Effect of decoherence on the dynamics of Bose-Einstein condensates in a double-well potential

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We study the dynamics of a Bose-Einstein condensate in a double-well potential in the mean-field approximation. Decoherence effects are considered by analyzing the couplings of the condensate to the environment. Two kinds of coupling are taken into account. With the first kind of coupling dominating, decoherence can enhance self-trapping by increasing the damping of the oscillations in the dynamics, while decoherence from the second kind of condensate-environment coupling leads to spoiling of the quantum tunneling and self-trapping.

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Bose-Einstein condensates (BECs) in a double-well potential exhibit many fascinating phenomena that are absent in thermal atomic ensembles, for example, quantum tunneling and self-trapping [1–6]. Quantum tunneling through a barrier is a paradigm of quantum mechanics and usually takes place on a nanoscopic scale, as in two superconductors separated by a thin insulator [7] or two reservoirs of superfluid helium connected by nanoscopic apertures [8,9]. Recently, tunneling on a macroscopic (micrometer) scale in two weakly linked Bose-Einstein condensates in a double-well potential has been observed [10]. Similar to tunneling oscillations in superconducting and superfluid Josephson junctions, Josephson oscillations are observed when the initial population difference is chosen to be below a critical value. When the initial population difference exceeds a critical value, an interesting feature of the coherent quantum tunneling between the two BECs is observed, i.e., tunneling oscillations are suppressed due to nonlinear condensate self-interactions. This phenomenon is known as macroscopic quantum self-trapping.

The interactions between the condensate and noncondensate atoms lead to decoherence. Describing decoherence by fully including the quantum effects requires sophisticated theoretical studies that include the effect of noncondensate atoms. Treating the noncondensate atoms as a Markovian reservoir, master equations that govern the dynamics of the condensate atoms might be derived [11–13]. In fact, in the experiments on BECs, trapped atoms are evaporatively cooled and they continuously exchange particles with the noncondensate atoms. Thus, standard procedure in quantum optics for open systems would naturally lead to master equations for treating atomic BECs. This Markovian treatment for BECs also can be understood as the presence of lasers for trapping or detection of atoms, which will polarize the atoms and thus couple them to the vacuum modes of the electromagnetic field [14]. On the other hand, due to the unavoidable interaction of a BEC with its environment, decoherence is always there in a BEC; hence the characterization of decoherence in this system is interesting. Because different decoherence may have different effects on the dynamics of the BECs, the character of decoherence in BECs may be read out from the dynamics of the BECs. Indeed, as we shall show, different BEC-environment coupling leads to different final population imbalance of the BEC in a double-well potential. This may be used to characterize the decoherence in double-well systems.

In this Brief Report, we study the effect of decoherence on the dynamics of a BEC in a double-well potential by studying the evolution of the master equation for the BEC within a mean-field framework, where the number of atoms in the condensates is supposed to be infinity and the quantum fluctuation is negligible. To derive the master equation, we need to model the environment and BEC-environment coupling. However, this is not an easy task and we do not address it at present. Instead, we write the master equation by analyzing the effects of environmentally induced decoherence. When analyzing decoherence effects on the dynamics of a BEC in a double-well potential, we are interested in answering two basic questions: (1) What effects are caused by the decoherence on self-trapping in the BEC? And (2) how does the decoherence affect the quantum tunneling in a BEC in a double-well potential?

Consider a BEC in a double-well potential. The wave function of the BEC can be expressed as the superposition of the individual wave functions in each well,

$$|\phi\rangle = a_R|R\rangle + a_L|L\rangle, \tag{1}$$

where $|R\rangle$ and $|L\rangle$ denote the wave functions of the right and left wells, respectively. The coefficients a_R and a_L of the expansion satisfy the two-mode Gross-Pitaevskii equation [1,2] (setting $\hbar=1$),

$$i\frac{\partial}{\partial t}\begin{pmatrix} a_R \\ a_L \end{pmatrix} = H\begin{pmatrix} a_R \\ a_L \end{pmatrix}. \tag{2}$$

The Hamiltonian is given by

$$H = \begin{pmatrix} \frac{\gamma}{2} + \frac{c}{2}(|a_R|^2 - |a_L|^2) & \frac{V}{2} \\ \frac{V}{2} & -\frac{\gamma}{2} - \frac{c}{2}(|a_R|^2 - |a_L|^2) \end{pmatrix}, \tag{3}$$

where γ is the energy bias between the two wells, c stands for the nonlinear parameter describing the condensate self-

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interaction, and V, which depends on the height of the barrier, is the coupling constant between the two condensates. In this paper, we shall focus on $\gamma=0$, i.e., the case of a BEC in a symmetric double-well potential. This situation is interesting because the amplitude distributions of all eigenstates are symmetric, leading to Josephson oscillations in the absence of decoherence. With the Markov approximation, the master equation that results from the condensate-environment coupling takes the form

$$i\frac{\partial}{\partial t}\rho=[H_\rho,\rho]+\mathcal{L}(\rho),$$

$$\mathcal{L}(\rho)=i\frac{\Gamma}{2}(2A\rho A^\dagger-\rho A^\dagger A-A^\dagger A\rho), \qquad (4)$$

where Γ denotes the decoherence rate of the condensate, and A stands for an operator of the condensate. This master equation can be derived by assuming that the condensate-environment couplings take the form $H_I \sim \Sigma_j g_j(Ab_j^\dagger+\text{H.c.})$, where g_j denotes the constant of interaction between the condensate and the environmental mode b_j. The condensate operator A in general is expressed as a linear superposition of three Pauli operators, i.e., $A=\lambda_x\sigma_x+\lambda_y\sigma_y+\lambda_z\sigma_z$, with the notation $\sigma_z=|R\rangle\langle R|-|L\rangle\langle L|$, $\sigma_x=|R\rangle\langle L|+|L\rangle\langle R|$, and similarly for σ_y. The values of λ_α $(\alpha=x,y,z)$ depend on the source of decoherence and its couplings to the environment. For example, for $\lambda_y=\lambda_x=0$ the environment couples dephasingly to the condensate, while for $\lambda_z=0$ the environment causes the BEC to dissipate. H_ρ takes the same form as in Eq. (3), except for the change $|a_x|^2 \to \rho_{xx}=\langle x|\rho|x\rangle$, $x=R,L$.

To start with, we consider the case of $A=\sigma_+=\sigma_x+i\sigma_y$. This situation happens in the case where the double-well potential is formed by using a Raman scheme to couple two hyperfine states in a spinor BEC. The condensate in the upper hyperfine states decays into the lower one, reminiscent of atomic spontaneous emission. The dynamics of the master equation is studied by numerical simulations; the results are presented in Figs. 1 and 2. In Figs. 1(a) and 1(b), we have plotted the population of condensates in the left and right wells, respectively. In the initial state all the condensate atoms are in the right well, and the decoherence rate has been set to be $\Gamma=0.1V$. In contrast, the dynamics of the condensate in the left well without decoherence ($\Gamma=0$) is presented in Fig. 1(c). Clearly, the decoherence increases the damping of the oscillations. When the nonlinearity characterized by c is small compared to the tunneling V, the oscillations of the population are suppressed, and the condensate finally remains in the two wells with equal probability. If the nonlinearity is large with respect to the tunneling V and the population imbalance exceeds a critical value, the condensate will be locked in one of the wells, depending on the initial population. We note that the population changes drastically in the vicinity of the critical value $c=2V$; this is due to the suppression of population oscillations by decoherence. With increasing decoherence, the jumplike change near the critical value in the population becomes unclear, as Fig. 2 shows. That means the decoherence may determine the final population imbalance between the two wells. On the other hand, the

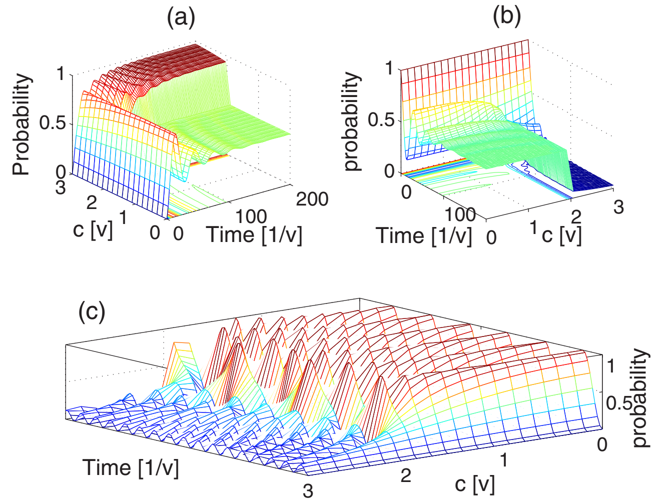

FIG. 1. (Color online) Populations of the condensates in the left well $|L\rangle$ [(a), (c)] and right well $|R\rangle$ [(b)]. The condensates were initially prepared in the right well; the decoherence rate was chosen $\Gamma=0.1V$ in (a) and (b), and $\Gamma=0$ in (c). A jumplike change at $c=2V$ in (a) and (b) clearly appears due to the decoherence effect. The nonlinear coupling constant is plotted in units of V, and the time in units of $1/V$ in all figures in this paper.

nonlinear interaction together with the initial population and the relative phase can affect the decoherence, which may be characterized by $|\rho_{12}|$, i.e., the norm of the off-diagonal element of the density matrix. This effect is shown in Fig. 3. We note that the jumplike change in Fig. 1 might appear at different c, depending on the fixed points around which the population imbalance and the relative phase oscillate. For

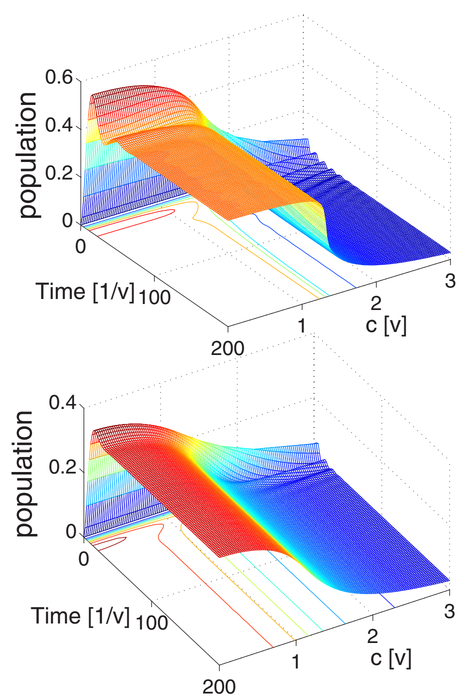

FIG. 2. (Color online) As Fig. 1(b) but with larger Γ, $\Gamma=0.3V$ for the upper panel, $0.5V$ for the lower panel. The jumplike change in the population disappears with increasing Γ.

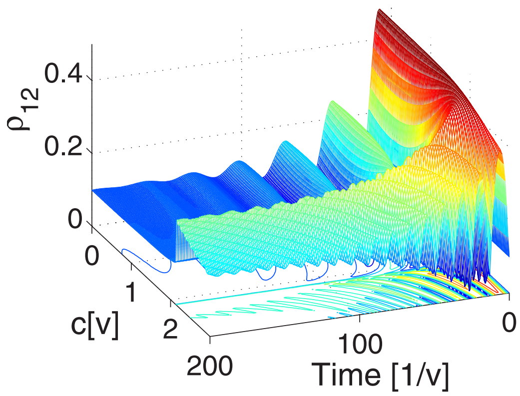

FIG. 3. (Color online) Norm of the off-diagonal element of the density matrix $|\rho_{12}|$ as a function of time and c, which is usually used to characterize the decoherence. The parameter chosen is $\Gamma=0.1V$, and the condensates were initially in the right well.

example, our simulations show that the jumplike change could appear at $c=V$ with initial relative phase $\theta=\pi$ and nonzero population imbalance [6].

Next, we take $A=\sigma_x$, corresponding to a BEC in a spatial double-well potential. The tunneling is driven by an environment (or by fluctuational fields), leading to decay in the quantum tunneling. An alternative BEC system can be formed by using a Raman scheme to couple two degenerate hyperfine states in a spinor BEC. The driving fields may fluctuate, resulting in decoherence in quantum tunneling. We have performed extensive numerical simulations for the master equation (4). Selected results, divided into three regimes by the nonlinearity, are presented in Figs. 4–6. Figure 4 shows the dynamics of the condensate in the self-trapping regime. The decoherence clearly increases the amplitude of oscillations in the population first, then increases the damping of the amplitude of oscillations; meanwhile on average it decreases the population imbalance, and finally spoils the self-trapping. In the self-trapping regime, the frequency of the oscillation depends on the nonlinear parameter c, the initial population imbalance and relative phase, as well as the coupling constant V between the BECs. This can be found by comparing Figs. 4–6. With c and V fixed, the decoherence changes the population imbalance; this results in a frequency change as shown in Fig. 4(a). In the quantum tunneling regime (Fig. 5), decoherence increases the damping of the oscillations, as expected. Finally, in Fig. 6, we have plotted the dynamics of the condensate in the regime between quantum tunneling and self-trapping. We see that decoherence increases the tunneling at the beginning of its evolution, and then destroys the quantum tunneling or self-trapping after a few cycles of evolution.

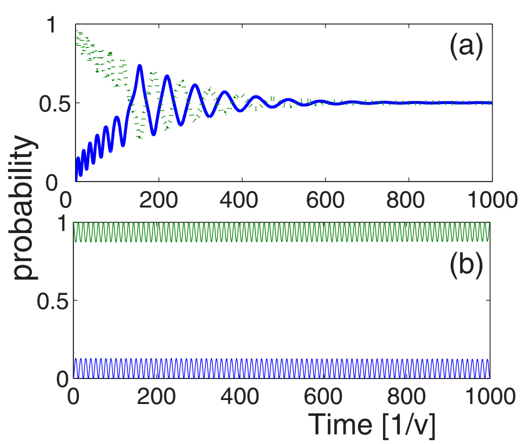

FIG. 4. (Color online) Change of population of the condensate with time. The parameters chosen are (a) $c=3V$, $\Gamma=0.01V$, and (b) $c=3V$, $\Gamma=0$. Self-trapping occurs in the absence of decoherence, as (b) shows. The decoherence first drives the BEC from the self-trapping regime to the quantum tunneling regime, and then it destroys the quantum tunneling.

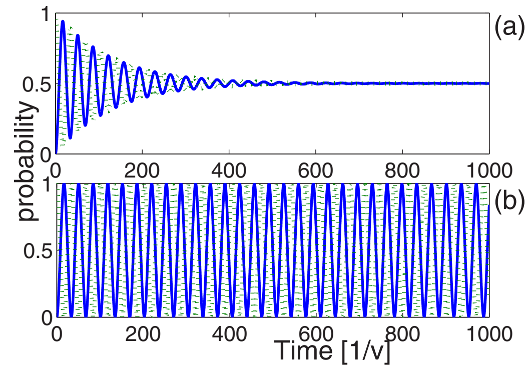

FIG. 5. (Color online) As Fig. 4, but with $c=V$. Solid line is plotted for the population of BEC in the right well, while dotted line is for the left.

In summary, we have studied the dynamics of a Bose-Einstein condensate in a double-well potential. The dynamics is governed by master equations with a condensate operator that comes from the condensate-environment coupling. Two kinds of decoherence characterized by σ_+ and σ_x are considered. By numerically solving the master equation, we show that there is a jumplike change in the BEC population due to the first kind of decoherence (σ_+). With increasing decoherence rate, the jumplike change in the population becomes unclear, resulting in decoherence-rate-dependent self-trapping. When the second kind of decoherence (σ_x) domi-

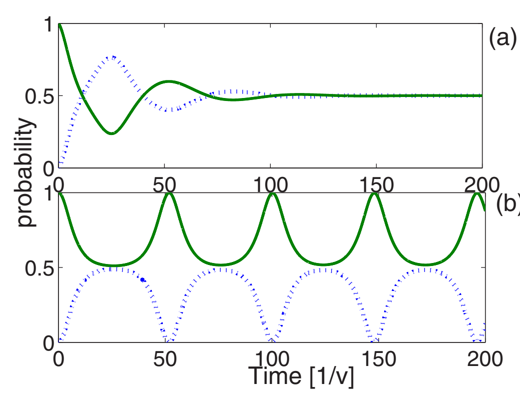

FIG. 6. (Color online) Population of the condensate at the critical value $c=2V$. (a) is plotted for $\Gamma=0.01V$, while (b) is for $\Gamma=0$. It confirms that the decoherence first leads the BEC from the self-trapping regime to the quantum tunneling regime, and then spoils the quantum tunneling.

nates, the decoherence first drives the BEC from the self-trapping regime to the quantum tunneling regime; then it destroys the quantum tunneling in the double-well system. The limitation of this paper is that we have treated the environment as Markovian and have ignored the quantum fluctuation in the condensate atoms. This may limit the application of the formalism to real double-well systems.

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