Dynamical evolution and analytical solutions for multiple degenerate dark states in the tripod-type atomic system

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We study both dynamical and steady responses of a coherently driven four-level tripod-type atomic system, which may be either open or closed. With all three driving fields being on Raman resonance, this tripod system can quickly evolve into a complicated steady dark state decoupled from both coherent and dissipative interactions. In the open model, this tripod system is sensitive to all parameters in its dynamical evolution while the steady dark state only depends on the initial population distribution and the relative Rabi frequency values. In the closed model, however, relative values of spontaneous decay rates also play an important role in generating the steady dark state. Analytical results based on the quantum jump theory show that the steady dark state is always multiple (from twofold to sixfold) degenerate and usually comes into existence through both coherent and incoherent superpositions.

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I. INTRODUCTION

In the past few decades, quantum coherence was shown to be essential for many applications of multilevel quantum systems driven by laser fields and have resulted in a lot of interesting phenomena. Electromagnetically induced transparency (EIT) [1,2], stimulated Raman adiabatic passage (STIRAP) [3,4], subluminal and superluminal light propagation [5,6], resonant enhancement of optical nonlinearity [7–9], spontaneous emission control [10–12], switching between normal and anomalous dispersion [13–15], and light induced chiralty in a nonchiral medium [16] are a few examples demonstrating the significance of quantum coherence. Besides the common atomic media, quantum coherence based phenomena can also be realized in certain solid media such as N-*V* color center materials [17,18], rare-earth-ion-doped crystals [19,20], and low-dimension semiconductor nanostructures [21,22]. These solid materials are certainly preferred for real applications due to the obvious advantages of compactness and stability, absence of atomic diffusion, simplicity and scalability during assembling, etc.

EIT is usually regarded as the basis of other quantum phenomena mentioned above and thus of great importance for modifying light-matter interactions. In a Λ-type EIT medium, when all particles evolve into a dark state [23,24] decoupled from both coherent and dissipative interactions, the cw driving fields (one *probe* and one *coupling*) can propagate through them without loss. In particular, a weak *probe* pulse can have an ultraslow group velocity due to the extremely steep spectral dispersion, or become at rest by mapping its quantum state onto collective atomic excitations as a result of adiabatica modulation of the *coupling* field and vice versa [25,26]. Disturbing weakly the EIT system to deviate a little from the dark state by a third field, one can obtain the so-called interacting double dark states, an ultranarrow spectral line, and greatly enhanced nonlinear optical processes [27–29]. But these interacting double dark states cannot be simultaneously established because they correspond to different frequencies of the driving fields, and each of them has the same structure or composition as in the typical Λ system. So far the dark state in a Λ system is clear for us as a coherent superposition of two ground levels determined just by field Rabi frequencies, while those composed of three or more ground levels in relatively complicated multilevel quantum systems are not well examined or identified yet.

In this paper, we will try to derive the general analytical expression for the dark states in a coherently driven tripod-type atomic system, a good candidate for exploring effective approaches to problems on dark states involving more than two ground levels. It has been shown that there may exist two degenerate dark states in the tripod system and adiabatic transitions between them may give rise to rich and complicated dynamics [30–32]. Aiming at the tripod system, Unanyan *et al.* described an efficient and robust technique for creating and probing an arbitrary coherent superposition of two atomic states [30], which was experimentally demonstrated by Vewinger *et al.* later on [31]. Paspalakis *et al.* investigated instead the pulse propagation dynamics in a coherently prepared tripod system and showed that a single incident pulse could parametrically generate two additional pulses [33]. Mazets further extended their work to the adiabatic regime and found two different modes (one *slow* and one *fast*) of shape-preserving pulse propagation [32]. The tripod system may also be used, as suggested by Rebic *et al.* [34], to realize a polarization quantum phase gate via coherently enhanced cross-Kerr nonlinearity. Last but not least, Petrosyan and Malakyan showed that optically dense vapors of tripod atoms could support ultrasensitive magneto-optical polarization rotation and entanglement of orthogonally polarized quantum fields [35]. To the best of our knowledge, however, no one has investigated the dynamical evolution of a tripod system from different initial states toward diverse steady dark states and how dark states of a tripod system

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depend on various parameters besides Rabi frequencies.

We first consider an open model where excited atoms only spontaneously decay to an external level of the tripod system. By numerical calculations based on the density matrix formalism, we find that the open system cannot definitely evolve into a simple dark state with two ground levels when two of the driving fields are on Raman resonance. If all driving fields are on two-photon resonance, however, a complicated dark state consisting of three ground levels will be established leading to a part of the population reserved in the open system. This complicated dark state depends critically on Rabi frequencies of the driving fields as well as the initial population distribution, but is irrelevant to field detunings and the outward spontaneous decay rate. Via the quantum jump method [36,37], we have derived the exact analytical expression for this complicated dark state and also determined the population loss as a result of spontaneous decay, which are consistent with the numerical calculations. Our analytical results reveal that this complicated dark state is a coherent superposition of two simple dark state when all atoms are initially at a single ground level, but in general it is contributed by all three simple dark states both coherently and incoherently. Using the same numerical and analytical methods, we then alternatively investigate the closed model where spontaneous emission occurs only toward internal levels of the tripod system. Numerical results show that the internal spontaneous decay rates can also alter the complicated dark state corresponding to three resonant fields having the same detunings, and the closed system will surely develop into a simple dark state in the case of two driving fields being on Raman resonance. These distinct characteristics relative to the open model can be attributed to the random population redistribution resulted from the internal spontaneous emission. Analytical results indicate that the complicated dark state is always superposed by all three simple dark states in both coherent and incoherent ways, and its structure is determined by the initial population distribution, relative values of Rabi frequencies, and relative values of spontaneous decay rates.

This paper is organized as follows. In Sec. II we describe our considered atomic system interacting with coherent fields and derive density matrix equations governing its dynamical evolution. In Sec. III, we discuss the generation of multiple degenerate dark states in an open model via both numerical simulations and quantum jump analysis. Section IV is devoted to the opposite closed tripod model where the same topic is examined again with quite different results yielded. Our conclusions are finally summarized in Sec. V.

II. THE TRIPOD SYSTEM AND DENSITY MATRIX EQUATIONS

We consider a coherently driven four-level atomic system in the tripod configuration as shown by Fig. 1, which has one excited level and three ground levels. The excited level $|3\rangle$ is coupled to the ground levels $|0\rangle$, $|1\rangle$, and $|2\rangle$ by three laser fields with carrier frequencies (amplitudes) ω_0 ($\vec{E}_0$), ω_1 ($\vec{E}_1$), and ω_2 ($\vec{E}_2$). $\Delta_0=\omega_0-\omega_{30}$, $\Delta_1=\omega_1-\omega_{31}$, and $\Delta_2=\omega_2-\omega_{32}$ are

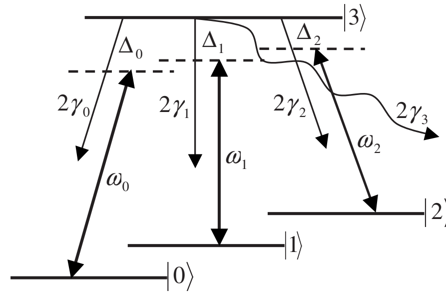

FIG. 1. Schematic diagram of a four-level tripod-type atomic system driven by three coherent laser fields.

one-photon detunings of the three driving fields from respective dipole-allowed optical transitions. Atoms at level $|3\rangle$ decay via spontaneous emission to levels $|0\rangle$, $|1\rangle$, $|2\rangle$, and the outside levels (not shown here) at rates $2\gamma_0$, $2\gamma_1$, $2\gamma_2$, and $2\gamma_3$, respectively. Simply setting $\gamma_3=0$ or $\gamma_0=\gamma_1=\gamma_2=0$, we can make the tripod system either completely closed or purely open.

Under the rotating-wave and electric-dipole approximations, with the assumption of $\hbar=1$, we can write the interaction Hamiltonian in the interaction picture as

$$H_I=\begin{bmatrix} 0 & 0 & 0 & -\Omega_0^* \\ 0 & \Delta_1-\Delta_0 & 0 & -\Omega_1^* \\ 0 & 0 & \Delta_2-\Delta_0 & -\Omega_2^* \\ -\Omega_0 & -\Omega_1 & -\Omega_2 & -\Delta_0 \end{bmatrix} \qquad (1)$$

where $\Omega_0=\vec{E}_0\cdot\vec{d}_{03}/2$, $\Omega_1=\vec{E}_1\cdot\vec{d}_{13}/2$, and $\Omega_2=\vec{E}_2\cdot\vec{d}_{23}/2$ denote Rabi frequencies of the respective laser fields. $\vec{d}_{03}$, $\vec{d}_{13}$, and $\vec{d}_{23}$ are dipole matrix elements on the optical transitions labeled $|0\rangle\leftrightarrow|3\rangle$, $|1\rangle\leftrightarrow|3\rangle$, and $|2\rangle\leftrightarrow|3\rangle$, respectively.

To examine both transient and steady responses of the tripod-type atomic system to the three driving fields, we should resort to the well-known density matrix formalism. By the standard procedures [38], we can derive from Eq. (1) and $\frac{\partial\rho}{\partial t}=-i[H_I,\rho]+\Lambda\rho$ the following density matrix equations

$$\frac{\partial\rho_{00}}{\partial t}=2\gamma_0\rho_{33}+i\Omega_0^*\rho_{30}-i\Omega_0\rho_{03},$$

$$\frac{\partial\rho_{11}}{\partial t}=2\gamma_1\rho_{33}+i\Omega_1^*\rho_{31}-i\Omega_1\rho_{13},$$

$$\frac{\partial\rho_{22}}{\partial t}=2\gamma_2\rho_{33}+i\Omega_2^*\rho_{32}-i\Omega_2\rho_{23},$$

$$\frac{\partial\rho_{33}}{\partial t}=-2\gamma_3\rho_{33}-\frac{\partial\rho_{00}}{\partial t}-\frac{\partial\rho_{11}}{\partial t}-\frac{\partial\rho_{22}}{\partial t},$$

$$\frac{\partial\rho_{01}}{\partial t}=-i(\Delta_0-\Delta_1)\rho_{01}+i\Omega_0^*\rho_{31}-i\Omega_1\rho_{03},$$

$$\frac{\partial\rho_{02}}{\partial t}=-i(\Delta_0-\Delta_2)\rho_{02}+i\Omega_0^*\rho_{32}-i\Omega_2\rho_{03},$$

$$\frac{\partial\rho_{12}}{\partial t} = -i(\Delta_1-\Delta_2)\rho_{12} + i\Omega_1^*\rho_{32} - i\Omega_2\rho_{13},$$

$$\frac{\partial\rho_{03}}{\partial t} = -\tilde{\gamma}_{03}\rho_{03} - i\Omega_1^*\rho_{01} - i\Omega_2^*\rho_{02} + i\Omega_0^*(\rho_{33}-\rho_{00}),$$

$$\frac{\partial\rho_{13}}{\partial t} = -\tilde{\gamma}_{13}\rho_{13} - i\Omega_0^*\rho_{10} - i\Omega_2^*\rho_{12} + i\Omega_1^*(\rho_{33}-\rho_{11}),$$

$$\frac{\partial\rho_{23}}{\partial t} = -\tilde{\gamma}_{23}\rho_{23} - i\Omega_0^*\rho_{20} - i\Omega_1^*\rho_{21} + i\Omega_2^*(\rho_{33}-\rho_{22}), \quad (2)$$

where $\tilde{\gamma}_{i3}=i\Delta_i+\gamma_{i3}$ with $\gamma_{i3}=\gamma_0+\gamma_1+\gamma_2+\gamma_3$ denoting the dephasing rate on transition $|i\rangle\leftrightarrow|3\rangle$. We note that the population conservation formula $\rho_{00}+\rho_{11}+\rho_{22}+\rho_{33}=1$ is not valid again if $\gamma_3\neq 0$, although above density matrix equations are still constrained by the Hermitian relation $\rho_{ij}=\rho_{ji}^*$.

From Eqs. (2) we find that, in the case of $\Delta_0=\Delta_1=\Delta_2=\Delta$ (i.e., all three driving fields have the same one-photon detunings), the tripod system will finally evolve into a steady state $|\Psi(\infty)\rangle$ where all density matrix elements related to level $|3\rangle$ become zero while others should satisfy

$$\Omega_0\rho_{00}(\infty)+\Omega_1\rho_{01}(\infty)+\Omega_2\rho_{02}(\infty)=0,$$

$$\Omega_0\rho_{10}(\infty)+\Omega_1\rho_{11}(\infty)+\Omega_2\rho_{12}(\infty)=0,$$

$$\Omega_0\rho_{20}(\infty)+\Omega_1\rho_{21}(\infty)+\Omega_2\rho_{22}(\infty)=0. \quad (3)$$

It is clear that the steady state $|\Psi(\infty)\rangle$ is irrelevant to level $|3\rangle$, i.e., decoupled from the spontaneous emission processes, so we can view it as a dark state (a nonabsorbing state). We cannot obtain analytical solutions for $\rho_{ij}(\infty)$ and thus exactly define this dark state just from Eqs. (3), which implies that the dark state may depend on other parameters, such as the initial population distribution and spontaneous decay rates, besides Rabi frequencies of the driving fields. In the next two sections, assuming the Rabi frequencies are real valued without loss of generality, we will first check how the tripod system evolves from a certain initial state into the final steady dark state by numerical simulations based on Eqs. (2), and then try to obtain the exact expression of the steady dark state for different initial atomic conditions and spontaneous decay rates via appropriate analytical methods.

III. DARK STATES IN THE OPEN TRIPOD MODEL

Here we focus on the relatively simple case of $\gamma_0=\gamma_1=\gamma_2=0$, i.e., excited atoms only spontaneously decay to levels outside of the tripod system. Just for simplicity, we assume in the following that atoms at level $|3\rangle$ decay to a single outside level $|f\rangle$.

Under the specific initial population condition $\rho_{00}(0)=1$, we plot the dynamical evolution of atomic population and coherence for some parameters in Fig. 2 and Fig. 3, respectively. As can be seen from Figs. 2(a) and 3(a), all atoms will decay to the external level $|f\rangle$ after a short damped oscillation and no population can be trapped in the tripod system if

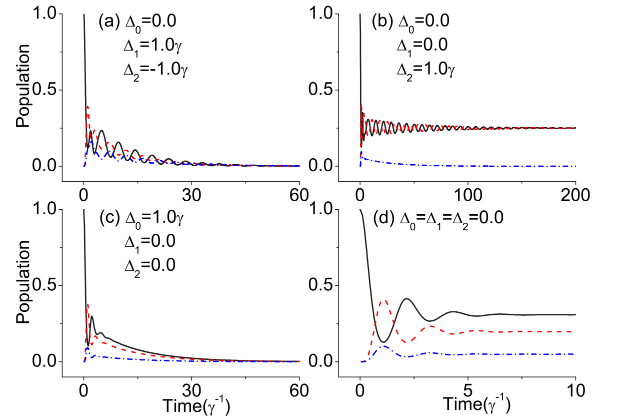

FIG. 2. (Color online) Dynamical evolution of atomic population at the ground levels with $\gamma_0=\gamma_1=\gamma_2=0$, $\gamma_3=1.5\gamma$, $\Omega_0=\Omega_1=2.0\gamma$, $\Omega_2=1.0\gamma$, $\rho_{00}(0)=1.0$, and $\rho_{11}(0)=\rho_{22}(0)=\rho_{33}(0)=0.0$. Solid curves—$\rho_{00}$; dashed curves—$\rho_{11}$; dash-dotted curves—$\rho_{22}$.

$\Delta_0\neq\Delta_1\neq\Delta_2$. But when $\Delta_0=\Delta_1\neq\Delta_2$, partial population trapping is realized, leading to some atoms coherently reserved at levels $|0\rangle$ and $|1\rangle$ [see Figs. 2(b) and 3(b)]. The dark state for trapped atoms is depicted by $|\Psi(\infty)\rangle=[\Omega_1|0\rangle-\Omega_0|1\rangle]/\sqrt{\Omega_0^2+\Omega_1^2}$ as in a Λ model, which can be confirmed by $\rho_{00}(\infty):\rho_{11}(\infty)=\Omega_1^2:\Omega_0^2$ and $\rho_{01}(\infty)=-\sqrt{\rho_{00}(\infty)\rho_{11}(\infty)}$. Contrary to our intuition, coherent population trapping cannot be achieved for $\Delta_0\neq\Delta_1=\Delta_2$ [see Figs. 2(c) and 3(c)]. This can be understood as follows: atoms at level $|0\rangle$ cannot evolve into $|\Psi(\infty)\rangle=[\Omega_2|1\rangle-\Omega_1|2\rangle]/\sqrt{\Omega_1^2+\Omega_2^2}$, a dark state composed of levels $|1\rangle$ and $|2\rangle$, before they spontaneously decay to $|1\rangle$ or $|2\rangle$ and simultaneously become dephased with $|0\rangle$, which is unfortunately forbidden here. If all driving fields are on two-photon resonance, we find from Figs. 2(d) and 3(d) that much more population can be trapped in the tripod system and the dark state becomes $|\Psi(\infty)\rangle=a_0|0\rangle+a_1|1\rangle+a_2|2\rangle$ with a_i undetermined yet. Although $\rho_{ii}(\infty):\rho_{jj}(\infty)=\Omega_j^2:\Omega_i^2$ is not true again, we note that the formula $\rho_{0i}(\infty)=-\sqrt{\rho_{00}(\infty)\rho_{ii}(\infty)}$ still holds true for levels $|1\rangle$ and $|2\rangle$, which is a signal for $|\Psi(\infty)\rangle$ being purely coherent.

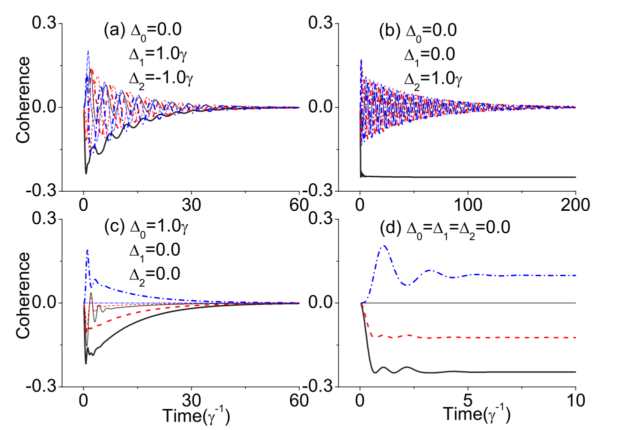

FIG. 3. (Color online) Dynamical evolution of atomic coherence at the ground levels with the same parameters as in Fig. 2. Thick solid, dashed, dash-dotted curves—Re(ρ_{01}), Re(ρ_{02}), Re(ρ_{12}); thin solid, dashed, dash-dotted curves—Im(ρ_{01}), Im(ρ_{02}), Im(ρ_{12}).

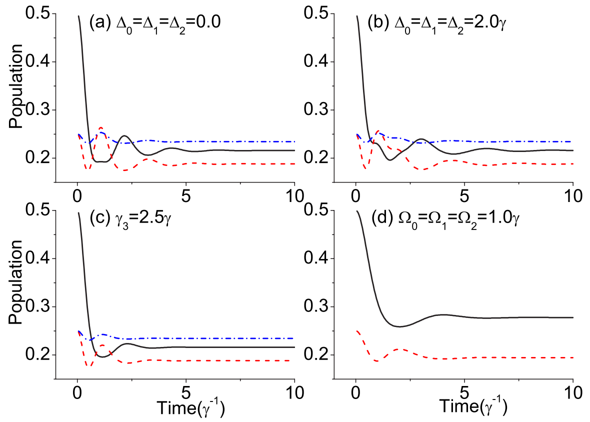

FIG. 4. (Color online) Dynamical evolution of atomic population at the ground levels with $\rho_{00}(0)=0.5$, $\rho_{11}(0)=\rho_{22}(0)=0.25$, and $\rho_{33}(0)=0.0$. Other parameters, if not shown, are the same as in Fig. 2(d). Solid curves—ρ_{00}; dashed curves—ρ_{11}; dash-dotted curves—ρ_{22}. In Fig. 4(d), curves for ρ_{11} and ρ_{22} fully overlap due to the symmetrical parameters.

In Fig. 4 and Fig. 5, we further investigate the dark state with $\Delta_0=\Delta_1=\Delta_2=\Delta$ under another initial population condition $\rho_{00}(0)=0.5$ and $\rho_{11}(0)=\rho_{22}(0)=0.25$. From Figs. 4(a) and 5(a), we can see that different initial population distributions will surely lead to distinct steady dark states. Just modulating the one-photon detuning Δ, we find from Figs. 4(b) and 5(b) that the tripod system finally evolves into the same dark state as in Figs. 4(a) and 5(a), though the dynamical evolution process becomes a little different. Figures 4(c) and 5(c) further show that the steady dark state and the population loss rate, defined as $[1-\Sigma_0^3\rho_{ii}(\infty)]$, do not depend on the unique spontaneous decay rate $2\gamma_3$. Figures 4(d) and 5(d) clearly demonstrates that, however, one can greatly modify the steady dark state by changing Rabi frequencies of the driving fields. Moreover, we remark that the formula $\rho_{ij}(\infty)=-\sqrt{\rho_{ii}(\infty)\rho_{jj}(\infty)}$ does not work again, which indicates that

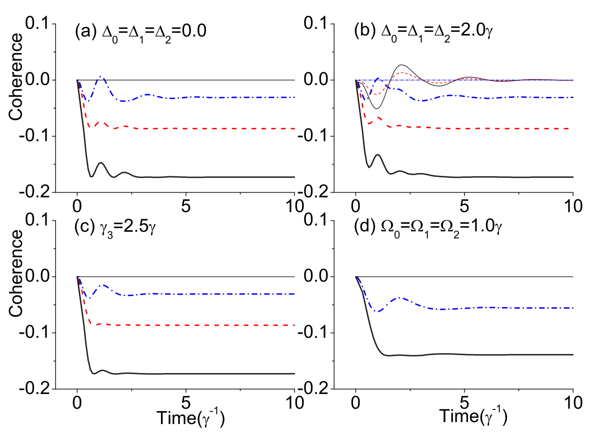

FIG. 5. (Color online) Dynamical evolution of atomic coherence at the ground levels with the same parameters as in Fig. 4. Thick solid, dashed, dash-dotted curves—Re(ρ_{01}), Re(ρ_{02}), Re(ρ_{12}); thin solid, dashed, dash-dotted curves—Im(ρ_{01}), Im(ρ_{02}), Im(ρ_{12}). In Fig. 5(d), curves for Re(ρ_{01}) and Re(ρ_{02}) fully overlap due to the symmetrical parameters. We also note Im(ρ_{01})=Im(ρ_{02})=Im(ρ_{12}) $\equiv 0$ when $\Delta=0$.

the steady dark state $|\Psi(\infty)\rangle$ be a partial coherent one involving incoherent interaction.

So far we have obtained some qualitative information about dark states in the open tripod system. To achieve a much deeper physical insight, we now try to obtain their exact solutions via analytical methods. One straightforward method is to diagonalize the interaction Hamiltonian and find its eigenstates having zero eigenvalues. The Hamiltonian shown by Eq. (1) has two null eigenvalues when $\Delta_0=\Delta_1=\Delta_2$, which implies that the corresponding dark state can be twofold degenerate. Meanwhile, probability amplitudes $a_i(\infty)$ should satisfy

$$\Omega_0 a_0(\infty)+\Omega_1 a_1(\infty)+\Omega_2 a_2(\infty)=0 \tag{4}$$

for the two degenerate dark states.

The tripod system can be regarded as three partially overlapped subsystems in the Λ configuration, whose dark states are respectively given by

$$|\Theta_0\rangle=[\Omega_2|1\rangle-\Omega_1|2\rangle]/\sqrt{\Omega_1^2+\Omega_2^2},$$

$$|\Theta_1\rangle=[\Omega_0|2\rangle-\Omega_2|0\rangle]/\sqrt{\Omega_0^2+\Omega_2^2},$$

$$|\Theta_2\rangle=[\Omega_1|0\rangle-\Omega_0|1\rangle]/\sqrt{\Omega_0^2+\Omega_1^2}. \tag{5}$$

Choosing $|\Theta_i\rangle$ as one of the two possible dark states because they fulfill Eq. (4), we can obtain the other one limited by the orthogonality requirement $\langle\Theta_i|\Phi_i\rangle=0$ as

$$|\Phi_0\rangle=[(\Omega_1^2+\Omega_2^2)|0\rangle-\Omega_0\Omega_1|1\rangle-\Omega_0\Omega_2|2\rangle]/\Omega_A^2,$$

$$|\Phi_1\rangle=[(\Omega_0^2+\Omega_2^2)|1\rangle-\Omega_1\Omega_0|0\rangle-\Omega_1\Omega_2|2\rangle]/\Omega_B^2,$$

$$|\Phi_2\rangle=[(\Omega_0^2+\Omega_1^2)|2\rangle-\Omega_2\Omega_0|0\rangle-\Omega_2\Omega_1|1\rangle]/\Omega_C^2, \tag{6}$$

with $\Omega_{A,B,C}^2=\sqrt{\Omega^2(\Omega_{1,0,0}^2+\Omega_{2,2,1}^2)}$ and the effective Rabi frequency $\Omega=\sqrt{\Omega_0^2+\Omega_1^2+\Omega_2^2}$. It is straightforward to find that $|\Phi_0\rangle$ is a coherent superposition of $|\Theta_1\rangle$ and $|\Theta_2\rangle$, i.e.,

$$|\Phi_0\rangle=[\Omega_1\sqrt{\Omega_0^2+\Omega_1^2}|\Theta_2\rangle-\Omega_2\sqrt{\Omega_0^2+\Omega_2^2}|\Theta_1\rangle]/\Omega_A^2, \tag{7}$$

and similar results can be obtained for $|\Phi_1\rangle$ and $|\Phi_2\rangle$. These results are clearly shown by the three-dimensional vectorgraphs in Fig. 6, where two of the bare state vectors generate a nondegenerate dark state vector, e.g., $|1\rangle$ and $|2\rangle$ generate $|\Theta_0\rangle$, and two of the nondegenerate dark state vectors further produce a twofold degenerate dark state vector, e.g., $|\Theta_1\rangle$ and $|\Theta_2\rangle$ produce $|\Phi_0\rangle$.

Currently we have found three pairs of orthogonal dark states $|\Theta_i\rangle$ and $|\Phi_i\rangle$, but their superposition way to generate the final dark state $|\Psi(\infty)\rangle$ is not clear for us yet. To solve this problem and determine how much population is lost through spontaneous emission, we can resort to the quantum jump approach [36,37], a poweful tool for distinguishing between coherent oscillation and dissipative processes.

Using a full quantum description, we can group the "atom+laser" system into manifolds $\xi(N_0,N_1,N_2)$ composed of four quasidegenerate states [see Fig. 7(a)]:

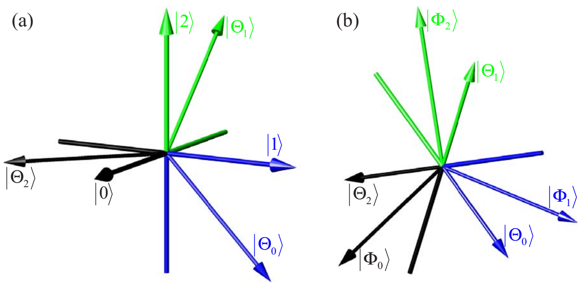

FIG. 6. (Color online) Three-dimensional schematic diagram of (a) the nondegenerate dark states $|\Theta_i\rangle$ in the basis of the orthogonal bare states $|0\rangle$, $|1\rangle$, and $|2\rangle$; (b) the twofold degenerate dark states $|\Phi_i\rangle$ as a coherent superposition of the nonorthogonal dressed states $|\Theta_0\rangle$, $|\Theta_1\rangle$, and $|\Theta_2\rangle$.

$$\xi(N_0,N_1,N_2) = \{|3,N_0,N_1,N_2\rangle,|0,N_0+1,N_1,N_2\rangle,|1,N_0,N_1+1,N_2\rangle,|2,N_0,N_1,N_2+1\rangle\}, \tag{8}$$

with N_i denoting photon numbers of laser field ω_i. Within a certain manifold, one atom can be transferred from a ground level $|i\rangle$ to the excited level $|3\rangle$ by absorbing a ω_i photon or directly to another ground level $|j\rangle$ by simultaneously emitting a second ω_j photon. In the absence of spontaneous emission, this reversible coherent oscillation can never stop and the "atom+laser" system will remain forever in the initial manifold. Only when the atom randomly decays to the external level $|f\rangle$, can the "atom+laser" system stops its coherent evolution by jumping into the decoupled state $|f,0,0,0\rangle$,

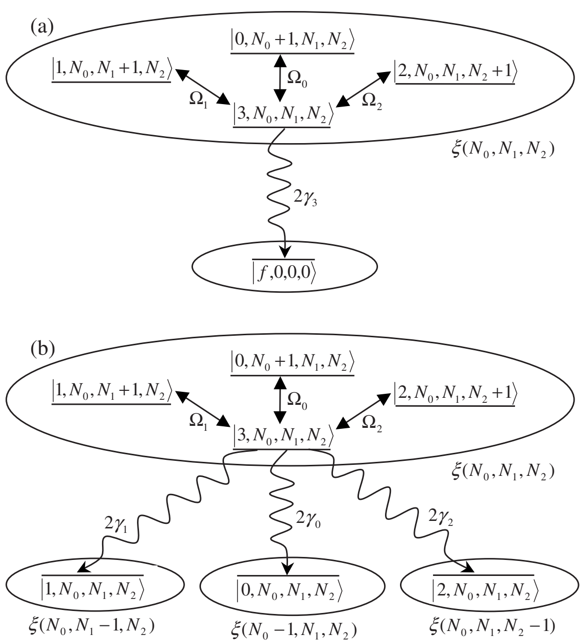

FIG. 7. Different manifolds of the "atom+laser" system and the allowed quantum jumps. Coherent evolutions in a manifold are characterized by Rabi frequencies while dissipative quantum jumps are denoted by spontaneous decay rates. Here (a) refers to an open tripod system while (b) is for a closed tripod system. State $|f,0,0,0\rangle$ denotes that atoms at the external level $|f\rangle$ are completely decoupled from the three driving fields.

so the initial state of a quantum jump must be level $|3\rangle$. Consequently, there are three possible coherent periods characterized by the entry state $|i\rangle$ and the exit state $|j\rangle$: period (0,3), period (1,3), and period (2,3). The dynamical evolution of atomic states within a manifold is governed by an effective non-Hermitian Hamiltonian H_{eff}, which is of the same form as Eq. (1) except the last element $-\Delta_0$ should be replaced by $-\Delta_0 - i\gamma_3$. From H_{eff}, one can calculate the probability amplitude $c_{ij}(\tau)=\langle j|\exp(-iH_{eff}\tau)|i\rangle$ for finding an atom in state $|j\rangle$ of $\xi(N_0,N_1,N_2)$ at time $t+\tau$ given that it is in state $|i\rangle$ of the same manifold at time t. Multiplying $|c_{ij}(\tau)|^2$ by $2\gamma_3 d\tau$ then gives the conditional probability

$$W_{ij}(\tau)d\tau = 2\gamma_3|c_{ij}(\tau)|^2 d\tau \tag{9}$$

that the system leaves $\xi(N_0,N_1,N_2)$ by a quantum jump from $|j\rangle$ at the time interval $d\tau$. The Schrödinger equation associated with H_{eff} leads to the following differential equations:

$$\frac{\partial c_{i0}(\tau)}{\partial \tau} = -i\Omega_0 c_{i3}(\tau),$$

$$\frac{\partial c_{i1}(\tau)}{\partial \tau} = -i(\Delta_0-\Delta_1)c_{i1}(\tau) - i\Omega_1 c_{i3}(\tau),$$

$$\frac{\partial c_{i2}(\tau)}{\partial \tau} = -i(\Delta_0-\Delta_2)c_{i3}(\tau) - i\Omega_2 c_{i3}(\tau),$$

$$\frac{\partial c_{i3}(\tau)}{\partial \tau} = -\tilde{\gamma}_3 c_{i3}(\tau) - i\Omega_0 c_{i0}(\tau) - i\Omega_1 c_{i1}(\tau) - i\Omega_2 c_{i2}(\tau), \tag{10}$$

with $\tilde{\gamma}_3 = \gamma_3 + i\Delta_0$. Under the two-photon resonance condition of $\Delta_0=\Delta_1=\Delta_2=\Delta$, one can find from Eq. (10) by integration that

$$\int_0^\infty |c_{i3}(\tau)|^2 d\tau = \frac{1}{2\gamma_3}\frac{\Omega_i^2}{\Omega^2}, \tag{11}$$

which clearly does not depend on the common one-photon detuning Δ.

For our purpose, two important quantities are $P(i)$ and $P(j|i)$, the probability of a coherent period starting at level $|i\rangle$ and the conditional probability of such a period ending at $|j\rangle$. In the open model, atoms can only spontaneously decay to the external level $|f\rangle$, the atoms decayed to $|f\rangle$ cannot go back to the tripod system again, and there is no population exchange among the internal levels via spontaneous emission, so the probability $P(i)$ is just determined by the initial population at level $|i\rangle$, i.e., we have $P(i)\equiv\rho_{ii}(0)$. Also due to the outward spontaneous emission, atoms cannot evolve into state $|\Theta_i\rangle$ if they are initially at level $|i\rangle$. As an example, we consider level $|0\rangle$ and state $|\Theta_0\rangle$. Since $|\Theta_0\rangle$ is a coherent superposition of $|1\rangle$ and $|2\rangle$, atoms initially at $|0\rangle$ have to first become decoupled with $|0\rangle$ by spontaneously decaying to $|1\rangle$ or $|2\rangle$ before they can coherently go into $|\Theta_0\rangle$. Unfortunately, the inward spontaneous decay is forbidden here so that atoms initially at $|0\rangle$ can only evolve into $|\Theta_1\rangle$ and $|\Theta_2\rangle$ with a coherent component of $|0\rangle$. Because both $|\Phi_1\rangle$ and $|\Phi_2\rangle$ have a coherent component of $|\Theta_0\rangle$, it is straightforward to say that atoms initially at $|0\rangle$ also cannot evolve into $|\Phi_1\rangle$ and $|\Phi_2\rangle$ since the atomic evolution toward $|\Theta_0\rangle$ is forbidden. Thus the steady dark state must be $|\Psi(\infty)\rangle = x_0|\Phi_0\rangle$ with $|x_0|^2$ $(1-|x_0|^2)$ being the totally trapped (lost) population if all atoms are initially at $|0\rangle$. In more general cases where $\rho_{ii}(0) \neq 0$ for $i \in [0,2]$, we have instead

$$|\Psi(\infty)\rangle = x_0|\Phi_0\rangle + x_1|\Phi_1\rangle + x_2|\Phi_2\rangle, \tag{12}$$

with $|x_i|^2$ being the population coherently trapped in $|\Phi_i\rangle$. Noting that all atoms trapped in $|\Phi_i\rangle$ comes from those initially at $|i\rangle$ and atoms initially at different ground levels are completely uncorrelated in phases of their probability amplitudes, we can judge that phases of the complex parameters x_i change with time independently and randomly, which means that Eq. (12) should be constrained by

$$\langle x_i x_j^* \rangle = |x_i|^2 \delta_{ij} \tag{13}$$

with $\langle x_i x_j^* \rangle$ being the measurable time-average result of $x_i x_j^*$. Equation (13) implies that $|\Psi(\infty)\rangle$ be the incoherent superposition of $|\Phi_0\rangle$, $|\Phi_1\rangle$, and $|\Phi_2\rangle$ and the coherece between any two of these dark states be exactly zero even if all x_i are nonzero. This fact allows us to treat x_i as real-valued parameters in the following discussion because phases of x_i are unimportant and meaningless. According to the definitions of $P(i)$ and $P(j|i)$, one can easily find that

$$x_i = \sqrt{P(i)[1 - P(3|i)]}. \tag{14}$$

The conditional probability $P(3|i)$, which is just equal to the population loss rate for atoms initially at level $|i\rangle$, is given from Eqs. (9) and (11) by

$$P(3|i) = \int_0^\infty W_{i3}(\tau) d\tau = \frac{\Omega_i^2}{\Omega^2}, \tag{15}$$

which together with Eq. (14) is consistent with numerical calculations in Fig. 2 and Fig. 4 as far as the steady population loss is concerned. Based on the above analysis, we finally derive the steady dark state,

$$|\Psi(\infty)\rangle = \sum_{i=0}^{2} \sqrt{\left[\frac{\Omega^2 - \Omega_i^2}{\Omega^2}\right] \rho_{ii}(0)} |\Phi_i\rangle, \tag{16}$$

as an incoherent superposition of the three twofold degenerate dark states $|\Phi_i\rangle$. That is, $|\Psi(\infty)\rangle$ is a sixfold degenerate dark state in general because it has six degenerate components $|\Theta_i\rangle$ [note that each $|\Theta_i\rangle$ appears twice in $|\Psi(\infty)\rangle$] connected via coherent or incoherent interactions. Clearly, $|\Psi(\infty)\rangle$ only depends on relative values (not absolute values) of Rabi frequencies Ω_i and the initial population distribution $\rho_{ii}(0)$, but is irrelevant to the spontaneous decay rate $2\gamma_3$ and the common one-photon detuning Δ. From Eqs. (6), (13), and (16), we can futher obtain

$$\rho_{00}(\infty) = \frac{\rho_{00}(0)[\Omega_1^2 + \Omega_2^2]^2 + \Omega_0^2[\rho_{11}(0)\Omega_1^2 + \rho_{22}(0)\Omega_2^2]}{\Omega^4},$$

$$\rho_{11}(\infty) = \frac{\rho_{11}(0)[\Omega_0^2 + \Omega_2^2]^2 + \Omega_1^2[\rho_{00}(0)\Omega_0^2 + \rho_{22}(0)\Omega_2^2]}{\Omega^4},$$

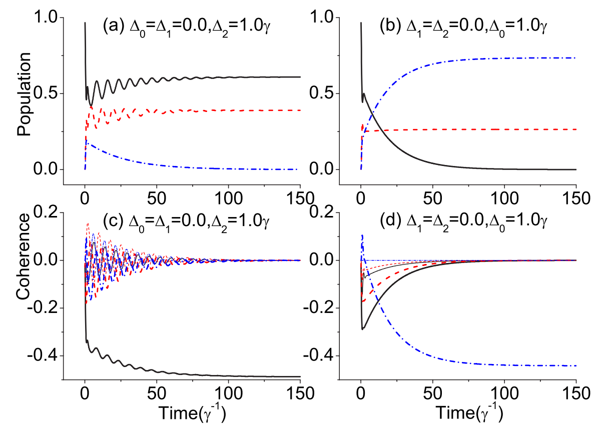

FIG. 8. (Color online) Dynamical evolution of atomic population and coherence at the ground levels with $\gamma_0=\gamma_1=\gamma_2=1.0\gamma$, $\gamma_3=0.0$, $\Omega_0=2.0\gamma$, $\Omega_1=2.5\gamma$, $\Omega_2=1.5\gamma$ $\rho_{00}(0)=1.0$, and $\rho_{11}(0)=\rho_{22}(0)=\rho_{33}(0)=0.0$. Thick solid, dashed, dotted curves—ρ_{00} and Re(ρ_{01}), ρ_{11} and Re(ρ_{02}), ρ_{22} and Re(ρ_{12}); thin solid, dashed, dash-dotted curves—Im(ρ_{01}), Im(ρ_{02}), Im(ρ_{12}).

$$\rho_{22}(\infty) = \frac{\rho_{22}(0)[\Omega_0^2 + \Omega_1^2]^2 + \Omega_2^2[\rho_{00}(0)\Omega_0^2 + \rho_{11}(0)\Omega_1^2]}{\Omega^4},$$

$$\rho_{01}(\infty) = \frac{-\Omega_0\Omega_1\{\Sigma_{i=0,1}[\rho_{ii}(0)(\Omega^2 - \Omega_i^2)] - \rho_{22}(0)\Omega_2^2\}}{\Omega^4},$$

$$\rho_{02}(\infty) = \frac{-\Omega_0\Omega_2\{\Sigma_{i=0,2}[\rho_{ii}(0)(\Omega^2 - \Omega_i^2)] - \rho_{11}(0)\Omega_1^2\}}{\Omega^4},$$

$$\rho_{12}(\infty) = \frac{-\Omega_1\Omega_2\{\Sigma_{i=1,2}[\rho_{ii}(0)(\Omega^2 - \Omega_i^2)] - \rho_{00}(0)\Omega_0^2\}}{\Omega^4}, \tag{17}$$

for atomic population and coherence in the steady state. These analytical results are fully consistent with the numerical calculations shown in Fig. 4 and Fig. 5. The clear demonstration of Eqs. (16) and (17) provides us an opportunity to well design and manipulate the final dark state of an open tripod-type atomic system.

IV. DARK STATES IN THE CLOSED TRIPOD MODEL

Now we consider an alternative tripod system where excited atoms only decay to the internal ground levels via spontaneous emission. As in the last section, we first try to obtain some qualitative informations about dark states in the closed model by numerical calculations.

From Fig. 8 we can see that, quite different from the open model, coherent population trapping always can be realized when two driving fields are on Raman resonance. The relevent simple dark states are just given by $|\Theta_2\rangle$ and $|\Theta_0\rangle$ as in a Λ model. Since no incoherent superposition is involved, it goes without saying that the formulas $\rho_{01}(\infty) = -\sqrt{\rho_{00}(\infty)\rho_{11}(\infty)}$ and $\rho_{12}(\infty) = -\sqrt{\rho_{11}(\infty)\rho_{22}(\infty)}$ are also right here. The fact that atoms can go into the dark state $|\Theta_0\rangle$ even if they are initially at level $|0\rangle$ is due to the population redis-

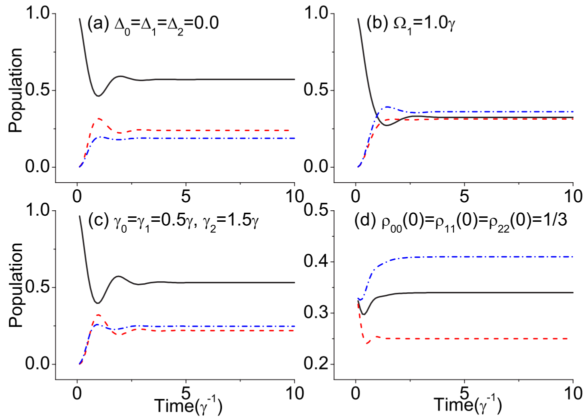

FIG. 9. (Color online) Dynamical evolution of atomic population at the ground levels. Other parameters, if not shown, are the same as in Fig. 8, except $\Delta_0=\Delta_1=\Delta_2=0.0$. Solid curves—$\rho_{00}$; dashed curves—$\rho_{11}$; dash-dotted curves—$\rho_{22}$.

tribution among the ground levels resulted from the inward spontaneous emission. That is, atoms pumped to level $|3\rangle$ from level $|0\rangle$ by field ω_0 can spontaneously decay to the other two ground levels and simultaneously become decoupled with level $|0\rangle$.

Figures 9 and 10 further show that, when all three driving fields are on Raman resonance, Rabi frequencies, spontaneous decay rates, and the initial population distribution all have important influence on the steady dark state $|\Psi(\infty)\rangle$. Also due to the population redistribution resulted from spontaneous emission, the formula $\rho_{ij}(\infty)=-\sqrt{\rho_{ii}(\infty)\rho_{jj}(\infty)}$ is always false even if all atoms are initially at a single ground level $|i\rangle$. These facts imply that the final dark state be contributed by the three ground leves in a more complicated way involving both coherent and incoherent superpositions.

To obtain the exact solution for this complicated dark state, we should use the same analytical methods as shown in Sec. III, by which we know that the steady dark state is still contributed by the three dark states $|\Phi_i\rangle$ [see Eqs. (6)] via the way shown in Eq. (12). Here a reasonable assumption has been made that atoms intially at level $|0\rangle$ can only directly evolve into $|\Phi_0\rangle$ but never directly into $|\Theta_0\rangle$. To go in $|\Theta_0\rangle$, they have to first spontaneously decay to another ground level, e.g., $|1\rangle$ (or $|2\rangle$), and then coherently evolve into $|\Phi_1\rangle$ (or $|\Phi_2\rangle$) as a coherent superposition of $|\Theta_0\rangle$ and $|\Theta_2\rangle$ (or $|\Theta_1\rangle$). Similar conclusions hold true for the simple dark states $|\Theta_1\rangle$ and $|\Theta_2\rangle$. Thus there should be completely random and independent phase relations among the complex parameters x_i so that we still have $\langle x_i x_j^*\rangle=|x_i|^2\delta_{ij}$ as shown by Eq. (13) and can assume x_i real valued. Applying the quantum jump approach to the closed tripod model, we can further make sure how x_i depends on the various parameters such as Rabi frequencies and spontaneous decay rates.

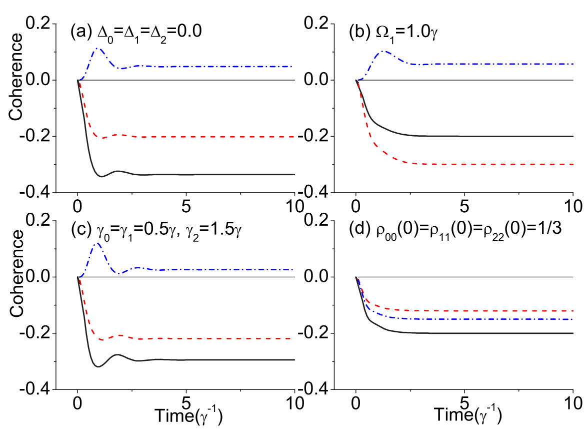

FIG. 10. (Color online) Dynamical evolution of atomic coherence at the ground levels with the same parameters as in Fig. 9. Thick solid, dashed, dash-dotted curves—$\mathrm{Re}(\rho_{01})$, $\mathrm{Re}(\rho_{02})$, $\mathrm{Re}(\rho_{12})$; thin solid curves—$\mathrm{Im}(\rho_{01})$, $\mathrm{Im}(\rho_{02})$, and $\mathrm{Im}(\rho_{12})$.

As in the open tripod model, atoms coherently oscillate among the four quasidegenerate states of a manifold $\xi(N_0,N_1,N_2)$ by absorbing or emitting coupled photons of the driving fields. When spontaneous emission occurs from level $|3\rangle$, the "atom+laser" system then stops its coherent evolution by jumping into a neighboring manifold, e.g., $\xi(N_0-1,N_1,N_2)$ and $\xi(N_0,N_1-1,N_2)$ [see Fig. 7(b)]. In this way, the final state of a coherent evolution, i.e., the initial state of a quantum jump, must be level $|3\rangle$. Therefore there are three different coherent periods in each manifold answering for a change in the coupled photon numbers: period (0, 3) with $\Delta N_0=-1$, period (1, 3) with $\Delta N_1=-1$, and period (2, 3) with $\Delta N_2=-1$. In the case of $\Delta_0=\Delta_1=\Delta_2=\Delta$, the conditional probability $P(3|i)$, which is just equal to the population redistribution rate for atoms initially at $|i\rangle$, is still governed by Eq. (15). The probability $P(i)$, however, becomes related to the spontaneous decay rates γ_i because atoms can go from one ground level to another one via quantum jump and thus begin a new coherent period there. Defining $Q(\text{in}:j/\text{in}:i)$ as the conditional probability of a coherent period starting from $|j\rangle$ given that the previous one has started from $|i\rangle$, we have

$$P(j)=\Sigma_{i=0}^{2}[P(i)Q(\text{in}:j/\text{in}:i)],$$

$$Q(\text{in}:j/\text{in}:i)=\gamma_j/\gamma_3, \tag{18}$$

with γ_3 being redefined as $\gamma_3=\gamma_0+\gamma_1+\gamma_2$ for the closed tripod model. Using the normalization condition $\Sigma_{i=0}^{2}P(i)=1$, we finally find from Eq. (18) that

$$P(j)=\gamma_j/\gamma_3, \tag{19}$$

which shows that $P(j)$ and $Q(\text{in}:j/\text{in}:i)$ have the same values for the closed tripod system. We note that it is not the case for a closed atomic system having two or more excited levels [39,40].

For atoms initially at level $|i\rangle$, after the first round of quantum jump, the population coherently trapped in the dark state $|\Phi_j\rangle$ (that experiences no spontaneous decay) becomes $\delta_{ij}[1-P(3|i)]\rho_{ii}(0)$, while the population randomly distributed at level $|j\rangle$ in the opposite bright state (that suffers spontaneous decay once) is $P(j)P(3|i)\rho_{ii}(0)$. Extended to the second round of quantum jump, the trapped population in $|\Phi_j\rangle$ becomes

$$x_j^2=[1-P(3|j)]\rho_{ii}(0)[\delta_{ij}+P(j)P(3|i)] \tag{20}$$

with that still reserved in the bright state being

$$y_j^2 = \Sigma_{i=0}^{2}[P(j)P(3|k)P(k)P(3|i)\rho_{ii}(0)] \tag{21}$$

for the respective ground levels $|j\rangle$. In fact, after each round of quantum jump, $[1-P(3|i)]$ of the total population remaining at level $|i\rangle$ in the bright state evolves into the opposite dark state $|\Phi_i\rangle$ with $P(j)$ of the residual population being distributed at level $|j\rangle$ in the bright state. Under this consideration, we finally obtain our concerned real parameters x_j as

$$x_j = \sqrt{[1-P(3|j)][\delta_{j0} + P(j)P(3|0)/\Pi]} \tag{22}$$

after infinite rounds of quantum jumps when $\rho_{ii}(0)=\delta_{i0}$. Here the normalization factor $\Pi=\Sigma_{j=0}^{2}P(j)[1-P(3|j)]$ can be easily derived from the unitary requirement $\Sigma_j x_j^2=1$.

For more general cases where all ground levels have nonzero population [$\rho_{ii}(0)\neq 0$ if $i\neq 3$], we obtain instead

$$x_j = \sqrt{[1-P(3|j)]\{\rho_{jj}(0) + P(j)\Sigma_{i=0}^{2}[\rho_{ii}(0)P(3|i)]/\Pi\}}. \tag{23}$$

From Eqs. (6), (12), and (23), we can exactly define the sixfold degenerate dark state $|\Psi(\infty)\rangle$ as a mixture of both coherent and incoherent superpositions of the simplest dark states $|\Theta_i\rangle$ for the closed tripod model. Equation (23) can be further written in a much clearer form as

$$x_j = \sqrt{\frac{\Omega^2-\Omega_j^2}{\Omega^2}\left\{\rho_{jj}(0) + \frac{\gamma_j\Sigma_{i=0}^{2}[\rho_{ii}(0)\Omega_i^2]}{\Sigma_{i=0}^{2}[\gamma_i(\Omega^2-\Omega_i^2)]}\right\}}. \tag{24}$$

It is clear that this sixfold degenerate dark state strongly depends on the initial population distribution, and is also sensitive to the relative values of field Rabi fequencies and spontaneous decay rates. Considering the restriction depicted by Eq. (13), we can easily obtain analytical expressions for population distribution $\rho_{ii}(\infty)$ and atomic coherernce $\rho_{ij}(\infty)$ in the steady state. These analytical expressions (not shown here due to their lengthy and complicated appearance) completely accord with numerical calcualtions demonstrated in Fig. 9 and Fig. 10. Thus according to above analytical results, we can easily achieve a preferred multiple degenerate dark state by modulating strengths of the driving fields when spontaneous decay rates and the initial population distribution are known.

A real tripod system can be either ultracold atoms in a magneto-optical trap (MOT) or rare-earth-ion-doped crystals in a cryogenerator such as Er^{3+}: yttrium aluminum garnet (YAG) and Pr^{3+}:YSO (Y_2SiO_5 crystal) [19]. For instance, we can utilize the magnetic sublevels of states $|5S_{1/2},F=1\rangle$ and $|5P_{3/2},F=0\rangle$ of cold ^{87}Rb atoms to construct the tripod scheme shown in Fig. 1. In this case, we have $\gamma_0=\gamma_1=\gamma_2=1.0$ MHz as a result of the equal branching ratio of the spontaneous decay rate from $|5S_{1/2},F=1\rangle$ to $|5P_{3/2},F=0\rangle$. To ensure that each field only acts on one transition, here we should use three laser beams with left circular (σ^-), linear (π), and right circular (σ^+) polarizations, respectively. The degeneracy of state $|5S_{1/2},F=1\rangle$ can be lifted by a magnetic field with moderate strength, and a magnetic field of $B=10$ G will cause a Zeeman splitting of $\Delta\omega=7.0$ MHz between adjacent magnetic sublevels of $|5S_{1/2},F=1\rangle$. Detailed informations of a sixfold degenerate dark state $|\Psi(\infty)\rangle$ can-

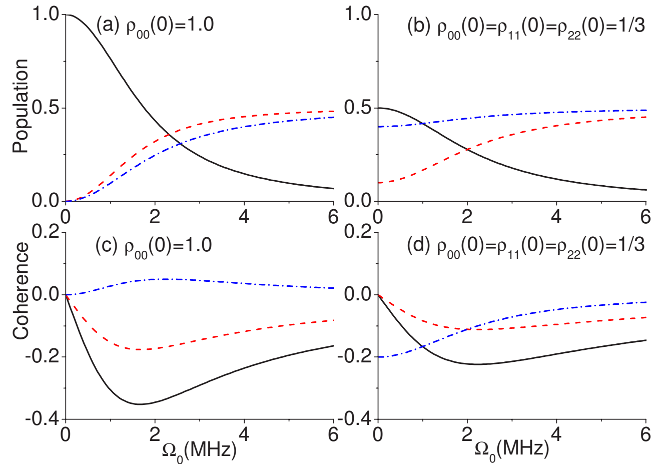

FIG. 11. (Color online) Steady population and coherence of cold ^{87}Rb atoms vs Ω_0 with $\gamma_0=\gamma_1=\gamma_2=1.0$ MHz, $\Omega_1=2.0$ MHz, $\Omega_2=1.0$ MHz, and $\Delta_0=\Delta_1=\Delta_2$. Solid curves—$\rho_{00}$ and ρ_{01}; dashed curves—ρ_{11} and ρ_{01}; dash-dotted curves—ρ_{22} and ρ_{12}.

not be examined directly in experiment, but can be inferred from the accessible ground state population and coherence (see Fig. 11) based on our analytical solutions. Fixing the ratio of field strengths and then decreasing them adiabatically to zero, one can obtain a specific partial coherent dark state of ^{87}Rb atoms with its structure determined by Eqs. (6), (12), and (24). Such a medium, when interacting with an incident laser pulse, can parametrically generate two additional light pulses [33], which are strongly dependent on the ground state population and coherence. Consequently, the parametric generation process may allow us to derive useful informations about the partial coherent and sixfold degenerate dark state.

V. CONCLUSION

In summary, we have investigated both dynamical and steady responses of a tripod-type atomic system to the applied laser fields, and analyzed how simple dark states having two ground levels contribute to generate the multiple degenerate steady dark state in a complicated way. The tripod system can be either open or closed depending on where atoms at the only excited level spontaneously decay. Numerical calculations based on density matrix equations show that the dynamical evolution of the tripod system depends critically on all involved parameters. Analytical results via the quantum jump method bring out the exact solutions of the steady dark state when all driving fields are on two-photon resonance, which is a function of the initial population distribution, field Rabi frequencies, and inward spontaneous decay rates. In particular, the dark state in an open tripod model is twofold degenerate and completely coherent if all atoms are initially at a single ground level, but becomes sixfold degenerate and partially coherent provided that all ground levels have nonzero population at the initial time. For the alternative closed model, the steady dark state is always sixfold degenerate no matter what the initial population distribution is. This distinct phenomenon can be attributed to the random population redistribution among the ground levels via the inward spontaneous emission. The exact identification of the steady dark state may allow us to manipulate and control it in a flexible and reliable way.

Our analytical methods can also be easily extended to explore the general expressions of dark states in other complicated atomic systems, e.g., a five-level M-type system [41,42]. We expect that the explicit analytical results about multiple degenerate dark states be instructive for the preparation of an arbitrary superposition of bare atomic states, and have potential applications in quantum nonlinear optics and multichannel quantum information processing based on dark-state polaritons [43,44]. Finally, we note that some analysis of the dark states with many ground levels in EIT-like systems has been done in the context of decoherence-free subspaces [45].

ACKNOWLEDGMENTS

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