Limits to differences in active and passive charges

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We explore consequences of a hypothetical difference between active charges, which generate electric fields, and passive charges, which respond to them. A confrontation with experiments using atoms, molecules, or macroscopic matter yields limits on their fractional difference at levels down to 10^{-21}, which at the same time corresponds to an experimental confirmation of Newton's third law.

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I. INTRODUCTION

In electrodynamics, one may distinguish between two types of charge: The *active charge* q_a is the source of the electric field,

$$\nabla \cdot \boldsymbol{E} = 4\pi q_a \delta(x), \tag{1}$$

whereas the *passive charge* q_p reacts to it,

$$m\ddot{\boldsymbol{x}} = q_p \boldsymbol{E}. \tag{2}$$

Here, m is the inertial mass and $\boldsymbol{x}$ the position of the particle. In gravitational physics, a hypothetical difference between active and passive gravitational mass has been considered and confronted with laboratory and astrophysical observations. However, as yet nothing similar has been done for the electric and magnetic analogs, in spite of a long history of precision experiments that includes tests of the $1/r$ Coulomb potential, searches for an electrostatic fifth force [1,2], the photon mass [3], and violations of Lorentz invariance [4,5]. Current Maxwell theory tacitly assumes the equality of passive and active charges and is fundamental to a broad range of theoretical and experimental physics; any inequality would thus have serious consequences throughout science, from the standard model of particle physics to practical applications like precision metrology. Here, we show that limits as low as $10^{-21}e$ can be derived for protons and electrons by introducing the concepts of active and passive neutrality. Furthermore, we identify signatures for such a difference in atomic spectroscopy. We extend the analysis to active and passive magnetic moments and find corresponding limits from hyperfine spectroscopy. Our limits appear important in the context of recent quantum gravity scenarios, where all sorts of symmetries (like Lorentz and *CPT* invariance or the equivalence principle) are expected to be violated [4].

A. Model

The dynamics of two particles located at $\boldsymbol{x}_{1,2}$ in their mutual electric fields are described by the equations

$$m_1\ddot{\boldsymbol{x}}_1 = q_{1p}q_{2a}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3} + q_{1p}\boldsymbol{E}(\boldsymbol{x}_1),$$

$$m_2\ddot{\boldsymbol{x}}_2 = q_{2p}q_{1a}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3} + q_{2p}\boldsymbol{E}(\boldsymbol{x}_2), \tag{3}$$

where $\boldsymbol{E}$ denotes a homogenous external electric field, and q_{1p}, q_{1a}, q_{2p}, and q_{2a} are the corresponding passive and active charges. (This nonrelativistic description will be sufficient for our purpose, which is to identify stringent constraints on any inequality of active and passive charges from experiments. In the light of these limits, it is not necessary to consider the relativistic equations of motion.) For the equation of motion of the center of mass $\boldsymbol{X}$, we find

$$\ddot{\boldsymbol{X}} = \frac{q_{1p}q_{2p}}{M}C_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} + \frac{1}{M}(q_{1p}+q_{2p})\boldsymbol{E}, \tag{4}$$

where $M=m_1+m_2$, $\boldsymbol{x}$ is the relative coordinate, and

$$C_{21} = \frac{q_{2a}}{q_{2p}} - \frac{q_{1a}}{q_{1p}}. \tag{5}$$

Thus, if active and passive charges are different, the center of mass shows a self-acceleration along the direction of $\boldsymbol{x}$, in addition to the acceleration caused by the external field $\boldsymbol{E}$. This can be interpreted as a violation of Newton's third law that action equals reaction for electric forces. $C_{21}=0$ means that the ratio between the active and passive charges is the same for both particles. If this ratio is the same for all particles, then it can be absorbed into a redefinition of the electric charges and has no observable consequences. The dynamics of the relative coordinate is given by

$$\ddot{\boldsymbol{x}} = -\frac{1}{m_{\text{red}}}q_{1p}q_{2p}D_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}, \tag{6}$$

where

$$D_{21} = \frac{m_1}{M}\frac{q_{1a}}{q_{1p}} + \frac{m_2}{M}\frac{q_{2a}}{q_{2p}} = \frac{q_{1a}}{q_{1p}} + \frac{m_2}{M}C_{21}, \tag{7}$$

and m_{red} is the reduced mass. In the standard framework, $D_{21}=1$. Bound solutions of the equation of motion (6) are ellipses; by choosing suitable coordinates, they can be normal to the z axis. The simplest case is circular motion, where $\boldsymbol{x}(t)=x_0(\cos(\omega t),\sin(\omega t),0)$. The center of mass oscillates at a frequency ω, which is related to the energy of the system.

The acceleration of the center of mass vanishes on average, $\langle\ddot{\boldsymbol{X}}\rangle=\boldsymbol{0}$. Thus, it is not necessarily observable. These considerations extend to many-particle systems, e.g., to atoms having many electrons.

Of course, a large C_{21} would likely be observed routinely in chemistry and physics. For example, the Born-Mayer model predicts the bond energy of ionic crystals to 1–10 % accuracy, so $C_{21}\gtrsim 1\%$ would be noticed; also a change of bond lengths would result [6] and would be detectable at this level. The limit to the accuracy is the complex nature of the crystals. However, the experiments to be discussed below can provide sensitivity up to 19 orders of magnitude better.

B. Active and passive mass

The analogous case of active and passive *masses* was first discussed by Bondi [7]. The equations of motion for a gravitationally bound two-body system have the same structure as for electric bound charges:

$$\ddot{\boldsymbol{x}}_1 = G\frac{m_{1\mathrm{p}}m_{2\mathrm{a}}}{m_1}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3},$$

$$\ddot{\boldsymbol{x}}_2 = G\frac{m_{2\mathrm{p}}m_{1\mathrm{a}}}{m_2}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}, \tag{8}$$

where the indices “p” and “a” denote the passive and active gravitational masses, respectively, and G is the gravitational constant. (As throughout, m without these indices denotes the inertial mass.) Thus, an inequality of active and passive masses results in a self-acceleration of the center of mass if $\bar{C}_{21}=(m_{2\mathrm{a}}/m_{2\mathrm{p}})-(m_{1\mathrm{a}}/m_{1\mathrm{p}})\neq 0$, which again can be interpreted as a violation of Newton’s third law for gravitational forces. A limit has been derived by lunar laser ranging: because no self-acceleration of the moon has been observed, the limit of $|\bar{C}_{\text{Al-Fe}}|\leqslant 7\times 10^{-13}$ is obtained [8]. The dynamics of the relative coordinate

$$\ddot{\boldsymbol{x}} = -G\frac{m_{1\mathrm{p}}m_{2\mathrm{p}}}{m_1 m_2}\left(m_1\frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}}+m_2\frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}}\right)\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} \tag{9}$$

has been probed in a laboratory experiment by Kreuzer [9], with the result $|\bar{C}_{21}|\leqslant 5\times 10^{-5}$. Note that these experiments are purely gravitational ones. For the astrophysical observations, this is because astronomical bodies do not carry active electric charges. (Otherwise, they would attract passively charged particles. If these carry active charges of the same sign, this will eventually neutralize the active charge.) In laboratory experiments, electric neutrality is ensured by grounding. These experiments thus confirm the equality of active and passive mass, independent of any inequality between active and passive electric charge.

Owing to the extreme relative weakness of the gravitational force compared to the electrical one, it would take a huge violation of the equality of active and passive mass to mimic a signal for an inequality of active and passive charge. For example, the gravitational force between the electron and the proton in a hydrogen atom is $\sim 10^{-39}$ times the electrical one. Thus, to mimic an electrical $C_{21}\sim 10^{-20}$ in the hydrogen spectroscopy experiments discussed later, it would take a gravitational $\bar{C}_{21}\sim 10^{+19}$. A $\bar{C}_{21}$ of this magnitude is clearly ruled out. Other experiments discussed by us are based on measuring the active charge of a macroscopic number of atoms by use of an electrometer. The electrometer is based on the Coulomb force caused by the charge to be measured on a number of electrons. Again, the gravitational interaction with these electrons is much weaker than the electrical, and it would take a huge gravitational $\bar{C}_{21}$ to mimic an electrical C_{21}. Because of this we can neglect any inequality of gravitational mass (as well as standard gravitational effects) in the remainder of this paper. The limits we will find are thus independent of an inequality of active and passive mass.

Moreover, since the acceleration of charges is proportional to q_{p}/m (where m is the inertial mass), one might ask whether measurements may be insensitive to changes in the passive charge that are accompanied by proportional changes in the inertial mass. This question is best answered by an explicit example. We shall do so in Sec. III.

C. Comparison of the gravitational and electrical cases

The electric case differs from the gravitational one in three important ways. (i) In the gravitational case, the weak equivalence principle $m=m_{\mathrm{p}}$ implies that paths of particles depend on the active gravitational mass only. (ii) Since the time scale of electric phenomena is much shorter than that of gravitational ones, the motion of the center of mass cannot be monitored. This kind of test is therefore not at our disposal. (iii) Contrary to the gravitational case, electric charges can have different signs. Therefore, we can define active neutrality $q_{1\mathrm{a}}+q_{2\mathrm{a}}=0$ as well as passive neutrality $q_{1\mathrm{p}}+q_{2\mathrm{p}}=0$. This allows us to find alternative tests of the equality of active and passive charges: An actively neutral system may not be passively neutral, and vice versa. The two definitions of neutrality are compatible, but a system can be actively and passively neutral if and only if $C_{21}=0$. Therefore, a self-acceleration of the center of mass occurs only if the system possesses a nonzero total active or passive charge.

II. EXPERIMENTS WITH MACROSCOPIC MATTER

In order to interpret tests of the neutrality of atoms and molecules as tests of the equality of active and passive charges, we study compound particles that are actively or passively neutral. If we assume passive neutrality, there will be no acceleration due to the external field $\boldsymbol{E}$, and C_{21} reduces to $(q_{2\mathrm{a}}+q_{1\mathrm{a}})/q_{2\mathrm{p}}$. The difference between active and passive charges is now related to the active neutrality of the composed system: a passively neutral system may still generate an electric field according to

$$\phi(\boldsymbol{x}) = \frac{q_{1\mathrm{a}}}{|\boldsymbol{x}-\boldsymbol{x}_1|}+\frac{q_{2\mathrm{a}}}{|\boldsymbol{x}-\boldsymbol{x}_2|}=\frac{q_{1\mathrm{a}}+q_{2\mathrm{a}}}{|\boldsymbol{x}|}+\cdots\approx C_{21}\frac{q_{2\mathrm{p}}}{|\boldsymbol{x}|} \tag{10}$$

(where the ellipsis denotes dipole and higher-order multipole contributions that are neglected here). On the other hand, an

TABLE I. Various tests of the neutrality of atoms. If no particle is specified, q_p refers to the passive charge of the atoms or molecules used in the experiment, divided by the charge number of that particle (and analogously for q_a).

Method	Limit ($10^{-20}e$)
Gas efflux (350 g CO_2) [16]	$q_{p,a}-q_{e,a}=0.1(5)$
Gas efflux (Ar-N) [17]	$q_{H,a}=1(3)$, $q_{n,a}=-1(3)$
Gas efflux [18]	$q_{He,a}=-4(2)$
Superfluid He [11]	$q_{He,a}=-0.22(15)$
Levitator [12]	$\|q_p\| \lesssim 1000$
Acoustic resonator (SF_6) [13]	$\|q_p\| \leqslant 0.13$
Cs beam [14]	$q_p=90(20)$
Neutron beam [15]	$q_{n,p}=-0.4(1.1)$

actively neutral system in a homogenous external electric field experiences a force

$$M\ddot{\boldsymbol{X}} = (q_{1p}+q_{2p})\boldsymbol{E} = \frac{q_{2p}}{q_{2a}} q_{1a} C_{12} \boldsymbol{E}. \tag{11}$$

Thus, we can distinguish two types of tests of neutrality: (i) tests of active neutrality, which measure the electric monopole field created by a passively neutral system, and (ii) tests of passive neutrality, which measure the force imposed by an external field onto an actively neutral system.

A review of tests of the neutrality of atoms can be found in [10]. One type of experiments, gas efflux experiments, tests the active neutrality. They are based on observing the charge of a metallic container during an in-or outflow of gas or liquid. (The charge measurement is based on the electric field caused by the charge and is therefore sensitive to active charge only. This also applies to the modern electrometers that use field-effect transistors.) With each of N atoms or molecules containing a number n_p of protons and electrons and n_n neutrons, the charge $N[n_p(q_{e,a}+q_{p,a})+n_n q_{n,a}]$ is measured. The indices e, p, and n denote the electron, the proton, and the neutron. An interesting modern variant [11] uses superfluid He as a medium.

The passive neutrality has been tested by a variety of methods (see Ref. [10] for details). (i) Levitation experiments [12] follow the famous experiment by Millikan for the measurement of the electric charge of atoms. (ii) In acoustic resonator experiments [13], one applies an alternating electric field within an acoustic resonator and listens for the sound that would result due to a passively charged medium. (iii) Atom [14] or neutron [15] beam experiments measure the deflection of a beam of atoms or neutrons that traverses an electric field.

The limits assembled in Table I are at levels down to 10^{-21} elementary charges for the active and passive charges of various combinations of electrons, protons, and neutrons. If we assume that there are no cancellations, we can thus conclude that $|C_{pe}| \leqslant 10^{-21}$, which can be regarded as a verification of Newton's third law for electric forces at the 10^{-21} level.

III. SPECTROSCOPY

We now study the shift of atomic transition frequencies due to an inequality of active and passive charges. Although the center-of-mass motion of the two-particle system cannot be quantized in general, the relative motion can. The Hamiltonian is

$$H = \frac{\boldsymbol{p}^2}{2m_{\text{red}}} + D_{21}\frac{q_{1p}q_{2p}}{|\boldsymbol{x}|}. \tag{12}$$

The energy levels for a single-electron atom are proportional to the square of a modified fine structure constant:

$$\alpha_{12} = \frac{q_{1p}q_{2p}D_{12}}{\hbar c} = \frac{q_{1p}q_{2p}}{\hbar c}\left(\frac{q_{1a}}{q_1} + \frac{m_2}{M}C_{21}\right). \tag{13}$$

Therefore, a comparison of the energy levels in atoms having different nuclear masses yields a test of the equality of active and passive charges. Since the accuracy is influenced by the accuracy of the theoretical prediction of the transition frequencies, comparison of simple atoms gives the most accurate results. Let us thus compare hydrogen (H) and ionized helium (He^+): For H, we have $q_1=q_p$, $q_2=q_e$, and $M\approx m_p$; for He^+, $q_1=2q_p$, $q_2=q_e$, and $M\approx 4m_p$. For the ratio of the transition frequencies, we thus find

$$\frac{\nu_{12}(\text{He}^+)}{4\nu_{12}(\text{H})} = \frac{\alpha_{12}^2(\text{He}^+)}{4\alpha_{12}^2(\text{H})} \approx 1 - \frac{3}{2}\frac{m_e}{m_p}C_{21}, \tag{14}$$

where we have neglected terms of the order $(m_e/m_p)^2$. The $1S_{1/2}-2S_{1/2}$ transition in hydrogen has been measured to a precision of 1.9×10^{-14} [19]; however, the theoretical prediction of the Lamb shift has an error bar of 6.9×10^{-13}, in part due to the uncertainty in the charge radius of the proton. See Appendix A of Ref. [20]. Since He^+ ions can be laser cooled [21], an even higher precision is expected for them [22]. Also, the theoretical uncertainty for He^+ can be lower since the properties of the He nucleus are better known. If no discrepancy like Eq. (14) between H and He^+ at 7×10^{-13} were found, we could deduce a limit of $|C_{21}| \leqslant 8.3\times10^{-10}$.

The accuracy of this limit is less than that of the frequencies by the nucleus-to-electron mass ratio of hydrogen, m_p/m_e. Thus, it is interesting to consider positronium, where the mass ratio is unity. Positronium's $1\,^3S_1-2\,^3S_1$ frequency is known to 2.6×10^{-9} [23]. The deviation from the theoretical prediction is $\sim1.3\sigma$ ([26], Fig. 3), but we consider this insignificant. Comparison to hydrogen thus yields a limit on $\alpha_{ee^+}^2/\alpha_{12}^2(\text{H})\approx 1+C_{ee^+}$, where we neglect a term that is suppressed by m_e/m_p. Thus, we obtain $|C_{ee^+}| \leqslant 2.6\times10^{-9}$ (the Lamb shift in positronium can be predicted to sufficient accuracy [20]). These limits are not as precise as the ones derived from bulk matter experiments, which gain sensitivity from the macroscopic number of particles. However, they are particularly clean: The physics of light atoms is known in detail to the same precision as the limit derived.

The explicit form of the experimental signature Eq. (14) also makes clear that the measurement is not insensitive to changes in the passive charge even if they are accompanied by changes in the inertial masses. These masses solely enter the factor m_e/m_p that sets the sensitivity of the experiment to

C_{21}. However, $C_{21} \neq 0$ will always be detected, regardless of small variations in this factor. The fundamental reason for this is that the experiment is based on comparing two atoms that have different nuclei and thus different charge-to-mass ratios.

IV. MAGNETIC MOMENTS

Since moving charges create magnetic fields and magnetic fields act on moving charges, one may extend the above analysis to the question of the equality of active and passive magnetic moments. In analogy to the above considerations, we first calculate the force between two magnetic moments:

$$m_1 \ddot{\boldsymbol{x}}_1 = \boldsymbol{\nabla}_1[\boldsymbol{\mu}_{1\mathrm{p}} \cdot \boldsymbol{B}_2(\boldsymbol{x}_1)],$$

$$m_2 \ddot{\boldsymbol{x}}_2 = \boldsymbol{\nabla}_2[\boldsymbol{\mu}_{2\mathrm{p}} \cdot \boldsymbol{B}_1(\boldsymbol{x}_2)], \quad (15)$$

where

$$\boldsymbol{B}_j(\boldsymbol{x}_k) = \frac{3[(\boldsymbol{x}_k - \boldsymbol{x}_j) \cdot \boldsymbol{\mu}_{j\mathrm{a}}](\boldsymbol{x}_k - \boldsymbol{x}_j) - \boldsymbol{\mu}_{j\mathrm{a}}|\boldsymbol{x}_k - \boldsymbol{x}_j|^2}{|\boldsymbol{x}_k - \boldsymbol{x}_j|^5}. \quad (16)$$

In a classical picture, the magnetic moments can be considered as being created by a current loop, and the direction of the magnetic moment is given by the orientation of the loop. Therefore, a difference between active and passive magnetic moments is a difference between their magnitudes only, which are related to the charges making up the current. Thus, we assume $\boldsymbol{\mu}_{1,2\mathrm{a,p}} = \mu_{1,2\mathrm{a,p}} \hat{\boldsymbol{\mu}}_{1,2}$, where $\hat{\boldsymbol{\mu}}_{1,2}$ are unit vectors indicating the direction of the magnetic moments. If we introduce

$$\widetilde{C}_{21} = \frac{\mu_{2\mathrm{a}}}{\mu_{2\mathrm{p}}} - \frac{\mu_{1\mathrm{a}}}{\mu_{1\mathrm{p}}},$$

$$\widetilde{D}_{21} = \frac{m_1}{M}\frac{\mu_{1\mathrm{a}}}{\mu_{1\mathrm{p}}} + \frac{m_2}{M}\frac{\mu_{2\mathrm{a}}}{\mu_{2\mathrm{p}}} = \frac{\mu_{1\mathrm{a}}}{\mu_{1\mathrm{p}}} + \frac{m_2}{M}\widetilde{C}_{21}, \quad (17)$$

then we obtain for the center-of-mass and relative coordinates

$$\ddot{\boldsymbol{X}} = -\frac{\mu_{2\mathrm{p}}\mu_{1\mathrm{p}}}{M}\widetilde{C}_{21}\boldsymbol{\nabla}\frac{3(\boldsymbol{x}\cdot\hat{\boldsymbol{\mu}}_2)(\hat{\boldsymbol{\mu}}_1\cdot\boldsymbol{x}) - \hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2|\boldsymbol{x}|^2}{|\boldsymbol{x}|^5},$$

$$\ddot{\boldsymbol{x}} = \frac{\mu_{1\mathrm{p}}\mu_{2\mathrm{p}}}{m_{\mathrm{red}}}\widetilde{D}_{21}\boldsymbol{\nabla}\frac{3(\boldsymbol{x}\cdot\hat{\boldsymbol{\mu}}_2)(\hat{\boldsymbol{\mu}}_1\cdot\boldsymbol{x}) - \hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2|\boldsymbol{x}|^2}{|\boldsymbol{x}|^5}. \quad (18)$$

For different ratios of active and passive magnetic moments, the center of mass will show self-acceleration. The equations of the torque describe the orientation of the magnetic moments only: $\dot{\boldsymbol{\mu}}_{1\mathrm{p}} = -\boldsymbol{B}_2(\boldsymbol{x}_1) \times \boldsymbol{\mu}_{1\mathrm{p}}$, and $\dot{\boldsymbol{\mu}}_{2\mathrm{p}} = -\boldsymbol{B}_1(\boldsymbol{x}_2) \times \boldsymbol{\mu}_{2\mathrm{p}}$. This gives rise to an additional spin-orbit coupling. We will not consider this.

Atomic spectroscopy

Again, the relative motion can contribute to the energy of a hydrogen atom: The Hamiltonian for the hyperfine interaction reads

$$H_{\mathrm{hf}} = -\frac{\mu_{1\mathrm{p}}\mu_{2\mathrm{p}}}{m_{\mathrm{red}}}\widetilde{D}_{21}\left(\frac{8\pi}{3}\delta(x)\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2 + \frac{3(\boldsymbol{x}\cdot\hat{\boldsymbol{\mu}}_2)(\hat{\boldsymbol{\mu}}_1\cdot\boldsymbol{x}) - \hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2|\boldsymbol{x}|^2}{|\boldsymbol{x}|^5}\right), \quad (19)$$

where the δ function describes the contribution from the local interaction of the electron with the nucleus. The $\hat{\boldsymbol{\mu}}_2$ are now total angular momentum operators.

To obtain experimental limits, we compare the hyperfine splitting of atoms having different nuclei. The hyperfine splitting of muonium has been measured to an accuracy of 1.1×10^{-8} [24]. As summarized in [25], it is compatible with the theoretical prediction, which has an uncertainty of 1.2×10^{-7}. For positronium, two precision measurements of the $1S$ hyperfine splitting have been reported by Mills *et al.* [27] and by Ritter *et al.* [28]. They agree within the experimental error. However, they deviate by $-6.3(2.9)$ MHz and $-4.7(1.7)$ MHz, respectively, from the theoretical prediction of 203.3917(6) GHz [26].

In our framework, this difference may be modeled by a difference of passive and active magnetic moments of $\widetilde{C}_{ee^+} = -1.7(7) \times 10^{-5}$ for the Mills *et al.* measurement and $\widetilde{C}_{ee^+} = -1.3(4) \times 10^{-5}$ for Ritter *et al.* It is worth noting that such a discrepancy between theory and experiment exist for this $1S$ hyperfine splitting only. All other spectroscopic quantities discussed in [26], which are approximately independent of a difference in active and passive magnetic moments, agree with their theoretical prediction (e.g., the $1S$-$2S$ interval already used above to find a limit on C_{ee^+}, and the fine structure of four different transitions).

On the other hand, the discrepancy between theory and experiment could be due to systematic influences in the experiments or an incomplete theoretical understanding of positronium (the latter point of view is expressed in [26]). Even two different experiments can be influenced by the same systematic effects if the measurement principle is similar or in part similar. For example, the line shape of positronium is still being investigated [14].

This would make it interesting to find alternative measurements, for example, from the hyperfine structure of nonleptonic atoms. An overview for hydrogen, deuterium, tritium, and the ^{3}He ion can be found in [25]. Precise experiments (with error bars in the 10^{-12} range) do exist, but unfortunately there are rather large discrepancies with the theory. These are attributed to the uncertainty of the nuclear contributions [25]. Moreover, the high nucleus-to-electron mass ratio suppresses the influence of $\widetilde{C}_{12}$ on the hyperfine splitting of nonleptonic atoms. For example, hydrogen and tritium show the lowest discrepancy between theory and experiment of -33 and -38 ppm, respectively. We take the difference of 5 ppm as a signal for active and passive magnetic moments, and the geometric sum of 50 ppm of the discrepancies as an estimate of the error. This gives $|\widetilde{C}_{21}(\mathrm{H}) - \frac{1}{3}\widetilde{C}_{21}(\mathrm{T})| \leq \frac{1}{2}\frac{m_p}{m_e} \times (5 \pm 50)\ \mathrm{ppm} = 0.005 \pm 0.045$. This is not suitable for ruling out the significant value from positronium and muonium spectroscopy.

To sum up, in the present state of theory and experiment we have to regard the possibility of a difference of theory and experiment as a hypothesis that is probably wrong, though it is supported by two independent experiments. However, it would be interesting to check it against other systems. Unfortunately, the hyperfine structure of nonleptonic atoms depicts even larger theoretical uncertainties.

V. SUMMARY AND OUTLOOK

In summary, we have introduced the concept of active and passive electric charges and magnetic moments, which in standard electrodynamics are assumed to be equal. The best limits (of the order of 10^{-21} for C_{pe}) come from experiments testing the neutrality of macroscopic matter, which gain sensitivity from a large number of particles. Spectroscopy of hydrogen and positronium provides $|C_{ee^+}|\leq 2.6\times 10^{-9}$. These limits can also be interpreted as experimental verifications of Newton's third law for electric forces at the 10^{-21} level. For magnetic moments, comparison of the hyperfine structure of positronium and muonium suggests a difference between active and passive of $\tilde{C}_{ee^+}=-1.4\times 10^{-5}$, which is significant at the 3σ level. However, in the present state of theory and experiment, this is more likely an artifact, even if two independent experiments agree. The best limit from nonleptonic atoms (hydrogen and tritium) is at the 5% level of accuracy.

The relativistic quantum theory of electrodynamics is quantum electrodynamics (QED), and one may ask how the question of active and passive charges can be formulated in this context. The most straightforward way to do so starts at the level of the field equations. We use the passive charge in the Dirac equation for an electron with minimal coupling to the electromagnetic field. The active charge enters the source term in the inhomogeneous Maxwell equation. The nonrelativistic Pauli equation and the classical limit can then be found in the usual way, as described in textbooks. As a result, this relativistic quantum description contains our above classical results. Further experimental signatures of a difference between active and passive charges might be sought within such a model. This might be possible, for example, by comparing the value of the fine-structure constant obtained from the measurement of the electron's anomalous magnetic moment $g-2=\alpha/(2\pi)+\cdots$ (to 7×10^{-10} accuracy [29]) to other measurements of α. However, even without a QED version of our question, we were able to answer it in terms of experimental limits, some of them very stringent.

The C and $\tilde{C}$ coefficients for most particle combinations are ~12 orders of magnitude less stringent than C_{pe} and C_{ne}. Thus, it might be interesting to seek further experimental limits. For example, certain selection rules in spectroscopy, which are normally imposed by symmetry arguments, might be broken. New versions of the macroscopic matter experiments could simultaneously measure the active and passive charges in order to suppress some of the systematic effects.

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[1] E. Fischbach and C. L. Talmadge, *The Search for Non Newtonian Gravity* (Springer-Verlag, New York, 1999).
[2] G. Spavieri *et al.*, Metrologia **41**, S159 (2004).
[3] L.-C. Tu, J. Luo, and G. T. Gillies, Rep. Prog. Phys. **68**, 77 (2005).
[4] G. Amelino-Camelia *et al.*, in *Gravitation and Cosmology*, edited by A. Macias, C. Lämmerzahl, and D. Nunez, AIP Conf. Proc. No. 758 (AIP, Melville, NY, 2005).
[5] D. Mattingly, Living Rev. Relativ. **8**, 20055 (2005).
[6] H. Müller, C. Braxmaier, S. Herrmann, A. Peters, and C. Lämmerzahl, Phys. Rev. D **67**, 056006 (2003); H. Müller, S. Herrmann, A. Saenz, A. Peters, and C. Lämmerzahl, *ibid.* **68**, 116006 (2003); H. Müller, *ibid.* **71**, 045004 (2005).
[7] H. Bondi, Rev. Mod. Phys. **29**, 423 (1957).
[8] D. F. Bartlett and D. Van Buren, Phys. Rev. Lett. **57**, 21 (1986).
[9] L. B. Kreuzer, Phys. Rev. **169**, 1007 (1968).
[10] C. S. Unnikrishnan and G. T. Gillies, Metrologia **41**, S125 (2004).
[11] J. Classen, C. K. Su, M. Mohazzab, and H. J. Maris, Phys. Rev. B **57**, 3000 (1998).
[12] M. Marinelli and G. Morpurgo, Phys. Rep. **85**, 161 (1982).
[13] H. F. Dylla and J. G. King, Phys. Rev. A **7**, 1224 (1973).
[14] V. W. Hughes, Phys. Rev. **105**, 170 (1957).
[15] J. Baumann, R. Gahler, T. Kalus, and W. Mampe, Phys. Rev. D **37**, 3107 (1988).
[16] A. Piccard and E. Kessler, Arch. Sci. Phys. Nat. **7**, 340 (1925).
[17] A. M. Hillas and T. E. Cranshaw, Nature (London) **186**, 459 (1960).
[18] J. G. King, Phys. Rev. Lett. **5**, 562 (1960).
[19] Th. Udem, A. Huber, B. Gross, J. Reichert, M. Prevedelli, M. Weitz, and T. W. Hänsch, Phys. Rev. Lett. **79**, 2646 (1997); J. Reichert, M. Niering, R. Holzwarth, M. Weitz, Th. Udem, and T. W. Hänsch, *ibid.* **84**, 3232 (2000); M. Niering *et al.*, *ibid.* **84**, 5496 (2000).
[20] P. Mohr and M. Taylor, Rev. Mod. Phys. **77**, 1 (2005).
[21] B. Roth, U. Fröhlich, and S. Schiller, Phys. Rev. Lett. **94**, 053001 (2005).
[22] S. A. Burrows *et al.*, in *Laser Spectroscopy 14*, edited by R. Blatt *et al.* (World Scientific, Singapore, 1999); see also http://www.mpq.mpg.de/~haensch/comb/research/helium.html
[23] M. S. Fee, S. Chu, A. P. Mills, R. J. Chichester, D. M. Zuckerman, E. D. Shaw, and K. Danzmann, Phys. Rev. A **48**, 192 (1993).

[24] W. Liu *et al.*, Phys. Rev. Lett. **82**, 711 (1999).
[25] S. G. Karshenboim and V. G. Ivanov, Phys. Lett. B **524**, 259 (2002); Eur. Phys. J. D **19**, 13 (2002).
[26] S. G. Karshenboim, Int. J. Mod. Phys. A **19**, 3879 (2004).
[27] A. P. Mills, Jr., and G. H. Bearman, Phys. Rev. Lett. **34**, 246 (1975).
[28] M. W. Ritter, P. O. Egan, V. W. Hughes, and K. A. Woodle, Phys. Rev. A **30**, 1331 (1984).
[29] B. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse, Phys. Rev. Lett. **97**, 030801 (2006); G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *ibid.* **97**, 030802 (2006).