Genuine multiqubit entanglement and controlled teleportation

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We construct a genuine $(2N+1)$-qubit entangled state to perform controlled teleportation of an arbitrary N-qubit state. The constructed state is a complementarity to the genuine $2N$-qubit entangled state constructed by Yeo and Chua for $N=2$ [Phys. Rev. Lett. **96**, 060502 (2006)] and by Chen, Zhu, and Guo for any N [Phys. Rev. A **74**, 032324 (2006)]. We also quantify the entanglement of the state and classify it with the well-known *GHZ* and *W* states by means of the recently proposed generalized global entanglement and the associated auxiliary measures [Phys. Rev. A **74**, 022314 (2006)]. Our study is of general importance with respect to exploring and exploiting the genuine multiqubit entanglement.

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Historically the notion of entanglement dates back to 1935 by Schrödinger, but only during the last two decades one could witness its positive power manifested in various intriguing tasks of quantum information processing and quantum computing. Although the general theory of entanglement has blossomed enormously [1], only bipartite entanglement is well understood with a good range of quantitative measure of entanglement available [2]. As for multipartite entanglement the complexity increases greatly with the number of parties involved and one can by no means simply extend the useful tools of the bipartite case to the multipartite case because there are many possible ways in which a multipartite state can be looked upon as an entangled state (see, e.g., Ref. [3]). Intuitively, one can properly explore multipartite entanglement if one finds various inequivalent entanglement formats, i.e., genuine entangled states. To seek for genuine entangled states we can resort to particular quantum schemes since in many cases sharing a unique entanglement allows ones to do things that ones cannot otherwise do. A paradigmatic example is the well-known teleportation [4]. To teleport a unknown $N=1$ qubit state from a sender Alice to a remote receiver Bob, the two parties should share *a priori* an EPR pair. However, to teleport a unknown $N=2$ qubit state, two EPR pairs [5] or generally a quadqubit entanglement should be employed [6,7]. Differently from Ref. [6], where an explicit protocol is not yet given, the authors of Ref. [7] construct a concrete structure of a genuine quadqubit entangled state. Shortly after, the idea of Ref. [7] was generalized to a genuine $2N$-qubit entanglement which can be used to teleport an arbitrary N-qubit state [8]. However, restricted by the process of teleportation they [7,8] cannot construct the genuine entanglement of $(2N+1)$-qubit system. In this paper, we present an explicit genuine $(2N+1)$-qubit entangled state motivated by the so-called controlled teleportation (CT) of a unknown N-qubit state, i.e., the state is to be teleported between Alice and Bob under control of a third remote party Charlie. As is well known, for $N=1$ [9] the CT can be performed via a triqubit *GHZ* state [10]. However, for $N>1$ the multiqubit *GHZ* state cannot do the job, but our $(2N+1)$-qubit entangled state can. This is a general important issue regarding the way to seek for potential candidates for CT as well as understanding genuine multiqubit entanglement in systems consisting of many qubits, either even or odd number of them. Here, to quantify our $(2N+1)$-qubit entangled state in comparison with other known states we adopt an operational method by means of the so-called generalized global entanglement $E_G^{(n)}$ recently developed in Ref. [11].

As mentioned above, CT of a single-qubit can be done via a single-*GHZ* trio [9]. A trivial way for CT of an arbitrary N-qubit state is to use N *GHZ* trios [12]. However, this can be done with less consumed quantum resources via just $(N-1)$ EPR pairs plus one *GHZ* trio, as we shall illustrate in what follows. Suppose that the state to be teleported is

$$|\Psi\rangle_{a_1\cdots a_N} = \sum_{\{l_n\}=0}^{1} \alpha_{l_1\cdots l_N}|l_1\rangle_{a_1}|l_2\rangle_{a_2}\cdots|l_N\rangle_{a_N}. \tag{1}$$

Alice, Bob, and Charlie share beforehand a quantum channel of the form

$$|M_{2N+1}\rangle_{A_1\cdots A_N B_1\cdots B_N C} = \otimes_{n=1}^{N-1}|\mathcal{B}^0\rangle_{A_n B_n} \otimes |GHZ_3\rangle_{A_N B_N C} \tag{2}$$

which is in fact a tensor product of $(N-1)$ EPR pairs

$$|\mathcal{B}^0\rangle_{A_n B_n} = \frac{1}{\sqrt{2}}\sum_{j_n=0}^{1}|j_n\rangle_{A_n}|j_n\rangle_{B_n} \tag{3}$$

and a *GHZ* trio

$$|GHZ_3\rangle_{A_N B_N C} = \frac{1}{\sqrt{2}}\sum_{j_N=0}^{1}|j_N\rangle_{A_N}|j_N\rangle_{B_N}|j_N\rangle_C. \tag{4}$$

In terms of the basis $|\tilde{k}\rangle=(1/\sqrt{2})\Sigma_{s=0}^{1}(-1)^{ks}|s\rangle$ for Charlie's qubit C, the quantum channel (2) can be expressed as

$$|M_{2N+1}\rangle_{A_1\cdots A_N B_1\cdots B_N C} = \frac{1}{\sqrt{2^{N+1}}}\sum_{\{j_n\},k=0}^{1}|j_1,\cdots,j_N\rangle_{A_1\cdots A_N}\sigma_{B_N}^{(k)} \times|j_1,\cdots,j_N\rangle_{B_1\cdots B_N}|\tilde{k}\rangle_C, \tag{5}$$

where $\{\sigma^{(0)},\sigma^{(1)},\sigma^{(2)},\sigma^{(3)}\}\equiv\{I,\sigma_z,\sigma_x,i\sigma_y\}$ with I the identity operator and $\sigma_{x(y,z)}$ the Pauli operators. Let the four orthonormal states constituting the so-called Bell basis of qubits a_n and A_n be

$$|\mathcal{B}^{i_n}\rangle_{a_nA_n}=\sigma^{(i_n)}_{a_n}|\mathcal{B}^0\rangle_{a_nA_n} \tag{6}$$

with $i_n\in\{0,1,2,3\}$ for each $n\in 1,2,\ldots,N-1$. If Alice performs N Bell measurements on the pairs (a_n,A_n) in the basis $|\mathcal{B}^{i_n}\rangle$ with outcomes i_n and Charlie measures her qubit C in the basis $|\tilde{k}\rangle$ with a outcome k, then Bob's qubits $B_1,B_2,\ldots,B_N$ are projected onto the state

$$\frac{1}{\sqrt{p_k\otimes^N_{n=1}p_{i_n}}}{}_C\langle\tilde{k}|\otimes^N_{n=1}{}_{a_nA_n}\langle\mathcal{B}^{i_n}|\Psi\rangle_{a_1\cdots a_N}|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}, \tag{7}$$

where p_{i_n} is probability of Alice's obtaining the outcome i_n and p_k is probability of Charlie's obtaining the outcome k. Substituting Eqs. (1), (5), and (6) into Eq. (7) yields

$$\frac{1}{\sqrt{p_k\otimes^N_{n=1}p_{i_n}}}\frac{1}{\sqrt{2^{2N+1}}}\sum_{\{j_n,l_n\}=0}^{1}\alpha_{l_1\cdots l_N}\otimes^N_{n=1}{}_{a_n}\langle j_n|\sigma^{(i_n)}_{a_n}|l_n\rangle_{a_n}\sigma^{(k)}_{B_N}|j_n\rangle_{B_n}$$
$$=\sum_{\{l_n\}=0}^{1}\alpha_{l_1\cdots l_N}\sigma^{(k)}_{B_N}\otimes^N_{n=1}\sigma^{(i_n)}_{B_n}|l_n\rangle_{B_n}=\sigma^{(k)}_{B_N}\otimes^N_{n=1}\sigma^{(i_n)}_{B_n}|\Psi\rangle_{B_1\cdots B_N} \tag{8}$$

since $p_k=1/2$ and $p_{i_n}=1/4\,\forall\, n$. Thus, upon receiving Alice's and Charlie's outcomes $\{i_n\}$ and k, Bob is always able to obtain the desired state (1) by applying on his qubits the operators $\otimes^N_{n=1}\sigma^{(i_n)}_{B_n}\sigma^{(k)}_{B_N}$.

We now consider the same CT task but using a new kind of quantum channel, i.e., the genuine $(2N+1)$-partite entangled state. To avoid reducibility to a tensor product of EPR pairs and a *GHZ* trio and to ensure faithful teleportation of the arbitrary N-qubit state, the $(2N+1)$-partite entangled state to be shared between Alice, Bob and Charlie is constructed in the form

$$|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}=\frac{1}{\sqrt{2^{N+1}}}\sum_{J=0}^{2^N-1}\sum_{k=0}^{1}|\bar{J}\rangle_{A_1\cdots A_N}|\bar{J}^{(k)}\rangle_{B_1\cdots B_N}|\tilde{k}\rangle_C \tag{9}$$

with

$$|\bar{J}\rangle_{A_1\cdots A_N}=U_{A_1\cdots A_N}|J\rangle_{A_1}\cdots{}_{A_N}, \tag{10}$$

$$|\bar{J}^{(k)}\rangle_{B_1\cdots B_N}=V_{B_1\cdots B_N}\sigma^{(k)}_{B_N}|J\rangle_{B_1}\cdots{}_{B_N}, \tag{11}$$

where $|0\rangle_{1\cdots N}=|0\rangle_1\cdots|0\rangle_N, |1\rangle_{1\cdots N}=|0\rangle_1\cdots|1\rangle_N,\ldots,|2^N-1\rangle_{1\cdots N}=|1\rangle_1\cdots|1\rangle_N$ are the binary form of N qubits (i.e., $|J\rangle_{1\cdots N}=|j_1,\ldots,j_N\rangle_{1\cdots N}$ with $j_n\in\{0,1\}$) and $U_{1\cdots N},V_{1\cdots N}$ are unitary operators acting jointly on N qubits $1,2,\ldots,N$ such that $U_{1\cdots N}\neq\otimes^N_{n=1}U_n$ and $V_{1\cdots N}\neq\otimes^N_{n=1}V_n$. We note that if $U_{1\cdots N}=\otimes^N_{n=1}U_n$ and $V_{1\cdots N}=\otimes^N_{n=1}V_n$, state (9) reduces to a tensor product of $(N-1)$ pairs $(A_1,B_1),(A_2,B_2),\ldots,(A_{N-1},B_{N-1})$ and a trio (A_N,B_N,C). In particular, if $U_{1\cdots N}=V_{1\cdots N}=\otimes^N_{n=1}I_n$, state (9) is nothing else but Eq. (5), which is a tensor product of $(N-1)$ EPR pairs and a *GHZ* trio. In general, $U_{1\cdots N}(V_{1\cdots N})$ are nonlocal with respect to qubits $A_1,A_2,\ldots,A_N(B_1,B_2,\ldots,B_N)$, state (9) is irreducible to a tensor product of $(N-1)$ pairs plus a trio and thus may be a genuine multipartite entangled state. In terms of the basis states $|J\rangle_{a_1\cdots a_N}$, the unknown state (1) can be represented as

$$|\Psi\rangle_{a_1\cdots a_N}=\sum_{L=0}^{2^N-1}\alpha_L|L\rangle_{a_1\cdots a_N}. \tag{12}$$

To accomplish the CT task one can define the following basis of 4^N orthonormal states for the $2N$ qubits $a_1,\ldots,a_N,A_1,\ldots,A_N$

$$|\bar{\Pi}^{i_1\cdots i_N}\rangle_{a_1\cdots a_NA_1\cdots A_N}=\frac{1}{\sqrt{2^N}}\otimes^N_{n=1}\sigma^{(i_n)}_{a_n}\sum_{K=0}^{2^N-1}\sum_{k=0}^{1}|K\rangle_{a_1\cdots a_N}|\bar{K}\rangle_{A_1\cdots A_N}. \tag{13}$$

Now, using the quantum channel (9), the CT is carried out by Alice's performing a complete joint measurement on the $2N$ qubits $a_1,\ldots,a_N,A_1,\ldots,A_N$ that project them on one of the state (13), while Charlie, as before, needs just to measure her qubit C in the basis $\{|\tilde{k}\rangle\}$. Let Alice's (Charlie's) outcomes be $\{i_1,i_2,\ldots,i_N\}(\{k\})$ which occurs with an equal probability $p_{i_1i_2\cdots i_N}=1/4^N(p_k=1/2)$. After Alice's and Charlie's measurements the state of Bob's N qubits $B_1,B_2,\ldots,B_N$ collapses into

$$\frac{1}{\sqrt{p_kp_{i_1i_2\cdots i_N}}}{}_C\langle\tilde{k}|_{a_1\cdots a_NA_1\cdots A_N}\langle\bar{\Pi}^{i_1\cdots i_N}|\Psi\rangle_{a_1\cdots a_N}|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}=\sum_{L,J=0}^{2^N-1}\alpha_L{}_{a_1\cdots a_N}\langle J|\otimes^N_{n=1}\sigma^{(i_n)}_{a_n}|L\rangle_{a_n}V_{B_1\cdots B_N}\sigma^{(k)}_{B_N}|J\rangle_{B_1\cdots B_N}$$
$$=\sum_{L=0}^{2^N-1}\alpha_LV_{B_1\cdots B_N}\sigma^{(k)}_{B_N}\otimes^N_{n=1}\sigma^{(i_n)}_{B_n}|L\rangle_{B_1\cdots B_N}=V_{B_1\cdots B_N}\sigma^{(k)}_{B_N}\otimes^N_{n=1}\sigma^{(i_n)}_{B_n}|\Psi\rangle_{B_1\cdots B_N}. \tag{14}$$

Expression (14) indicates that the CT always succeeds if the outcomes $\{i_1,i_2,\ldots,i_N,k\}$ are broadcasted. Namely, upon receiving the necessary information Bob is able to obtain an exact replica of Alice's original state (12) by applying on his qubits the operators $\otimes_{n=1}^{N}\sigma_{B_n}^{(i_n)}\sigma_{B_N}^{(k)}V_{B_1\cdots B_N}$.

To study properties of our state (9), let us restrict ourselves to $N=2$, for simplicity, i.e., we shall be explicitly concerned with the pentaqubit entangled state

$$|\bar{M}_5\rangle_{A_1A_2B_1B_2C}=\frac{1}{\sqrt{2^3}}\sum_{J=0}^{3}\sum_{k=0}^{1}|\bar{J}\rangle_{A_1A_2}|\bar{J}^{(k)}\rangle_{B_1B_2}|\tilde{k}\rangle_C. \quad (15)$$

Suppose that, in the basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}_{A_1A_2(B_1B_2)}$, $U_{A_1A_2}$ and $V_{B_1B_2}$ are chosen as

$$U_{A_1A_2}=V_{B_1B_2}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\0&1&1&0\\-1&0&0&1\\0&-1&1&0\end{pmatrix}. \quad (16)$$

Then, we have from Eq. (15)

$$\begin{aligned}|\bar{M}_5\rangle_{A_1A_2B_1B_2C}=\frac{1}{4}(&|00000\rangle+|00001\rangle+|00110\rangle-|00111\rangle\\&+|01010\rangle+|01011\rangle-|01100\rangle+|01101\rangle\\&-|10010\rangle+|10011\rangle+|10100\rangle+|10101\rangle\\&+|11000\rangle-|11001\rangle+|11110\rangle\\&+|11111\rangle)_{A_1A_2B_1B_2C}. \end{aligned}\quad (17)$$

According to definition 2 in Ref. [11], a pure state of $\mathcal{N}$ qubits $|\Psi\rangle$ is a genuine multiqubit entangled state if $\forall n<\mathcal{N}$, $\mathrm{Tr}(\rho_{j_1}^2)$, $\mathrm{Tr}(\rho_{j_1j_2}^2),\ldots,$ $\mathrm{Tr}(\rho_{j_1,j_2,\ldots,j_n}^2)\leqslant 1/2$, where $\rho_{j_1,j_2,\ldots,j_n}=\mathrm{Tr}_{\overline{j_1,j_2,\ldots,j_n}}(|\Psi\rangle\langle\Psi|)$, $1\leqslant j_1<j_2<\cdots<j_n\leqslant\mathcal{N}$ and $\overline{j_1,j_2,\ldots,j_n}$ is the complement of the subsystem $j_1,j_2,\ldots,j_n$. For our state (17), calculations yield $\mathrm{Tr}(\rho_{j_1}^2)=1/4$ for $j_1\in\{A_1,A_2,B_1,B_2\}$, $\mathrm{Tr}(\rho_k^2)=1/2$ for $k\equiv C$, $\mathrm{Tr}(\rho_{j_1j_2}^2)=1/4$ for $j_1j_2\in\{A_1A_2,A_2B_1,B_1B_2,B_2C,B_1C,A_1B_2,A_2C,A_1C\}$, $\mathrm{Tr}(\rho_{k_1k_2}^2)=1/2$ for $k_1k_2\in\{A_1B_1,A_2B_2\}$, $\mathrm{Tr}(\rho_{j_1j_2j_3}^2)=1/4$ for $j_1j_2j_3\in\{A_1A_2B_1, A_2B_1B_2,B_1B_2C,A_1A_2B_2,A_2B_1C,A_1A_2C,A_1B_1B_2,A_1B_2C\}$, $\mathrm{Tr}(\rho_{k_1k_2k_3}^2)=1/2$ for $k_1k_2k_3\in\{A_2B_2C,A_1B_1C\}$, and $\mathrm{Tr}(\rho_{j_1j_2j_3j_4}^2)=1/2$ for $j_1j_2j_3j_4\in\{A_1A_2B_1B_2,A_2B_1B_2C, A_1A_2B_1C,A_1A_2B_2C,A_1B_1B_2C\}$. Therefore, our state $|\bar{M}_5\rangle_{A_1A_2B_1B_2C}$ is authentically a genuine pentaqubit entangled state. To identify our state we adopt the concept of the generalized global entanglement $E_G^{(n)}$ and the auxiliary measures $G(n,i_1,i_2,\ldots,i_{n-1})$ defined as [11]

$$E_G^{(n)}=\frac{(\mathcal{N}-n)!(n-1)!}{(\mathcal{N}-1)!}\sum_{i_1=1}^{\mathcal{N}-1}\sum_{i_2=i_1+1}^{\mathcal{N}-1}\sum_{i_3=i_2+1}^{\mathcal{N}-1}\times\cdots\times\sum_{i_{n-1}=i_{n-2}+1}^{\mathcal{N}-1}G(n,i_1,i_2,\ldots,i_{n-1}), \quad (18)$$

where all the parameters are natural numbers $n<\mathcal{N},1\leqslant i_1<i_2<\cdots<i_{n-1}\leqslant\mathcal{N}-1$ and $i_0=0$;

$$G(n,i_1,i_2,\ldots,i_{n-1})=\frac{d}{d-1}\left[1-\frac{1}{\mathcal{N}-i_{n-1}}\times\sum_{j=1}^{\mathcal{N}-i_{n-1}}\mathrm{Tr}(\rho_{j,j+i_1,j+i_2,\ldots,j+i_{n-1}}^2)\right], \quad (19)$$

where $\rho_{j,j+i_1,j+i_2,\ldots,j+i_{n-1}}$ is obtained by tracing out all the subsystems $\{S_i\}$ but $S_A=\{S_j,S_{j+i_1},S_{j+i_2},\ldots,S_{j+i_{n-1}}\},d=\min\{\dim S_A,\dim\bar{S}_A\}$ with $\dim S_A$ and $\dim\bar{S}_A$ the Hilbert space dimension of S_A and its complement $\bar{S}_A$, respectively. In summation, the index n is for the number of subsystems in the A partition and the indices $i_1,i_2,\ldots,i_{n-1}$ are the neighborhood addresses for each of the involved subsystems. The other well-known genuine pentaqubit entangled states to be compared with ours are the *GHZ* state

$$|GHZ_5\rangle_{A_1A_2B_1B_2C}=\frac{1}{\sqrt{2}}(|00000\rangle+|11111\rangle)_{A_1A_2B_1B_2C} \quad (20)$$

and the W state

$$|W_5\rangle_{A_1A_2B_1B_2C}=\frac{1}{\sqrt{5}}(|10000\rangle+|01000\rangle+|00100\rangle+|00010\rangle+|00001\rangle)_{A_1A_2B_1B_2C}. \quad (21)$$

The results are shown in Table I. From the table, $E_G^{(1)}$ discriminates $|\bar{M}_5\rangle$ from $|W_5\rangle$ but identifies it with $|GHZ_5\rangle$. However, for $n>1$ all the three considered states are well distinguished. A transparent fact of belonging to different classes of genuine entanglement is that the multiqubit *GHZ* state, Eq. (20), and W state, Eq. (21), cannot be served as a quantum channel for CT, but our state can. This fact can be proved as follows. Let the total entanglement amount of a $(2N+1)$-partite state be $E_G=\sum_{n=1}^{2N}E_G^{(n)}$. The computed values in Table I clearly indicates that $E_G(|\bar{M}_5\rangle)>E_G(|GHZ_5\rangle),E_G(|W_5\rangle)$. In general, we have $E_G(|\bar{M}_{2N+1}\rangle)>E_G(|GHZ_{2N+1}\rangle),E_G(|W_{2N+1}\rangle)$. Suppose that Alice and Charlie share an entangled state $|\bar{M}_{2N+1}\rangle_{a_1'\cdots a_N'a_1\cdots a_NC'}$ of which qubits $a_1',\ldots,a_N',a_1,\ldots,a_N$ belong to Alice and qubit C' belongs to Charlie. If the quantum channel $|GHZ_{2N+1}\rangle_{A_1\ldots A_NB_1\ldots B_NC}(|W_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC})$ could be used for CT of qubits $a_1,\ldots,a_N$ to qubits $B_1,\ldots,B_N$, then, as a result, Alice, Bob, and Charlie would share the entangled state $|\bar{M}_{2N+1}\rangle_{a_1'\cdots a_N'B_1\cdots B_NC'}$. This would mean that the parties could increase their shared entanglement amount just by local operations and classical communication (LOCC), which is impossible in quantum information processing. Therefore, we conclude that neither state $|GHZ_{2N+1}\rangle$ nor $|W_{2N+1}\rangle$ can implement CT. This also implies that state $|\bar{M}_{2N+1}\rangle$ is both sufficient and necessary for CT of an arbitrary unknown N-qubit state. Since ours is the most entangled state among the three, the multiqubit *GHZ* and W states cannot be con-

TABLE I. Generalized global entanglement $E_G^{(n)}$ and their associated measures $G(n,i_1,i_2,\ldots,i_{n-1})$ for three genuine pentaqubit entangled states $|GHZ_5\rangle$, $|W_5\rangle$, and $|\bar{M}_5\rangle$.

	$\lvert GHZ_5\rangle_{A_1A_2B_1B_2C}$	$\lvert W_5\rangle_{A_1A_2B_1B_2C}$	$\lvert \bar{M}_5\rangle_{A_1A_2B_1B_2C}$
$E_G^{(1)}$	1	$16/25=0.640$	1
$E_G^{(2)}$	$2/3\simeq 0.667$	$16/25=0.640$	$17/18\simeq 0.944$
$G(2,1)$	2/3	16/25	1
$G(2,2)$	2/3	16/25	$7/9\simeq 0.778$
$G(2,3)=G(2,4)$	2/3	16/25	1
$E_G^{(3)}$	2/3	16/25	$11/12\simeq 0.917$
$G(3,1,2)=G(3,1,3)=G(3,1,4)$	2/3	16/25	1
$G(3,2,3)$	2/3	16/25	$5/6\simeq 0.833$
$G(3,2,4)$	2/3	16/25	2/3
$G(3,3,4)$	2/3	16/25	1
$E_G^{(4)}$	2/3	16/25	1
$G(4,1,2,3)=G(4,1,2,4)=G(4,1,3,4)$ $=G(3,2,3,4)$	2/3	16/25	1

verted to ours by any LOCC. Yet, the question of whether or not our state can be converted by LOCC to either *GHZ* or *W* states requires further investigations.

In conclusion, we have explicitly constructed a genuine $(2N+1)$-qubit entangled state making use of the controlled teleportation of any unknown N-qubit state. We have showed that it is a genuine multiqubit entanglement and compared its properties with those of the corresponding *GHZ* and *W* states. Our state is not only important as potential candidates for controlled teleportation, but together with the genuine $2N$-qubit entangled states studied in Refs. [7,8] it also provides a complementary insight into the understanding of genuine entanglement in systems consisting of both even and odd numbers of qubits in general. It is also noted that although our state manifests genuine entanglement among qubits, it is not the most general one for any possible kinds of entanglement among $(2N+1)$ qubits. It was especially constructed here for the purpose of controlled teleportation and in fact it exhibits quantum correlations between three remote locations of Alice, Bob and Charlie. In this sense, our state can as well be looked upon as an extended tripartite maximally entangled state (i.e., $\rho_{A_1\ldots A_N},\rho_{B_1\ldots B_N},\rho_C\propto I$) in an enlarged Hilbert space of dimension $2^N\otimes 2^N\otimes 2$.

Finally, it is worth mentioning again that our primary purpose is to explore possible different inequivalent $(2N+1)$-partite entangled quantum channels to perform CT (the same purpose was also pursued in [7,8] for candidates to perform teleportation exploring $2N$-partite entangled quantum channels). Because CT was explored here to define a multiqubit entangled state, the entanglement content of this state can be viewed as a resource for CT. Thus, it would be interesting to show the advantages/disadvantages by using the genuine multiqubit entangled state $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ instead of a tensor product of $(N-1)$ EPR pairs and one *GHZ* trio (i.e., the state $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\ldots B_NC}$) or of N *GHZ* trios. In view of the quantum resource, N *GHZ* trios are most expensive since they consume $3N$ qubits, while $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ and $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ are more economical since only $(2N+1)$ qubits are needed. As far as CT is concerned, $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ seems to have no superior advantage to $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$. In fact, given $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ Alice (Bob) can, in principle, implement $U^{-1}_{A_1A_2\cdots A_N}(V^{-1}_{B_1B_2\cdots B_N})$ on the qubits $A_1,A_2,\cdots,A_N(B_1,B_2,\cdots,B_N)$ to convert $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ into $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ and then use $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ to perporm the CT as described in Eqs. (7) and (8). However, since $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ is a genuine multiqubit entangled state which possesses a richer structure compared to $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$, it would find more power in other tasks of quantum networking, i.e., quantum information processing and distributed quantum computing involving many remote parties. In other words, it would not be excluded that there might exist tasks that cannot be done by means of $|M_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$ but can be done by means of $|\bar{M}_{2N+1}\rangle_{A_1\cdots A_NB_1\cdots B_NC}$.

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