Quantum phase gate of photonic qubits in a cavity QED system

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A scheme is proposed to realize a two-qubit quantum phase gate for the intracavity modes. In the scheme, two qubits are encoded in zero- and one-photon Fock states of two intracavity modes, and the three-level Λ-type atom trapped in a high-Q cavity mediates the conditional phase gate within a given interaction time. The influence of cavity decay and atomic spontaneous emission on the gate fidelity is also discussed.

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The existence of quantum algorithms for specific problems shows that a quantum computer can in principle provide a tremendous speedup compared to classical computers [1]. This discovery motivated an intensive research into this mathematical concept which is based on quantum logic operations on multiqubit systems. It is well known that two-qubit controlled phase gate and one-qubit gate are universal for constructing quantum computer, i.e., any unitary transformation can be decomposed into these elementary gates [2]. In order to implement quantum computer into a real physical system, a quantum system is needed, which makes the storage and the readout of quantum information and the implementation of the required set of quantum gates possible. The isolation of the quantum system from the environment should be very well in order to suppress decoherence processes. Several physical systems were suggested to implement the concept of quantum computing: Trapped ions [3], cavity quantum electrodynamics (QED) [4,5], liquid state nuclear magnetic resonance [6], etc.

Among these systems being explored for hardware implementations of quantum computers, cavity QED is favored because of their demonstrated advantage when subjected to coherent manipulation [4,5]. By encoding quantum information into atomic states, a number of proposals for realizing quantum phase gate (QPG) have been proposed by using cavity mode as data bus [7]. Since cavity QED systems also hold great promise as basic tools for quantum networks since they provides an interface between computation and communication, i.e., between atoms and photons, it is necessary to look for techniques to manipulate and control quantum state of the intracavity fields [8–12]. In fact, two different schemes have been proposed to use cascade-, or V-type three-level atom to realize quantum phase gate of two intracavity modes [8,9]. References [10–12] showed that a single Λ-type three-level atom can be used to realize quantum logic gate and quantum state transfer between atom and cavity mode.

In this paper, we present a scheme to realize the quantum phase gate of two polarization modes by using a Λ-type three-level atom as data bus. Our scheme differs from Ref. [10] in two points. First, in their scheme, the internal states of an atom represent one qubit and the quantum states of the field inside the cavity represent the other. However, experimental realization of typical quantum algorithms may require the two qubits to be treated on equal footing, so in our scheme the two qubits are represented by the zero- and single-photon states of two different polarization modes of the radiation field inside the cavity. Second, the quantum phase gate operation can be performed successfully by requiring the two-photon resonance condition. A precise control of the interaction time between the three-level atom and the cavity modes will yield the conditional evolution needed to implement the quantum phase gate.

We now consider the system of a Λ-configuration three-level atom interacting with two orthogonal polarization modes of the same frequency. The energy level configuration of the atom is depicted in Fig. 1. For concreteness, we consider a possible implementation using ^{87}Rb in an optical Fabry-Perot (FP) cavity. As shown in Fig. 1, the ground states $|g_{-1}\rangle$ and $|g_1\rangle$ correspond to Zeeman levels $|F=1,m=-1\rangle$ and $|F=1,m=1\rangle$ of $5S_{1/2}$ electronic ground state, while the excited state $|e_0\rangle$ is denoted by the Zeeman level $|F'=1,m'=0\rangle$ of the $5P_{1/2}$ electronic excited state. The lifetime of the atomic levels $|g_{-1}\rangle$ and $|g_1\rangle$ is comparatively long so that spontaneous decay of these states can be neglected. The atomic transitions $|e_0\rangle\leftrightarrow|g_{-1}\rangle$ and $|e_0\rangle\leftrightarrow|g_1\rangle$ are coupled to two opposite circular polarization modes $\sigma_\pm$ with coupling strengths g_L and g_R, respectively. Similar model has been invoked for generating multiphoton entangled states [13,14].

In the rotating-wave approximation and the interaction picture, the Hamiltonian can be written as

$$H=\hbar[g_L|e_0\rangle\langle g_{-1}|a_Le^{-i\Delta_Lt}+g_R|e_0\rangle\langle g_1|a_Re^{-i\Delta_Rt}+\text{H.c.}], \tag{1}$$

where a_L and a_R are the annihilation operators of the two opposite circular polarizations $\sigma_\pm$ respectively; $\Delta_J=\omega_{e_0l}-\omega_c$

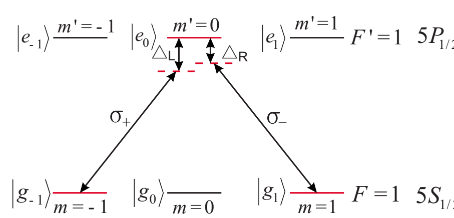

FIG. 1. (Color online) Λ-type three-level atom interacting with a bimodal cavity field, where Δ_L and Δ_R are the respective one-photon detunings.

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is the detuning between the atomic transition and the relevant cavity mode a_L or a_R ($J=L,R$); ω_c is the frequency of the cavity mode and $\omega_{e_0,l}$ is the atomic transition frequency between states $|e\rangle$ and $|l\rangle$ ($l=g_{-1},g_1$), which can be adjusted by external magnetic field. To realize the quantum phase gate, we assume $\Delta_L=\Delta_R=\Delta$. The effect of the difference between two detunings on the quantum phase gate will be studied at the end of the paper.

In this paper, we adopt the notation $|i,j,k\rangle\equiv|i\rangle|j\rangle_{a_L}|k\rangle_{a_R}$, where $|i\rangle$ ($i=g_{-1},g_1,e_0$) denotes atomic state while $|j\rangle_{a_L}$ and $|k\rangle_{a_R}$ denote that cavity fields have j photons in mode a_L and k photon in mode a_R. In the scheme, two qubits are encoded in zero- and one-photon Fock states of two different polarization intracavity modes. Initially, we assume that the atom is prepared in ground state $|g_{-1}\rangle$, and two cavity modes are in computational basis, i.e., system is initially prepared in the states $|g_{-1},j,k\rangle$ ($j,k\in 0,1$). The quantum phase gate we want to realize is given by

$$\hat{U}=\exp(i\pi\delta_{j1}\delta_{k1})|g_{-1},j,k\rangle\langle g_{-1},j,k|, \tag{2}$$

which means that the cavity states remain unaffected for the initial states $|0,\,0\rangle$, $|0,\,1\rangle$, and $|1,\,0\rangle$, and acquire a π phase shift only for the state $|1,\,1\rangle$.

It is apparent that the states $|g_{-1},0,0\rangle$ and $|g_{-1},0,1\rangle$ do not evolve with the time, since these states remain completely decoupled from interaction of Eq. (1)

$$H|g_{-1},0,0\rangle=0, \tag{3a}$$

$$H|g_{-1},0,1\rangle=0. \tag{3b}$$

In the following, we only give the detail analysis of the time evolution of states $|g_{-1},1,0\rangle$ and $|g_{-1},1,1\rangle$.

If the initial state of the system is $|g_{-1},1,0\rangle$, the Hamiltonian of Eq. (1) can be reduced to the form

$$H_1=g_L|e_0,0,0\rangle\langle g_{-1},1,0|e^{-i\Delta t}+g_R|e_0,0,0\rangle\langle g_1,0,1|e^{-i\Delta t}+\text{H.c.} \tag{4}$$

After interaction time t, the state of the system is given by

$$|\Psi(t)\rangle=b_{10}|g_{-1},1,0\rangle+c_{00}|e_0,0,0\rangle+d_{01}|g_1,0,1\rangle. \tag{5}$$

On the other hand, if the initial state of the system is $|g_{-1},1,1\rangle$, the Hamiltonian can take the form

$$H_2=g_L|e_0,0,1\rangle\langle g_{-1},1,1|e^{-i\Delta t}+\sqrt{2}g_R|e_0,0,1\rangle\langle g_1,0,2|e^{-i\Delta t}+\text{H.c.} \tag{6}$$

and time evolution of initial state is given by

$$|\Psi(t)\rangle=b_{11}|g_{-1},1,1\rangle+c_{01}|e_0,0,1\rangle+d_{02}|g_1,0,2\rangle. \tag{7}$$

For different initial states, we solve Schrödinger equation $i\partial_t|\Psi(t)\rangle=H|\Psi(t)\rangle$, and obtain expression of $b_{jk}(t)$ as follows

$$b_{jk}(t)\equiv\langle g,j,k|\Psi(t)\rangle=\frac{e^{i\Delta t/2}}{2}\left[\left(1-\frac{\Delta}{\Omega_{jk}}\right)e^{i\Omega_{jk}t/2}+\left(1+\frac{\Delta}{\Omega_{jk}}\right)e^{-i\Omega_{jk}t/2}\right], \tag{8}$$

with $\Omega_{10}=\sqrt{\Delta^2+4(g_L^2+g_R^2)}$ and $\Omega_{11}=\sqrt{\Delta^2+4(g_L^2+2g_R^2)}$.

TABLE I. Best numerical solutions to Eq. (10), sorted by the required interaction time gt.

q	n	m	p	Δ/g_1	g_1t
1	10	6	10.9891	2.12	35.58
1	17	5	20.0183	0.87	72.14
1	21	11	27.0153	1.74	79.45
1	20	0	23.9949	0	88.83

Therefore, to implement the quantum phase gate in Eq. (2), one needs to choose the experimental parameters to satisfy the following conditions

$$\frac{\Delta t}{2}=2m\pi,\quad \frac{\Omega_{10}t}{2}=2n\pi,\quad \frac{\Omega_{11}t}{2}=(2p+1)\pi, \tag{9}$$

where using the definition of Ω_{10} and Ω_{11}, the integers m, n, and p should satisfy the inequality $2p+1>2n>2m\geqslant 0$. If we set $g_L=g_R=g$, we have

$$(2p+1)^2=6n^2-2m^2. \tag{10}$$

From Eq. (9), for a fixed n, the value of m is determined through the detuning Δ and g according to the relation $\Delta^2/(2g^2)=4m^2/(n^2-m^2)$. The problem is then reduced to finding the p closest to an integer value that satisfies Eq. (10).

We report in Table I the best numerical solutions of Eq. (10). Note that the values for the detuning and the interaction time given in Table I are made dimensionless through the vacuum Rabi frequency g, which means that these results are general in the sense that they do not rely on any specific physical implementation.

In order to check the validity of our proposal, we define the following fidelity [9] to characterize the deviation of how much the output states $|\Psi(t)\rangle$ deviate in amplitude and phase from the ideal phase gate transformation for the four different input states $|g_{-1},0,0\rangle$, $|g_{-1},0,1\rangle$, $|g_{-1},1,0\rangle$, and $|g_{-1},1,1\rangle$:

$$F=\left\langle\left|\sum_{j,k=0,1}|b_{j,k}(t)|^2e^{i\delta\phi_{jk}}\right|^2\right\rangle, \tag{11}$$

where $\delta\phi_{j,k}$ is the difference between the phase acquired during the gate operation and the ideal phase of the gate defined in Eq. (2). Figure 2 plots the numerical calculation of Eq. (11), in which the time evolution of the fidelities for two different Δ are shown. Fidelities oscillate with peak values close to $F=1$ for the particular interaction time values predicted in Table I.

It is necessary to give a brief discussion on the dissipative processes [15] to examine with how much efficiency the desired outcome can be produced because the interaction of the atom and the cavity with the environment causes them to decay and results in decoherence. The decoherence mechanisms arise through two dominant channels: (i) Cavity decay rates κ_a and κ_b and (ii) atomic spontaneous emission rate γ. A single trajectory in the quantum jump mode [16] is well suitable for evaluating the effects on the gate fidelity. Subject

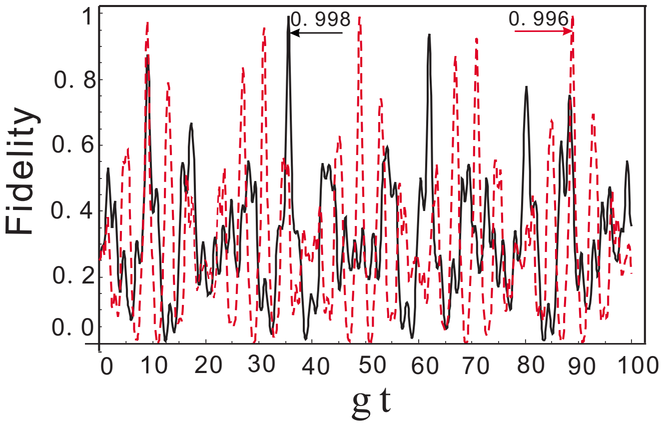

FIG. 2. (Color online) Time evolution of the fidelity F for the quantum phase gate, where $\Delta=2.12$ (solid curve) and $\Delta=0$ (dashed curve).

to no decay being recorded in the detectors, under the rotating wave approximation, the system conditionally evolves in the interaction picture according to a non-Hermitian Hamiltonian which is given by

$$
\begin{aligned}
H_{\mathrm{con}} = {} & g_L|e_0,0,0\rangle\langle g_{-1},1,0| + g_R|e_0,0,0\rangle\langle g_1,0,1| + g_L|e_0,0,1\rangle \\
& \times\langle g_{-1},1,1| + \sqrt{2}g_R|e_0,0,1\rangle\langle g_1,0,2| + \mathrm{H.c.} \\
& - \left(\Delta + \frac{i\kappa_{a_L}}{2}\right)a_L^\dagger a_L - \left(\Delta + \frac{i\kappa_{a_R}}{2}\right)a_R^\dagger a_R - \frac{i\gamma}{2}(|e_0,0,0\rangle \\
& \times\langle e_0,0,0| + |e_0,0,1\rangle\langle e_0,0,1|),
\end{aligned} \tag{12}
$$

where κ_{a_L} and κ_{a_R} denote the rate of decay of the cavity mode fields a_L and a_R; γ is spontaneous emission rate of the excited states. Figure 3 plots the numerical calculation of the fidelity vs κ/g. For $\Delta/g=2.12$, Table I shows that the maximum fidelity is obtained at $gt=35.58$. We see the gate fidelity remains more than 0.90 for $\kappa=0.001g$ and $\gamma=0.001g$. It monotonically decreases when κ/g and γ/g grows.

In reality the detunings Δ_L and Δ_R are likely to be different for both transitions due to the little difference between the two cavity polarization modes or the nondegenerate atomic ground states, where the magnetic field plays in general a negative role as it breaks the degeneracy between the atomic ground states. Figure 4 shows the fidelity for implementation for the QPG as a function of δ_Δ/Δ_L, where $\delta_\Delta=\Delta_L-\Delta_R$.

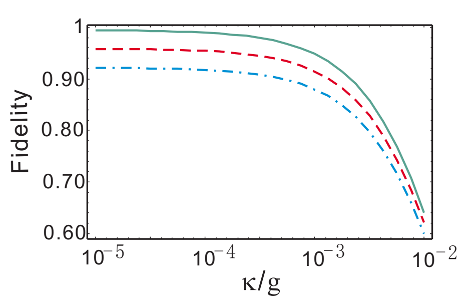

FIG. 3. (Color online) Variation of the fidelity F vs κ/g when $\kappa_{a_L}=\kappa_{a_R}=\kappa$, $\Delta/g=2.12$, and $gt=35.58$. Other parameters: For solid curve, $\gamma/g=0.0001$, for dashed curve, $\gamma/g=0.001$, and for dot-dashed curve, $\gamma/g=0.002$.

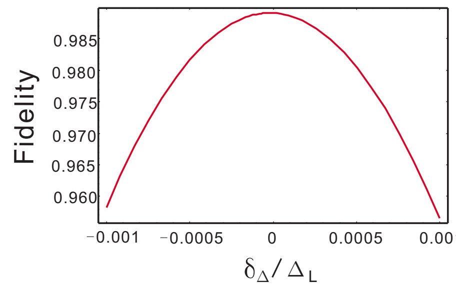

FIG. 4. (Color online) Variation of the fidelity F vs δ_Δ/Δ_L when $\kappa_{a_L}/g=\kappa_{a_R}/g=\kappa/g=0.0001$, $\Delta/g=10$, $gt=62.8$, and $\gamma/g=0$, where $\delta_\Delta=\Delta_L-\Delta_R$.

Finally, We give a brief discussion of the experimental feasibility of the proposed scheme within optical cavity QED. As with any proposal for quantum computing implementation, its success ultimately depends on being able to complete many coherent dynamics during the decoherence time, so the atomic and cavity lifetimes should be larger than the interaction time of the atoms with the cavity fields. The three important cavity QED parameters describing the cavity-atom interaction system have been obtained as $(g,\kappa,\gamma)/2\pi=(16,1.4,3)$ MHz [17], which would imply cavity lifetime τ_{cav} is of the order of 1 μs. For the QPG discussed in this paper the gate times t_{gate} is also of the order of 1 μs. Thus to match the condition $t_{\mathrm{gate}}<\tau_{\mathrm{cav}}$ and guarantee higher fidelity, κ should at least be smaller than g by approximately two orders of magnitude. This can be realized through adjusting the length L and finesse F of the cavity: $g\sim L^{-3/4}$ and $\kappa\sim FL^{-1}$. If we can increase the cavity finesse F of Ref. [17] by one order of magnitude, $(g,\kappa)/2\pi=(16,0.14)$ MHz can be obtained, which satisfies the requirement of the present scheme.

Furthermore as the vacuum Rabi frequency g is not constant throughout the cavity mode volume in current optical cavity QED systems, the FP cavity has a general mode function described by $\chi(\vec{r})=\sin(kz)\exp[-(x^2+y^2)/\omega_0^2]$ [18], and $g(\vec{r})=g_0\chi(\vec{r})$, where ω_0 and $k=2\pi/\lambda$ are, respectively, the waist and the wave vector of the Gaussian cavity mode, and $\vec{r}(x,y,z)$ describes the atomic locations; z is assumed to be along the axis of the cavity. In the above description we need the coupling rate $g_L(\vec{r})=g_L$ and $g_R(\vec{r})=g_R$. Optimal results will be obtained for an atom trapped at the antinode of the

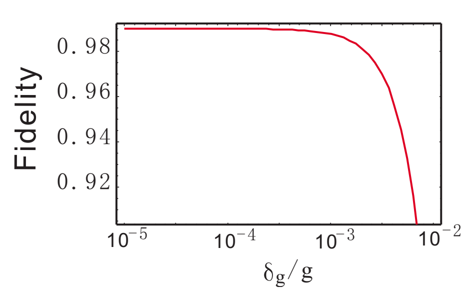

FIG. 5. (Color online) Variation of the fidelity F vs δ_g/g when $\kappa_{a_L}=\kappa_{a_R}=\kappa$, $\Delta/g=2.12$, $gt=35.58$, and $\kappa/g=\gamma/g=0.0001$.

cavity field. This is experimentally viable, as shown in Refs. [19], where trapping times up to 1 s have been reported in the strong-coupling regime. If we take the deviation of g into account by the value δ_g, that is $g' = g + \delta_g$, we can see in Fig. 5 the fidelity exceeds 0.98 when δ_g / g is smaller than 10^{-3}. We also note that our scheme requires an efficient source of single photons [20] and their injection into an optical cavity. Recently, Fattal, Beausoleil, and Yamamoto [21] proposed a significant scheme to achieve photon injection from the outside to the inside of an optical cavity and then store it in the atomic internal state even with highly imperfect hardware. However, the storage of single photon information and feeding of single photons into or out of cavities are still experimental challenges for large-scale quantum computation.

In conclusion, we have presented a scheme to carry out a two-qubit quantum phase gate in which a three-level Λ-type atom interacts with a high-Q bimodal optical cavity and the two qubits are represented by the vacuum and single-photon states of two polarization modes of the cavity. We also discuss the influence of the atomic spontaneous emission, cavity decay, and deviation of g on the gate fidelity.

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