frac{i}{\Omega}\left(\frac{a_0}{2} + \sum_{n=1}^{\infty}\frac{a_n}{2}(e^{in(2kz+\phi)} + e^{-in(2kz+\phi)})\right) \times \left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)[\sin\,\theta(\Psi^+ e^{ikz} + \Psi^- e^{-ikz})]. \tag{21}$$

From Eq. (21) we see that σ_{ba} can be written as

$$\sigma_{ba} = \sum_{n=-\infty}^{\infty} \sigma_{ba}^{(2n+1)} e^{i(2n+1)kz}. \tag{22}$$

To derive a set of wave equations for the polariton field components, we need to calculate the components σ_{ba}^{+1} and σ_{ba}^{-1} of the expansion (22) which we label $\sigma_{ba}^\pm$ for brevity. These components are given by

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba}^+ = \frac{i}{2\Omega}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)[\sin\,\theta(a_0\Psi^+ + a_1 e^{i\phi}\Psi^-)], \tag{23a}$$

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba}^- = \frac{i}{2\Omega}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)[\sin\,\theta(a_0\Psi^- + a_1 e^{-i\phi}\Psi^+)]. \tag{23b}$$

We see that we only need to calculate the first two Fourier coefficients a_0 and a_1 of the expansion (21). These are given by

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\mathrm{d}x}{1 + y\cos x} = \frac{2}{\sqrt{1 - y^2}}, \tag{24a}$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\cos x dx}{1 + y\cos x} = 2\frac{\sqrt{1 - y^2} - 1}{y\sqrt{1 - y^2}}. \tag{24b}$$

Inserting the adiabatic expression (23) for $\sigma_{ba}^\pm$ into the wave equations for the probe field components

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)E_p^+(z,t) = ig_pN_{\mathbf{r}}\sigma_{ba}^+(z,t), \tag{25a}$$

$$\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial z}\right)E_p^-(z,t) = ig_pN_{\mathbf{r}}\sigma_{ba}^-(z,t), \tag{25b}$$

we obtain a set of coupled wave equations for the DSP field components,

$$\frac{\partial\Psi^+}{\partial t} + c\cos^2\theta'\frac{\partial\Psi^+}{\partial z} = -\sin^2\theta'\left[\Gamma_{bc}\Psi^+ + se^{i\phi}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)\Psi^- - s\dot{\theta}\frac{\cos\,\theta}{\sin\,\theta}(y\Psi^+ - e^{i\phi}\Psi^-)\right], \tag{26a}$$

$$\frac{\partial\Psi^-}{\partial t} - c\cos^2\theta'\frac{\partial\Psi^-}{\partial z} = -\sin^2\theta'\left[\Gamma_{bc}\Psi^- + se^{-i\phi}\left(\Gamma_{bc} + \frac{\partial}{\partial t}\right)\Psi^+ - s\dot{\theta}\frac{\cos\,\theta}{\sin\,\theta}(y\Psi^- - e^{-i\phi}\Psi^+)\right], \tag{26b}$$

where we have introduced a new angle θ' defined by

$$\tan\,\theta' = \sqrt{\frac{a_0}{2}}\frac{g_p\sqrt{N_{\mathbf{r}}}}{\Omega} = \sqrt{\frac{a_0}{2}}\tan\,\theta, \tag{27}$$

as well as the constant

$$s = \frac{a_1}{a_0} = \frac{\sqrt{1 - y^2} - 1}{y}. \tag{28}$$

Since we are considering the adiabatic limit in which the coupling field Rabi frequency changes slowly in time, we shall neglect the last term on the right-hand side of Eq. (26) in the following.

D. Low-group-velocity limit

In the experimentally relevant *low-group-velocity limit* $\cos^2\theta \ll 1$, the wave equations (26) take the simpler form

$$\left(\Gamma_{bc}+\frac{\partial}{\partial t}\right)\Psi^+ + |\kappa^+|^2 v_g \frac{\partial \Psi^+}{\partial z} = \kappa^+\kappa^{-*} v_g \frac{\partial \Psi^-}{\partial z}, \quad (29a)$$

$$\left(\Gamma_{bc}+\frac{\partial}{\partial t}\right)\Psi^- - |\kappa^+|^2 v_g \frac{\partial \Psi^-}{\partial z} = -\kappa^{+*}\kappa^{-} v_g \frac{\partial \Psi^+}{\partial z}, \quad (29b)$$

in the case where $|\kappa^+| \geq |\kappa^-|$. In the opposite case, $|\kappa^+| \leq |\kappa^-|$, the wave equations are

$$\left(\Gamma_{bc}+\frac{\partial}{\partial t}\right)\Psi^+ + |\kappa^-|^2 v_g \frac{\partial \Psi^+}{\partial z} = \kappa^+\kappa^{-*} v_g \frac{\partial \Psi^-}{\partial z}, \quad (30a)$$

$$\left(\Gamma_{bc}+\frac{\partial}{\partial t}\right)\Psi^- - |\kappa^-|^2 v_g \frac{\partial \Psi^-}{\partial z} = -\kappa^{+*}\kappa^{-} v_g \frac{\partial \Psi^+}{\partial z}. \quad (30b)$$

We have introduced the group velocity $v_g = c\cos^2\theta$ in Eqs. (29) and (30), and have also made use of the fact that in the low-group-velocity limit, $\cos^2\theta' \simeq \sqrt{1-y^2}\cos^2\theta$.

E. Initial conditions

We shall consider the same kind of experiment as in [14] in which a probe pulse, propagating under the influence of a copropagating *traveling-wave* coupling field, is stored in the medium and subsequently retrieved by a *standing-wave* coupling field with $|\kappa^+| \geq |\kappa^-|$.

Assuming that the standing-wave coupling field is switched on at $t=0$, we need to find the initial conditions for the two components of the DSP field $\Psi^{\pm}(z,0)$. The initial condition for the Raman coherence is

$$\sqrt{N_{\mathbf{r}}}\sigma_{bc}(z,0) = -\Psi(z,0), \quad (31)$$

where $\Psi(z,0)$ is a known function of z determined by the DSP field prior to switching on the standing-wave coupling field. Using Eq. (13) and the definition of the DSP field in the standing wave case (17), along with the initial condition (31), we obtain

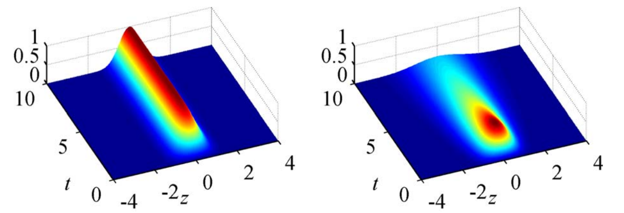
$$\Psi(z,0)(\kappa^+ e^{ikz} + \kappa^- e^{-ikz}) = \Psi^+(z,0)e^{ikz} + \Psi^-(z,0)e^{-ikz}. \quad (32)$$

From this expression we see that the initial conditions for the components of the DSP field $\Psi^{\pm}(z,0)$ are

$$\Psi^+(z,0) = \kappa^+\Psi(z,0), \quad \Psi^-(z,0) = \kappa^-\Psi(z,0). \quad (33)$$

With the initial conditions (33), we find the solution

$$\Psi^+(z,t) = \frac{\kappa^+}{2}\left[\left(1+\frac{\beta}{|\kappa^+|^2}\right)\Psi[z-\beta r(t),0] + \left(1-\frac{\beta}{|\kappa^+|^2}\right)\Psi[z+\beta r(t),0]\right]e^{-\Gamma_{bc}t}, \quad (34a)$$



(a) Stationary atoms (b) Thermal gas

FIG. 2. (Color online) Retrieval of a stored probe pulse with a standing-wave coupling field. The probe field energy density, in units of $\frac{\hbar\omega_p}{V}|E_0|^2$, is plotted for a medium comprised of stationary atoms (a) and thermal atoms (b) as a function of z in units of the pulse length L_p, and t in units of the switching time T_s. The absorption length of the media is taken to be $l_a = 0.1 \times L_p$.

$$\Psi^-(z,t) = \frac{\kappa^-}{2}\{\Psi[z-\beta r(t),0] + \Psi[z+\beta r(t),0]\}e^{-\Gamma_{bc}t}, \quad (34b)$$

where $\beta = \sqrt{|\kappa^+|^2(|\kappa^+|^2 - |\kappa^-|^2)}$ and $r(t) = \int_0^t c\cos^2\theta(t')dt'$.

F. Probe retrieval by a standing-wave coupling field

To determine the solution for a standing-wave coupling field, we let $\kappa^{\pm} \to \frac{1}{\sqrt{2}}$ in the solution (34). In this limit we find the solution

$$\Psi^+(z,t) = \frac{1}{\sqrt{2}}\Psi(z,0)e^{-\Gamma_{bc}t}, \quad (35a)$$

$$\Psi^-(z,t) = \frac{1}{\sqrt{2}}\Psi(z,0)e^{-\Gamma_{bc}t}. \quad (35b)$$

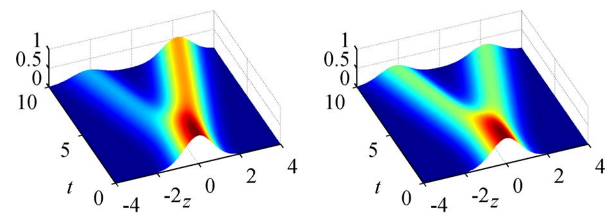
As an example, we take the initial condition for the DSP field to be $\Psi(z,0) = \Psi_0 \exp[-(z/L_p)^2]$, where L_p is the characteristic length of the stored probe pulse. The polariton amplitude Ψ_0 is related to the initial probe field amplitude E_0 by $\Psi_0 = E_0/\cos\theta_0$, where θ_0 is determined by the Rabi frequency of the traveling-wave coupling field prior to storage.

The components of the retrieved probe field found from Eq. (17) are

$$E_p^+(z,t) = \frac{1}{\sqrt{2}}\frac{\cos\theta(t)}{\cos\theta_0}E_0\exp[-(z/L_p)^2]e^{-\Gamma_{bc}t}, \quad (36a)$$

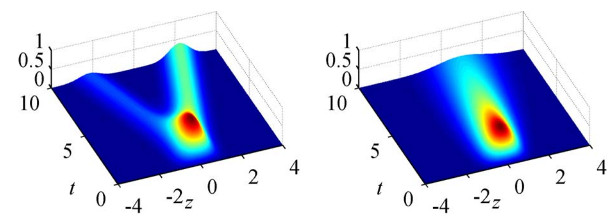
$$E_p^-(z,t) = \frac{1}{\sqrt{2}}\frac{\cos\theta(t)}{\cos\theta_0}E_0\exp[-(z/L_p)^2]e^{-\Gamma_{bc}t}. \quad (36b)$$

In Fig. 2 we compare the retrieval of an initially stored probe pulse by a standing-wave coupling field in the thermal gas and ultracold gas cases. The time dependence of the angle θ is assumed to be given by $\cos^2\theta(t) = \cos^2\theta_0 \tanh(t/T_s)$ for $t \geq 0$, where T_s is the characteristic switching time. For simplicity, we have assumed zero Raman dephasing ($\Gamma_{bc}=0$) and taken the characteristic length of the stored probe pulse to be



(a) $\Psi^{+}(z,t)$ (b) $\Psi^{-}(z,t)$

FIG. 3. (Color online) Retrieval of a stored probe pulse with a quasi-standing-wave coupling field ($\kappa^{+}=\sqrt{0.55}$, $\kappa^{-}=\sqrt{0.45}$). (a) and (b) show the polariton amplitudes $\Psi^{\pm}$ in units of Ψ_0 as a function of z and t, in units of L_p and T_s, respectively.



(a) Cold gas (b) Thermal gas

FIG. 4. (Color online) Retrieval of a stored probe pulse with a quasi-standing-wave coupling field. The probe field energy density, in units of $\frac{\hbar\omega_p}{V}|E_0|^2$, is shown for both the ultracold gas case (a) and for the thermal gas case (b) as a function of z and t, in units of L_p and T_s, respectively. Parameters are the same as in Fig. 3.

$L_p=v_{g,0}T_s$, where $v_{g,0}=c\cos^2\theta_0$. The probe field photon density averaged over many wavelengths $|E_p^{+}|^2+|E_p^{-}|^2$, in units of the photon density prior to storage $|E_0|^2$, is plotted as a function of z in units of L_p and t in units of T_s. In both the stationary atom case and the thermal gas case we see that the stored probe pulse is revived into a stationary probe field, but we note that in the stationary atom case, the diffusive broadening of the probe field, evident in the thermal gas case, is absent.

The solution for thermal gas media is based on the theory in [18], which is reviewed briefly in the Appendix. The medium is characterized by the absorption length in the absence of EIT $l_a=0.1\times L_p$, which roughly corresponds to the conditions in [14].

G. Probe retrieval by a quasi-standing-wave coupling field

We shall now study the situation in which the probe field is retrieved by a quasi-standing-wave coupling field. In Ref. [18], it was shown that in the thermal gas case a quasi-standing-wave coupling field leads to a drift of the revived probe pulse in the direction of the stronger of the two coupling field components, see also Eq. (A3) in the appendix.

In the ultracold gas case considered here, we find from the solution (34) that the revived probe pulse instead splits into two parts. A stronger part which propagates in the direction of the stronger of the coupling field components, and a weaker part which propagates in the opposite direction.

Figure 3 shows the solution (34) with the same initial conditions as in Fig. 2, but with $\kappa^{+}=\sqrt{0.55}$ and $\kappa^{-}=\sqrt{0.45}$. Figure 4 compares the retrieval of a stored probe pulse by a quasi-standing-wave coupling field in thermal and ultracold gas media. The splitting of the revived probe pulse is clearly evident in the cold gas case, indicating a qualitative difference between the thermal gas and the ultracold gas cases. The cause of this difference is the coupling to the high spatial-frequency components of the Raman coherence $\hat{\sigma}_{bc}$ in the ultracold gas case. This splitting of the probe pulse is very important when considering various schemes for interacting pulses. In the phase-gate proposal of André *et al.* [17], a small imbalance in the two components of the coupling field is used to propagate a quasistationary light pulse across a stored excitation in a thermal gas medium. As is evident from Fig. 4 this scheme would not work in media comprised of stationary atoms, since a large part of the revived probe field would then propagate in the wrong direction.

H. Calculation of the Raman coherence

To calculate the Raman coherence of the atoms, we use the zero-order expression (13) for σ_{bc}

$$\sqrt{N_{\mathbf{r}}}\sigma_{bc}=-\frac{g_p\sqrt{N_{\mathbf{r}}}E_p}{\Omega_c}. \tag{37}$$

By inserting the decompositions of the probe and coupling fields, as well as the definition (17) of the DSP field, we obtain

$$\sqrt{N_z}\sigma_{bc}=-\sin\theta\frac{\Psi^{+}(z,t)e^{ikz}+\Psi^{-}(z,t)e^{-ikz}}{\kappa^{+}e^{ikz}+\kappa^{-}e^{-ikz}}. \tag{38}$$

Inserting the solution (34) into this expression, we find by a binomial expansion

$$\sqrt{N_z}\sigma_{bc}=-\frac{1}{2}\sin\theta\Bigg[\Psi(z-\beta r,0)+\Psi(z+\beta r,0)+\frac{\beta}{|\kappa^{+}|^2}[\Psi(z-\beta r,0)-\Psi(z+\beta r,0)]\times\sum_{n=0}^{\infty}\left(-\frac{\kappa^{-}}{\kappa^{+}}\right)^{n}e^{-2inkz}\Bigg]e^{-\Gamma_{bc}t}. \tag{39}$$

From this expression we see that the Raman coherence can be written as

$$\sigma_{bc}(z,t)=\sum_{n=-\infty}^{\infty}\sigma_{bc}^{(2n)}(z,t)e^{2inkz}, \tag{40}$$

where the dc component is

$$\sqrt{N_{\mathbf{r}}}\sigma_{bc}^{(0)} = -\frac{1}{2}\sin\theta\left[\left(1+\frac{\beta}{|\kappa^+|^2}\right)\Psi[z-\beta r(t),0] + \left(1-\frac{\beta}{|\kappa^+|^2}\right)\Psi[z+\beta r(t),0]\right]e^{-\Gamma_{bc}t}. \quad (41)$$

For the rapidly varying components of the Raman coherence we find

$$\sqrt{N_{\mathbf{r}}}\sigma_{bc}^{(-2n)} = -\frac{1}{2}\sin\theta\frac{\beta}{|\kappa^+|^2}\{\Psi[z-\beta r(t),0]-\Psi[z+\beta r(t),0]\} \times\left(-\frac{\kappa^-}{\kappa^+}\right)^n e^{-\Gamma_{bc}t} \quad (42)$$

and

$$\sqrt{N_{\mathbf{r}}}\sigma_{bc}^{(2n)} = 0, \quad (43)$$

where, in both cases, $n>0$.

In the case of a perfect standing-wave coupling field, only the dc component of the Raman coherence is present which is given by

$$\sqrt{N_z}\sigma_{bc}^{(0)}(z,t) = -\sin\theta(t)\Psi(z,0)e^{-\Gamma_{bc}t}. \quad (44)$$

In the quasi-standing-wave case, the rapidly varying components of the Raman coherence $\sigma_{bc}^{(2n)}$ with *negative* values of n attain a small but nonvanishing value, becoming progressively smaller with decreasing n. The rapidly varying components of the Raman coherence with *positive* values of n all vanish. An asymmetry in the Raman coherence is to be expected, since neither the coupling field nor the revived probe field is symmetric in z.

III. NONADIABATIC CORRECTIONS

In [20] it was shown that the finite length of the probe pulse leads to a broadening of the pulse envelope due to dispersion. In this section we shall investigate the same effect in the standing-wave case and show that the dispersive broadening vanishes in the case of a pure standing-wave coupling field.

Our starting point is the differential equation (12) for the Raman coherence σ_{bc}. To first order in $(\gamma_{ba}T)^{-1}$ we find

$$\sigma_{bc} = -\frac{g_pE_p}{\Omega_c} + \frac{\Gamma_{ba}}{|\Omega_c|^2}\frac{\partial}{\partial t}\left(\frac{g_pE_p}{\Omega_c}\right), \quad (45)$$

where we have assumed $\Gamma_{bc}=0$ to simplify the calculations. Inserting this expression into Eq. (11a) and introducing the DSP fields defined in Eq. (17), we obtain

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba} = \frac{-\sin\theta}{i\Omega[1+2|\kappa^+||\kappa^-|\cos(2kz+\phi)]}\frac{\partial}{\partial t}(\Psi^+e^{ikz}+\Psi^-e^{-ikz}) + \frac{\Gamma_{ba}}{g_p^2N_{\mathbf{r}}}\frac{\sin\theta\tan^2\theta}{i\Omega[1+2|\kappa^+||\kappa^-|\cos(2kz+\phi)]^2}\frac{\partial^2}{\partial t^2} \times(\Psi^+e^{ikz}+\Psi^-e^{-ikz}), \quad (46)$$

where we have assumed that the coupling laser Rabi frequency changes slowly enough to set $\dot{\theta}=0$ in the equations.

As in Sec. II we need to find the Fourier components $\sigma_{ba}^{\pm}$. To do this we apply the Fourier series (20) and we introduce

$$\frac{1}{(1+y\cos x)^2} = \frac{d_0}{2} + \sum_{n=1}^{\infty} d_n\cos(nx), \quad (47)$$

where, as before, $y=2|\kappa^+||\kappa^-|$ and $x=2kz+\phi$. Inserting the Fourier series into Eq. (46), we obtain

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba}^{+} = \frac{\sin\theta}{2i\Omega}\left(-\frac{\partial}{\partial t}(a_0\Psi^+ + a_1e^{i\phi}\Psi^-) + \frac{\Gamma_{ba}}{g_p^2N_{\mathbf{r}}}\tan^2\theta\frac{\partial^2}{\partial t^2}(d_0\Psi^+ + d_1e^{i\phi}\Psi^-)\right), \quad (48a)$$

$$\sqrt{N_{\mathbf{r}}}\sigma_{ba}^{-} = \frac{\sin\theta}{2i\Omega}\left(-\frac{\partial}{\partial t}(a_0\Psi^- + a_1e^{-i\phi}\Psi^+) + \frac{\Gamma_{ba}}{g_p^2N_{\mathbf{r}}}\tan^2\theta\frac{\partial^2}{\partial t^2}(d_0\Psi^- + d_1e^{-i\phi}\Psi^+)\right). \quad (48b)$$

The Fourier coefficients $a_{0,1}$ have already been calculated and are given by Eq. (24), while the Fourier coefficients $d_{0,1}$ are given by

$$d_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{dx}{(1+y\cos x)^2} = \frac{2}{(1-y^2)^{3/2}}, \quad (49a)$$

$$d_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\cos x dx}{(1+y\cos x)^2} = -\frac{2y}{(1-y^2)^{3/2}}. \quad (49b)$$

We now insert the expressions (48) into Eq. (25) to obtain a set of coupled wave equations for the DSP fields

$$\frac{\partial\Psi^+}{\partial t} + c\cos^2\theta'\frac{\partial\Psi^+}{\partial z} = -\sin^2\theta'\left[se^{i\phi}\frac{\partial\Psi^-}{\partial t} - \frac{\Gamma_{ba}}{g_p^2N_{\mathbf{r}}} \times\tan^2\theta\frac{\partial^2}{\partial t^2}(s'\Psi^+ + s''e^{i\phi}\Psi^-)\right], \quad (50a)$$

$$\frac{\partial\Psi^-}{\partial t} - c\cos^2\theta'\frac{\partial\Psi^-}{\partial z} = -\sin^2\theta'\left[se^{-i\phi}\frac{\partial\Psi^+}{\partial t} - \frac{\Gamma_{ba}}{g_p^2N_{\mathbf{r}}} \times\tan^2\theta\frac{\partial^2}{\partial t^2}(s'\Psi^- + s''e^{-i\phi}\Psi^+)\right], \quad (50b)$$

where we have introduced the constants

$$s = \frac{a_1}{a_0}, \quad s' = \frac{d_0}{a_0}, \quad s'' = \frac{d_1}{a_0}. \quad (51)$$

Once again we consider the low-group-velocity limit $\cos^2\theta \ll 1$. With this approximation the wave equations simplify to

$$\frac{\partial\Psi^+}{\partial t} + c\cos^2\theta'\frac{\partial\Psi^+}{\partial z} = -se^{i\phi}\frac{\partial\Psi^-}{\partial t} + \frac{\Gamma_{ba}}{g_p^2 N_{\mathbf{r}}}\times\tan^2\theta\frac{\partial^2}{\partial t^2}(s'\Psi^+ + s''e^{i\phi}\Psi^-), \tag{52a}$$

$$\frac{\partial\Psi^-}{\partial t} - c\cos^2\theta'\frac{\partial\Psi^-}{\partial z} = -se^{-i\phi}\frac{\partial\Psi^+}{\partial t} + \frac{\Gamma_{ba}}{g_p^2 N_{\mathbf{r}}}\times\tan^2\theta\frac{\partial^2}{\partial t^2}(s'\Psi^- + s''e^{-i\phi}\Psi^+). \tag{52b}$$

To the same order of approximation, we can replace the second time derivatives of the DSP fields with the second time derivative of the zero-order solution. Differentiating both sides of Eq. (52) with respect to t, and discarding derivatives of order greater than two, we obtain

$$\frac{\partial^2\Psi^+}{\partial t^2} + c\cos^2\theta'\frac{\partial}{\partial z}\frac{\partial\Psi^+}{\partial t} = -se^{i\phi}\frac{\partial^2\Psi^-}{\partial t^2}, \tag{53a}$$

$$\frac{\partial^2\Psi^-}{\partial t^2} - c\cos^2\theta'\frac{\partial}{\partial z}\frac{\partial\Psi^-}{\partial t} = -se^{-i\phi}\frac{\partial^2\Psi^+}{\partial t^2}, \tag{53b}$$

where we once again assume that the coupling laser Rabi frequency changes slowly. Using Eq. (52) we can solve for the second time derivatives of the DSP field. We find

$$\frac{\partial^2\Psi^\pm}{\partial t^2} = \frac{1-y^2}{1-s^2}(c\cos^2\theta)^2\frac{\partial^2\Psi^\pm}{\partial z^2}, \tag{54}$$

where we exploited the fact that in the low-group-velocity limit $\cos^2\theta' \simeq \sqrt{1-y^2}\cos^2\theta$. Inserting Eq. (54) into Eq. (52), and assuming that $|\kappa^+| \geq |\kappa^-|$, the coupled-wave equations take the form

$$\frac{\partial\Psi^+}{\partial t} + |\kappa^+|^2 v_g\frac{\partial\Psi^+}{\partial z} = \kappa^+\kappa^{-*}v_g\frac{\partial\Psi^-}{\partial z} + \frac{|\kappa^+|^2 l_a v_g}{\sqrt{1-y^2}}\frac{\partial^2}{\partial z^2}(|\kappa^+|^2\Psi^+ - \kappa^+\kappa^{-*}\Psi^-), \tag{55a}$$

$$\frac{\partial\Psi^-}{\partial t} - |\kappa^+|^2 v_g\frac{\partial\Psi^-}{\partial z} = -\kappa^{+*}\kappa^- v_g\frac{\partial\Psi^+}{\partial z} + \frac{|\kappa^+|^2 l_a v_g}{\sqrt{1-y^2}}\frac{\partial^2}{\partial z^2}(|\kappa^+|^2\Psi^- - \kappa^{+*}\kappa^-\Psi^+). \tag{55b}$$

To solve the coupled-wave equations (55) we proceed by Fourier transforming with respect to z, such that $\Psi^\pm(z,t) \to \tilde{\Psi}^\pm(q,t)$, and find the solution

$$\tilde{\Psi}^+(q,t) = \frac{1}{2d}\{[b\tilde{\Psi}^-(q,0) - (|\kappa^+|^2 - d)\tilde{\Psi}^+(q,0)]\exp(iq\lambda_+ r(t)) + [(|\kappa^+|^2 + d)\tilde{\Psi}^+(q,0) - b\tilde{\Psi}^-(q,0)]\exp(iq\lambda_- r(t))\}, \tag{56a}$$

$$\tilde{\Psi}^-(q,t) = \frac{1}{2d}\{[-b^*\tilde{\Psi}^+(q,0) + (|\kappa^+|^2 + d)\tilde{\Psi}^-(q,0)]\times\exp(iq\lambda_+ r(t)) + [b^*\tilde{\Psi}^+(q,0) - (|\kappa^+|^2 - d)\tilde{\Psi}^-(q,0)]\exp(iq\lambda_- r(t))\}, \tag{56b}$$

where

$$b = \kappa^+\kappa^{-*}(1 - iq\xi), \quad \lambda_\pm = i|\kappa^+|^2\xi q \pm d, \tag{57}$$

and

$$\xi = \frac{|\kappa^+|^2 l_a}{\sqrt{1-y^2}}, \tag{58}$$

$$d = \sqrt{|\kappa^+|^2(|\kappa^+|^2 - |\kappa^-|^2) - |\kappa^+|^2|\kappa^-|^2\xi^2 q^2}. \tag{59}$$

Inserting the initial conditions (33) for the DSP field and considering the limit $\kappa^\pm \to \frac{1}{\sqrt{2}}$, corresponding to a pure standing-wave coupling field, the solution becomes

$$\Psi^+(z,t) = \frac{1}{\sqrt{2}}\Psi(z,0), \tag{60a}$$

$$\Psi^-(z,t) = \frac{1}{\sqrt{2}}\Psi(z,0). \tag{60b}$$

From this solution it is clear that the broadening of the pulse envelope due to dispersion is absent in the case of a pure standing-wave coupling field. The effect *is* present in the case of a quasi-standing-wave coupling field. If we consider the limiting case of a traveling-wave coupling field ($\kappa^- \to 0$), we find the same dispersion term in the wave equation (55) that is given in [20].

IV. SUMMARY

In this article we have presented a detailed theoretical treatment of stationary light pulses in media comprised of stationary atoms, such as ultracold gasses and solid-state media. We found that contrary to the thermal gas case, the achievable trapping time is limited only by the Raman dephasing rate of the atoms and such media are thus ideally suited for the kind of nonlinear optical interactions envisaged in [16,17]. It was also shown that the behavior of the probe pulse when employing quasi-stationary-coupling fields is significantly different for moving and nonmoving atoms. This fact must be taken into account when considering schemes for interacting pulses. Although, to the best of our knowledge, no experiment with stationary light pulses in ultracold media has yet been reported, several experiments on normal EIT and light storage have been performed with ultracold gasses [6] and solid-state media [11,12]. These experiments have also demonstrated the possibility of using beam geometries other than copropagating probe and coupling lasers. We therefore expect that the experimental demonstration of stationary light pulses in such media is within present day capability.

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APPENDIX: STANDING WAVE POLARITONS IN THERMAL GASSES

As shown in Sec. II the behavior of the stationary light pulses depends critically on whether the EIT medium is comprised of stationary or moving atoms. In this section we present a brief review of the theory for the thermal gas case presented in [18]. It is argued that the motion of the atoms in a thermal gas causes a rapid dephasing of the spatially rapidly varying components of the Raman coherence and it is therefore assumed that only the $n=0$ component in the expansion (40) is nonvanishing. Consequently, the only nonvanishing components of the optical coherence in the expansion (22) is the $n=-1$ and $n=0$ terms. With this approximation, the relevant Heisenberg-Langevin equations for the slowly varying operators are

$$\dot{\sigma}_{ba}^{+} = ig_pE_p^{+} + i\Omega_c^{+}\sigma_{bc} - \Gamma_{ba}\sigma_{ba}^{+}, \tag{A1a}$$

$$\dot{\sigma}_{ba}^{-} = ig_pE_p^{-} + i\Omega_c^{-}\sigma_{bc} - \Gamma_{ba}\sigma_{ba}^{-}, \tag{A1b}$$

$$\dot{\sigma}_{bc} = i(\Omega_c^{+*}\sigma_{ba}^{+} + \Omega_c^{-*}\sigma_{ba}^{-}) - \Gamma_{bc}\sigma_{bc}. \tag{A1c}$$

As shown in [18] these equations can be solved approximately by adiabatically eliminating the optical coherences $\sigma_{ba}^{\pm}$ and making an adiabatic expansion of Eq. (A1c). The resulting expressions for the components of the optical coherence $\sigma_{ba}^{\pm}$ is then inserted into the wave equations (25). Contrary to [18], which deals directly with the probe field operators $E_p^{\pm}$, we introduce the polariton field defined by Eq. (17) which enables us to treat time-dependent coupling fields in a consistent manner. To facilitate the solution of the resulting wave equations, sum and difference normal modes defined by

$$\Psi_S = \kappa^{+*}\Psi^{+} + \kappa^{-*}\Psi^{-}, \tag{A2a}$$

$$\Psi_D = \kappa^{-}\Psi^{+} - \kappa^{+}\Psi^{-}, \tag{A2b}$$

are introduced. In the case of an optically thick medium, the difference mode can be adiabatically eliminated, resulting in a diffusion equation for the sum normal mode

$$\frac{\partial\Psi_S}{\partial t} + (|\kappa^{+}|^2 - |\kappa^{-}|^2)c\cos^2\theta\frac{\partial\Psi_S}{\partial z} = 4|\kappa^{+}|^2|\kappa^{-}|^2 l_a c\cos^2\theta\frac{\partial^2\Psi_S}{\partial z^2} - \Gamma_{bc}\sin^2\theta\Psi_S, \tag{A3}$$

where we have assumed zero probe field detuning ($\delta_p=0$). The difference normal mode is given by

$$\Psi_D = -2\kappa^{+}\kappa^{-}l_a\frac{\partial\Psi_S}{\partial z}. \tag{A4}$$

The solution of Eq. (A3), subject to the initial conditions (33), is the basis for the comparison between thermal gas media and stationary atom media presented in Sec. II.

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