Nonclassicality of quantum excitation of classical coherent field in photon-loss channel

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We investigate the nonclassicality of photon-added coherent states in the photon-loss channel by exploring the entanglement potential and negative Wigner distribution. The total negative probability defined by the volume of the negative part of the Wigner function reduces with the decay time. The total negative probability and the entanglement potential of pure photon-added coherent states exhibit the similar dependence on the beam intensity. The reduce of the total negative probability is consistent with the behavior of entanglement potential for the dissipative single-photon-added coherent state at short decay times.

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Nonclassical optical fields play a crucial role in understanding fundamentals of quantum physics and have many applications in quantum information processing [1]. Usually, the nonclassicality manifests itself in specific properties of quantum statistics, such as the antibunching [2], sub-Poissonian photon statistics [3], squeezing in one of the quadratures of the field [4], partial negative Wigner distribution [5], etc.

When the nonclassical optical fields propagate in the medium, they inevitably interact with their surrounding environment, which causes the dissipation or dephasing [6]. It is well known that the dissipation or dephasing will deteriorate the degree of nonclassicality of the optical fields. A quantitative measure of nonclassicality of quantum fields is necessary for further investigating the dynamical behavior of their nonclassicality. Many authors have investigated the relations between nonclassicality of optical fields and the entanglement and shown that nonclassicality is a necessary condition for generating inseparable state via the beam splitter [7]. Based on them, a measure called the entanglement potential for quantifying the nonclassicality of the single-mode optical field has been proposed [8]. The entanglement potential is defined as the entanglement achieved by 50:50 beam splitter characterized by the unitary operation $U_{\mathrm{BS}}=e^{(\pi/4)i(a^{\dagger}b+ab^{\dagger})}$ acting on the target optical mode a and the vacuum mode b. Throughout this paper, log negativity is explored as the measure of entanglement potential. The log-negativity of a density matrix ρ is defined by [9]

$$N(\rho)=\log_2\|\rho^{\Gamma}\|, \tag{1}$$

where ρ^{Γ} is the partial transpose of ρ and $\|\rho^{\Gamma}\|$ denotes the trace norm of ρ^{Γ}, which is the sum of the singular values of ρ^{Γ}.

Nevertheless, experimental measurement of the entanglement potential is still a challenge task. How to quantify the variation of the nonclassicality of quantum fields based on the current mature laboratory technique is an interesting topic. The Wigner function is a quasiprobability distribution, which fully describes the state of a quantum system in phase space. The partial negativity of the Wigner function is indeed a good indication of the highly nonclassical character of the state. Reconstructions of the Wigner functions in experiments with quantum tomography [10–12] have demonstrated appearance of their negative values, which cannot be explained in the framework of the probability theory and have not any classical counterparts. Therefore, to seek certain possible monotonic relation between the partial negativity of the Wigner distribution and the entanglement potential may be an available first step for experimentally quantifying the variation of nonclassicality of quantum optical fields in dissipative or dephasing environments.

Here, for clarifying the feasibility of this idea, we investigate the nonclassicality of photon-added coherent states in the photon-loss channel by exploring the entanglement potential and negative Wigner distribution. The total negative probability defined by the volume of the negative part of the Wigner function is adopted, and our calculations show it reduces with the increase of decay time. The total negative probability and the entanglement potential of pure photon-added coherent states exhibit the similar dependence on the beam intensity. The reduce of total negative probability is consistent with the behavior of entanglement potential for the dissipative single-photon-added coherent state at short decay times.

The photon-added coherent state was introduced by Agarwal and Tara [13]. The single photon-added coherent state (SPACS) has been experimentally prepared by Zavatta *et al.* and its nonclassical properties have been detected by homodyne tomography technology [14]. Such a state represents the intermediate non-Gaussian state between quantum Fock state and classical coherent state (with well-defined amplitude and phase) [15]. For the SPACS, a quantum to classical transition has been explicitly demonstrated by ultrafast time-domain quantum homodyne tomography technique. Thus, it is timely to analyze how the photon loss affects the nonclassicality of such kind of optical fields.

Let us first briefly recall the definition of the photon-added coherent states (PACSs) [13]. The PACSs are defined by $\frac{1}{\sqrt{N(\alpha,m)}}a^{\dagger m}|\alpha\rangle$, where $|\alpha\rangle$ is the coherent state with the amplitude α and $a^{\dagger}$ is the creation operator of the optical mode. $N(\alpha,m)=m!L_m(-|\alpha|^2)$, where $L_m(x)$ is the mth-order Laguerre polynomial. When the PACS evolves in the photon-

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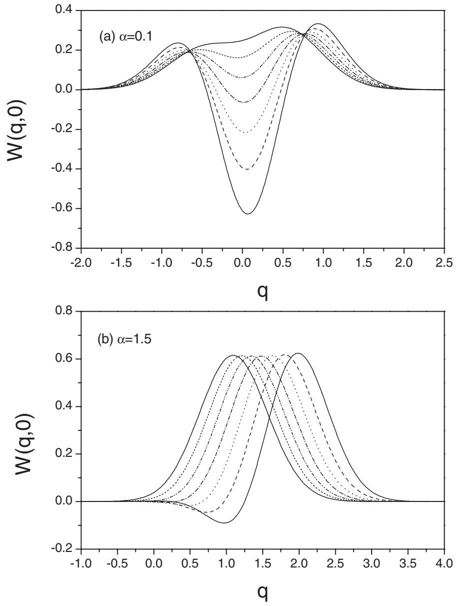

FIG. 1. The Wigner distribution function at $p=0$ of the SPACSs with two different values of initial amplitudes α [(a) $\alpha=0.1$; (b) $\alpha=1.5$] in the photon-loss channel are depicted for several decay times γt. From bottom to top, the decay times γt are 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, respectively. The absolute value of negative minimum of the Wigner function decreases with decay time for two cases.

loss channel, the evolution of the density matrix can be described by [6]

$$\frac{d\rho}{dt} = \frac{\gamma}{2}(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a), \tag{2}$$

where γ represents dissipative coefficient. The corresponding nonunitary time evolution density matrix can be obtained as

$$\rho(t) = \frac{1}{m!L_m(-|\alpha|^2)}\sum_{k=0}^{\infty}\frac{(1-e^{-\gamma t})^k}{k!}\hat{L}(t)a^k a^{\dagger m}|\alpha\rangle\langle\alpha|a^m a^{\dagger k}\hat{L}(t), \tag{3}$$

where $\hat{L}(t)=e^{-(1/2)\gamma t a^{\dagger}a}$. For the dissipative photon-added coherent state in Eq. (3), the total output state passing through a 50/50 beam splitter characterized by the unitary operation $e^{(\pi/4)i(a^{\dagger}b+ab^{\dagger})}$ with a vacuum mode b can be obtained or

$$\rho_{\mathrm{tot}} = D_a(t)D_b(t)\frac{e^{-m\gamma_2 t}e^{|\alpha|^2(e^{-\gamma_2 t}-1)}}{m!L_m(-|\alpha|^2)}\sum_{k=0}^{\infty}\frac{(e^{\gamma_2 t}-1)^k}{k!}\hat{E}^k\hat{E}^{\dagger m}|00\rangle \times\langle 00|\hat{E}^m\hat{E}^{\dagger k}D_a^{\dagger}(t)D_b^{\dagger}(t), \tag{4}$$

where $D_a(t)=e^{(\sqrt{2}/2)\alpha(t)a^{\dagger}-(\sqrt{2}/2)\alpha^*(t)a}$ and $D_b(t)=e^{(\sqrt{2}i/2)\alpha(t)b^{\dagger}+(\sqrt{2}i/2)\alpha^*(t)b}$ are the displacement operators of the modes a and b, respectively, where $\alpha(t)=\alpha e^{-\gamma_2 t/2}$. $\hat{E}=\frac{\sqrt{2}}{2}a-\frac{\sqrt{2}i}{2}b+\alpha e^{-(1/2)\gamma_2 t}$. The local unitary operators can not change entanglement, therefore, we only need to consider the entanglement of the mixed state given as follows:

$$\rho'_{\mathrm{tot}} = \frac{e^{-m\gamma_2 t}e^{|\alpha|^2(e^{-\gamma_2 t}-1)}}{m!L_m(-|\alpha|^2)}\sum_{k=0}^{\infty}\frac{(e^{\gamma_2 t}-1)^k}{k!}\hat{E}^k\hat{E}^{\dagger m}|00\rangle\langle 00|\hat{E}^m\hat{E}^{\dagger k}. \tag{5}$$

The log negativity of the above density matrix can be analytically solved for the case of single photon excitation, i.e., $m=1$, but its expression is still lengthy. The corresponding entanglement potential of the SPACSs quantified by log negativity is given in Fig. 4(b).

On the other hand, the presence of negativity in the Wigner function of the field is also the indicator of nonclassicality. The Wigner function, the Fourier transformation of characteristics function [16] of the state can be derived by [17]

$$W(\beta) = (2/\pi)\mathrm{Tr}[(\hat{O}_e - \hat{O}_o)\hat{D}(\beta)\rho\hat{D}^{\dagger}(\beta)], \tag{6}$$

where $\hat{O}_e \equiv \Sigma_{n=0}^{\infty}|2n\rangle\langle 2n|$ and $\hat{O}_o \equiv \Sigma_{n=0}^{\infty}|2n+1\rangle\langle 2n+1|$ are the even and odd parity operators, respectively. In the photon-loss channel described by the master equation (2), the time evolution Wigner function satisfies the following Fokker-Planck equation [18]:

$$(\partial/\partial t)W(q,p,t) = (\gamma/2)[(\partial/\partial q)q + (\partial/\partial p)p]W(q,p,t) + (\gamma/8)[(\partial^2/\partial q^2) + (\partial^2/\partial p^2)]W(q,p,t), \tag{7}$$

where q and p represent the real part and imaginary part of β, respectively. Substituting the initial Wigner function of a SPACS [13]

$$W(q,p,0) = \frac{-2L_1(|2q+2ip-\alpha|^2)}{\pi L_1(-|\alpha|^2)}e^{-2|q+ip-\alpha|^2} \tag{8}$$

and the initial Wigner function of a two photon-added coherent state (TPACS) [13]

$$W(q,p,0) = \frac{2L_2(|2q+2ip-\alpha|^2)}{\pi L_2(-|\alpha|^2)}e^{-2|q+ip-\alpha|^2} \tag{9}$$

into Eq. (7), we can obtain the time evolution Wigner function. In Fig. 1, the phase space Wigner distributions at $p=0$ of the SPACSs in the photon-loss channel are depicted for several different values of γt and α, from which the influence of photon loss on the partial negativity of the Wigner function is explicitly shown. For the cases of $\alpha=0.1, 1.5$, it is found that those curves $W(q,0)$ at decay times $\gamma t=0, 0.2, 0.4, 0.6$ exhibit partial negativity. The further photon loss will completely destroy the partial negativity. The pure TPACSs exhibit more nonclassicality than the pure SPACSs when the entanglement potential is adopted as the measure of nonclassicality [19]. Figure 2 shows that the photon loss deteriorates the partial negativity of the Wigner function of the TPACS. For more explicitly observing the details, we plot $W(q,0)$ of the TPACSs with different values of γt and α in Fig. 3. Different from the cases of SPACSs, here, $W(q,0)$ of these pure TPACSs with $|\alpha|\leqslant 1$ have two explicit negative local minimal values. As $|\alpha|$ increases, the absolute value of the negative local minimum at the left more rapidly de-

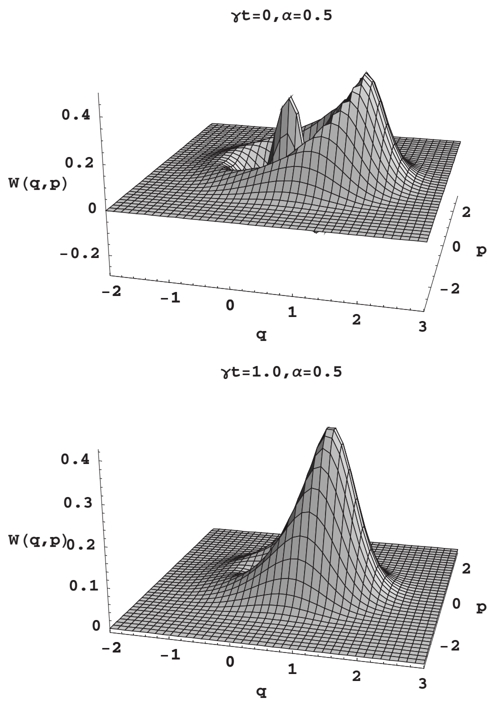

FIG. 2. The Wigner functions of the TPACS with $\alpha=0.5$ in photon-loss channel are depicted for two different values of decay time γt.

creases than the one at the right. As γt increases, the absolute value of negative minimum of $W(q,0)$ decreases which implies the decreases of the nonclassicality of the states. When γt exceeds a threshold value, the partial negativity of the Wigner distribution cannot be explicitly observed from this figure.

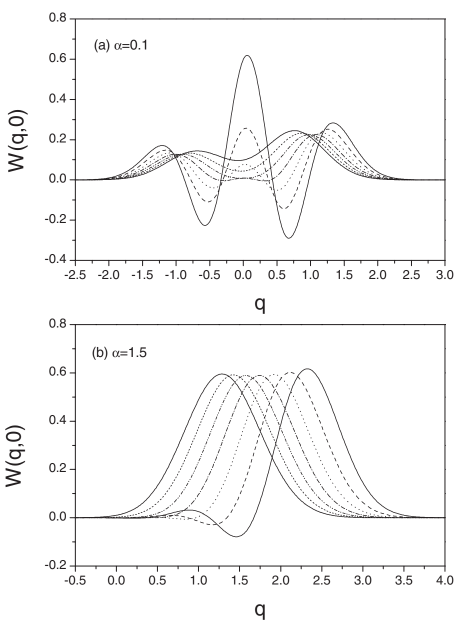

FIG. 3. The Wigner distribution function at $p=0$ of the TPACSs with two different values of initial amplitudes α [(a) $\alpha=0.1$; (b) $\alpha=1.5$] in the photon-loss channel are depicted for several decay times γt. From bottom to top, the decay times γt are 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, respectively. The absolute value of negative minimum of the Wigner function decreases with decay time for two cases.

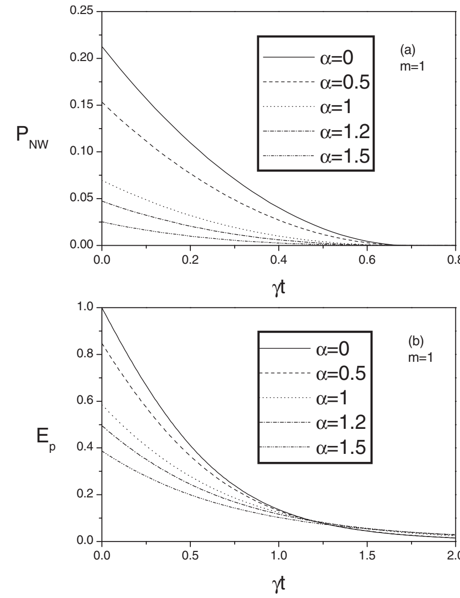

FIG. 4. (a) The absolute values P_{NW} of negative Wigner distribution probability of the SPACSs with several different values of initial amplitude α are plotted as the function of decay time γt in photon-loss channel. From top to bottom, $\alpha=0,0.5,1,1.2,1.5$. (b) The entanglement potential measured by log negativity of the SPACSs in photon-loss channel are plotted as the function of γt. From top to bottom, $\alpha=0,0.5,1,1.2,1.5$.

A natural question arises whether the partial negativity of the Wigner function can be used to quantitatively measure the nonclassicality of certain kinds of nonclassical fields. It is obvious that the negative minimum of the Wigner function cannot appropriately quantitatively measure the nonclassicality. For example, comparing Figs. 1(a) and 3(a), the absolute value of the negative minimum in the Wigner function of the pure SPACS with $\alpha=0.1$ is larger than the one of the pure TPACS with $\alpha=0.1$. The further calculations show the absolute value $|\mathrm{Min}[W]|$ of negative minimum of the Wigner function of the pure SPACSs with $|\alpha|<1.85$ is larger than the one of the pure TPACSs with the same $|\alpha|$, and $|\mathrm{Min}[W]|$ of the pure TPACSs is not the monotonically decreasing function of $|\alpha|$ [see Fig. 5(b)]. However, there was already the evidence that the pure TPACSs possesses larger nonclassicality (quantified by entanglement potential) than the pure SPACS with the same $|\alpha|$ [19]. Therefore, the absolute value of negative minimum of the Wigner function is not coincident with entanglement potential. Nevertheless, the absolute value P_{NW} of the total negative probability of the Wigner function may be a better choice for quantifying the nonclassicality than the negative minimum of the partial negative Wigner function [20–22]. P_{NW} is defined by

$$P_{\mathrm{NW}} = \left| \int_{\Omega} W(q,p)dqdp \right|, \tag{10}$$

where Ω is the negative Wigner distribution region and $|x|$ represents the absolute value of x. In Fig. 4(a), we can see that P_{NW} of the SPACSs with different values of α decreases with γt, and becomes zero after a threshold value of $\gamma t = \ln 2$. In Fig. 4(b), the entanglement potential quantified by

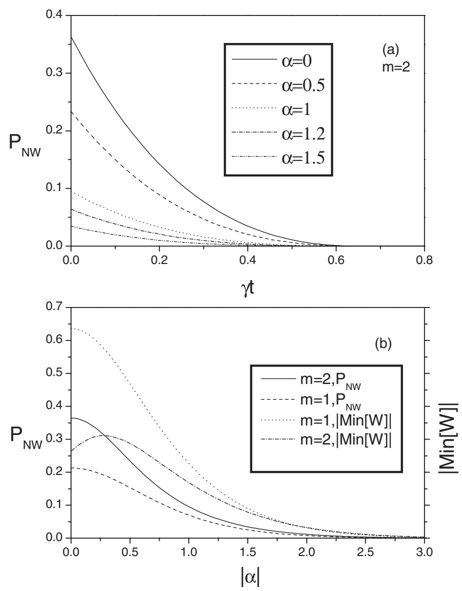

FIG. 5. (a) The absolute values P_{NW} of negative Wigner distribution probability of the TPACSs with several different values of initial amplitude α are plotted as the function of decay time γt in photon-loss channel. From top to bottom, $\alpha=0,0.5,1,1.2,1.5$. (b) The absolute values $|\mathrm{Min}[W]|$ of negative minimum of the Wigner functions and P_{NW} of the pure SPACSs and the pure TPACSs are plotted as the function of $|\alpha|$. (Solid line) P_{NW} of the pure TPACSs; (dash line) P_{NW} of the pure SPACSs; (dot line) $|\mathrm{Min}[W]|$ of the pure SPACSs; (dash-dot line) $|\mathrm{Min}[W]|$ of the pure TPACSs.

log negativity of SPACSs in the photon-loss channel is plotted. The entanglement potential also decreases with γt. These calculations partially elucidate the consistent between P_{NW} and the entanglement potential of the dissipative SPACSs at short time. Currently, the experimental quantitative investigation of the nonclassicality of the quantum optical fields is still an open issue, and the experimental measurement of entanglement potential has still several technical difficulties. So, the measurement of P_{NW} may be adopted as a replaced approach to investigate the influence of photon loss on the nonclassicality of the SPACSs. In Fig. 5(a), we also investigate the effect of photon loss on P_{NW} of the TPACSs with various values of α. At short decay times $\gamma t \leqslant 1$, the values of P_{NW} of the TPACSs are larger than those of the SPACSs with the same beam intensity $|\alpha|^2$. However, P_{NW} of the TPACSs is more fragile against photon loss than the one of the SPACS. As an illustration, for $\alpha=0.1$ and $\gamma t \geqslant 0.34$, the P_{NW} of TPACSs is smaller than the one of SPACS. From Fig. 4(b), we can also find that the entanglement potentials of SPACSs with different values of $|\alpha|<1$ more rapidly decrease than the ones of SPACSs with $|\alpha|>1$ in the photon-loss channel. Both the pure SPACSs and pure TPACSs are non-Gaussian nonclassical states with partial negativity of the Wigner distributions for any large but finite values of $|\alpha|$. In Fig. 5(b), the dependence of P_{NW} of the pure SPACSs and the pure TPACSs on $|\alpha|$ are shown. It is found that both the P_{NW} of the pure SPACSs and the pure TPACSs reduce with the increase of $|\alpha|$, and the TPACS possesses larger P_{NW} than the SPACS with the same value of $|\alpha|$. This property is also coincident with the dependence of entanglement potentials of the pure SPACSs and the pure TPACSs on $|\alpha|$ [19].

In summary, we have investigated the nonclassicality of photon excitation of classical coherent field in the photon-loss channel by exploring both the entanglement potential and the partial negativity of the Wigner function. Consistent behaviors of the total negative probability defined by the volume of the negative part of the Wigner function and the entanglement potential of single quantum excitation of classical coherent fields are found for the short time photon-loss process. Similar dependence of the total negative probability and the entanglement potential on the beam intensity is revealed for few photon-added coherent states. The partial negativity of the Wigner function can not be observed when the decay time exceeds a threshold value, while the entanglement potential always exists for any large but finite decay time. In the future tasks, it is interesting to investigate the variation of nonclassicality of general nonclassical states in Gaussian channel or non-Gaussian channel.

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