# Field-emitter bound states in structured thermal reservoirs

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We derive a master equation for a two-level emitter interacting with a band-gap reservoir at finite temperatures. This equation is able to capture effects of emitter-reservoir entanglement. We show that the entangled field-emitter bound state, which arises in the process of interaction, does not survive indefinitely at finite temperatures. However, such an entangled state may be effectively excited through an intensive incoherent driving.

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## I. INTRODUCTION

One of the most interesting and promising consequences of the emitter's interaction with band-gap reservoirs is the field-emitter bound state (FEBS). If the photonic density of states exhibits a complete gap, and the reservoir is in the vacuum state, then after some initial period of decay the population of the emitter's excited level reaches a stationary value. Such behavior was referred to as the "freezing" of spontaneous emission, or fractional decay [1–5]. In this process the state of the atom entangles with the reservoir and, ideally, survives indefinitely. The FEBS might be accompanied by the so-called "giant" Lamb shift of the emitter's excited level, which is much larger than the usual Lamb shift in unstructured reservoirs [2,6,7]. The field is localized in a space of a few unit cells of the photonic crystal (PC) around the emitter; thus, an emitter whose transition frequency falls in the vicinity of a band edge functions as a cavity without any defects in the structure of the PC to localize the field in. Such a cavity exists due to strong coupling between the emitter and the field and has a very high $Q$ factor (limited, in fact, only by nonideality of a realistic PC). This natural cavity effect has been already suggested as a basis for a single-emitter reversible quantum-optical memory cell and for all-optical switches for quantum information purposes [8–10]. One might readily imagine it as a building block of PC lasers, amplifiers, and quantum logical gates.

However, to implement the FEBS as a quantum cavity, one must check the FEBS existence in real PCs, i.e., those that are finite size, with incomplete gaps and losses, such as dephasing and absorption of the radiation by the PC material. Second, one needs to devise ways to access the FEBS, to excite it, and to control it. This is rather nontrivial, since the FEBS actually exists due to the impossibility of radiation's propagation inside the gap.

The existence of the FEBS was predicted first by simple theoretical models of densities of states [2]. So, lately even the very possibility of the FEBS for realistic structures were debated, until the calculations of densities of states with realistic parameters were correctly performed [7]. It was established that in lossless perfectly periodic PC structures the FEBS may exist. Very recently it has been shown that the FEBS is quite robust with respect to losses caused by the emitter's coupling with additional Markovian reservoirs or by nonzero density of states within the gap [11–13]. In particular, even the long-time rate of decay of the emitter's upper-state population in a pseudogap can be significantly shorter than the value predicted by Fermi's golden rule. Losses do not hinder the formation of the FEBS if their inverse rates are much larger than its typical formation time. Also, once the FEBS has been established, the whole emitter-reservoir system subjected to Markovian losses has a higher probability to be found in the FEBS than in any other excited state orthogonal to it. So, one can speak here about the decay of the FEBS as a single compound entity. It should also be noted that when the inverse loss rate approaches the FEBS formation time, the dynamics of the emitter's upper-level population becomes markedly nonexponential [11,12]. So, one might conjecture that the FEBS effect is the reason for the nonexponential behavior observed in a number of recent experiments with single quantum dots [14,15].

To manipulate the FEBS it has been suggested either to use multiphoton driving or to implement a multilevel emitter with transitions both outside and inside the gap [8,16]. It might seem that the most obvious way to manipulate the FEBS of a two-level emitter via a single-photon process is to create inside the gap a defect resonant with the emitter's transition frequency. Such a scheme was also suggested for probing the FEBS through its interaction with a monitored field mode [17]. However, recently it has been shown that there is no need to spoil the PC and, consequently, introduce additional losses to affect the FEBS. Actually, it could be effectively excited and controlled by a coherent field with a frequency inside the continuum and close to the band edge [10,18]. Indeed, if the interaction of an emitter with modes close to the band edge is sufficiently strong to induce the FEBS, then it is natural to expect the possibility of influencing the FEBS through these modes. Moreover, one might excite the FEBS by a coherent driving more effectively through the modes near the band edge than through the in-gap defect [10,18].

To the best of our knowledge, in this paper the dynamics of the FEBS in band-gap reservoirs at finite temperatures is studied for the first time. A non-Markovian time-local master equation is derived for the purpose. This equation is able to capture the FEBS and to describe its dynamics for arbitrary temperatures of the reservoir. We demonstrate that the emitter-field entanglement is eventually destroyed by the reservoir with nonzero temperature, which can be an obstacle for the FEBS observation at longer wavelengths. On the other hand, we show that the FEBS may also be excited through a sufficiently intensive thermal pumping, opening the possibility for building different lasing devices based on the FEBS excited by means of an incoherent out-of-gap driving.

The outline of the paper is as follows: in Sec. II we give a brief description of the collective operator method (COM) for the case of a two-level emitter interacting with a reservoir. We also define the FEBS, and describe conditions of the FEBS formation. Section III is about the derivation of the non-Markovian master equation for finite-temperature reservoirs. Section IV is devoted to the discussion of the dynamics of the FEBS under the influence of a thermally excited reservoir.

## II. COLLECTIVE OPERATOR METHOD

In this section we describe the method of collective operators, which is the basis of our approach. The method was extensively discussed in a number of publications [5,10,11,17–19] and reviewed in Ref. [12]. So, here we give only a brief outline of the method emphasizing points especially important for our subsequent discussion.

Let us begin by considering the interaction of a single two-level emitter with a set of reservoir modes. In the rotating-wave approximation, and in the frame rotating at the frequency of the atomic transition $\omega_0$ the Hamiltonian describing the interaction is written as (here we use for simplicity a system of units where $\hbar=1$)

$$H_{pc}=\sum_{j=1}\Delta_j b_j^{\dagger}b_j+\sum_{j=1}g_j(\sigma^{+}b_j+b_j^{\dagger}\sigma^{-}),\tag{1}$$

where $b_j$ and $b_j^{\dagger}$ are the annihilation and creation operators for modes of the reservoir (which may be either a continuous or a discrete set); $\sigma^{\pm}=|\pm\rangle\langle\mp|$ represent the raising and lowering emitter operators with vectors $|\pm\rangle$ corresponding to upper and lower states of the emitter; $g_j$ are interaction constants; $\Delta_j$ are the detunings of frequencies of reservoir modes from the atomic transition frequency. In all further consideration we assume the spectrum of the reservoir modes to be the discrete one. Practically, to apply the COM one needs to discretize the density of states with an appropriate discretization procedure (see Ref. [12]).

The essence of the collective operator method is to represent the Hamiltonian $H_{pc}$ in a diagonal form

$$H_{pc}=\sum_{j=0}\lambda_j C_j^{\dagger}C_j,\tag{2}$$

by using the collective operators

$$C_j=U_{j0}\sigma^{-}+\sum_{k=1}U_{jk}b_k,\tag{3}$$

with the help of the unitary matrix $U$ with the following elements:

$$U_{j0}=U_{0j}=\left[1+\sum_{k=1}g_k^2/(\lambda_j-\Delta_k)^2\right]^{-1/2},$$

$$U_{jk}=U_{j0}g_k/(\lambda_j-\Delta_k).\tag{4}$$

The elements of the transformation matrix satisfy the relations

$$\sum_{j=0}U_{ij}U_{jk}^{*}=\delta_{ik}.$$

In further consideration we take the elements $U_{ij}$ to be real. Eigenvalues are provided by the following equation:

$$\lambda_j=\sum_{k=1}g_k^2/(\lambda_j-\Delta_k).\tag{5}$$

For the reservoir of $N$ modes one would have $N+1$ solutions of Eq. (5). One also has

$$\sum_{j=0}\lambda_j U_{0j}U_{jk}=g_k,\quad \sum_{j=0}\lambda_j U_{lj}U_{jk}=\Delta_k\delta_{kl}$$

for $k,l>0$.

In the case of spontaneous emission of the completely excited emitter into the initially empty reservoir, for the upper-state emitter's population one finds [12]

$$\langle\sigma^{\dagger}(t)\sigma^{-}(t)\rangle=\sum_{j,k=0}U_{j0}U_{0k}\langle C_j^{\dagger}(t)C_k(t)\rangle=\left|\sum_{j=0}U_{0j}^2\exp(-i\lambda_j t)\right|^2.\tag{6}$$

When the emitter's transition frequency is in the gap or in the continuum but sufficiently close to the band edge, i.e., when the spontaneous decay of the atomic upper-state population can be "frozen," the COM allows one to demonstrate in a very simple and straightforward way the formal mechanism underlying the "freezing." The freezing of spontaneous emission may be easily understood through Eq. (6) by noting that for a transition frequency of the emitter within or in the vicinity of the gap one and only one (in the particular case of a two-level emitter) of the coefficients $U_{0j}$ (say, $U_{00}$) is much larger than the other $U_{0j}$. Then for a sufficiently long time the only term that survives in the above sum is one with $U_{00}$. This term defines the frozen population of the excited emitter's level, that is,

$$\langle\sigma^{\dagger}(t)\sigma^{-}(t)\rangle=\left|U_{00}^2\exp(-i\lambda_0 t)+\sum_{j=1}U_{0j}^2\exp(-i\lambda_j t)\right|^2\overset{t\to\infty}{\longrightarrow}U_{00}^4.$$

In Fig. 1 we illustrate the freezing behavior.

Exactly this feature makes the COM very useful for theoretical investigation of the FEBS dynamics. The dominant coefficient $U_{00}$ is a natural large parameter to divide the whole reservoir on parts strongly and weakly interacting with the emitter. This also allows one to identify the FEBS. As follows from Eq. (3), in the single-photon subspace eigenvectors of the Hamiltonian $H_{pc}$ are given by

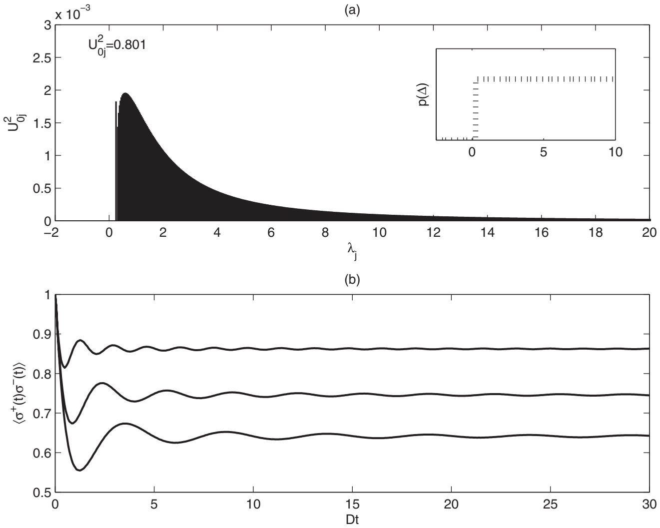


FIG. 1. Examples of the coefficients $U_{0i}^2$ for the emitter's transition frequency near the upper edge of an extended gap of the steplike density of states (a) and dynamics of emitter's upper-state population (b). The inset depicts the corresponding local projected density of states. For (a) the detuning between the atomic transition frequency and the gap edge $\omega_g$ is $0.25D$, in (b) curves correspond in descending order to the following detunings $D$, $0.5D$ and $0.25D$. In both figures the frequency interval used for simulation is $\omega_h-\omega_g=32D$, and the number of points used in the discretization is $N=1000$.

$$|\psi_j\rangle = C_j^\dagger|-\rangle|\text{vac}\rangle = U_{0j}|+\rangle|\text{vac}\rangle + \sum_{k=1} U_{kj}|-\rangle|1_k\rangle, \qquad (7)$$

where the vector $|\text{vac}\rangle$ denotes the vacuum of the reservoir, and the vector $|1_k\rangle$ corresponds to the single-photon state of the $k$th mode of the reservoir. Using states $|\psi_j\rangle$, the evolution operator $U(t)$ of the emitter-reservoir system without external perturbations can be written as

$$U(t) = \exp\{-iH_{pc}t\} = \sum_{j=0} |\psi_j\rangle\langle\psi_j|\exp(-i\lambda_j t).$$

When the freezing is possible, for the initially excited emitter decaying into the vacuum of the reservoir, the probability to find the system in the state $|\psi_0\rangle$ is much larger than the probabilities to find the system in any other state orthogonal to it.

$$|\langle\psi_0|U(t)|+\rangle|\text{vac}\rangle|^2 = U_{00}^2 \gg |\langle\psi_{j>0}|U(t)|+\rangle|\text{vac}\rangle|^2 = U_{0j}^2.$$

Also, only the state $|\psi_0\rangle$ shows a significant degree of emitter-field entanglement. Indeed, by introducing the bosonic collective operators

$$\sqrt{1-U_{0j}^2}B_j = C_j - U_{0j}\sigma^- = \sum_{k=1} U_{jk}b_k, \qquad (8)$$

one can rewrite each vector $|\psi_j\rangle$ as

$$|\psi_j\rangle = U_{0j}|+\rangle|\text{vac}\rangle + \sqrt{1-U_{0j}^2}|-\rangle|1_j\rangle_B,$$

representing it as a two-qubit state with the following concurrence of the emitter-field entanglement [20]:

$$c_j = 2U_{0j}\sqrt{1-U_{0j}^2}. \qquad (9)$$

So, one may label the state $|\psi_0\rangle$ as the "ideal FEBS," because any state of the form

$$|\Psi\rangle = \alpha|\psi_0\rangle + \beta|-\rangle|\text{vac}\rangle$$

is also a FEBS unaffected by the Hamiltonian $H_{pc}$. It is useful to emphasize that the value of the frozen upper-level population is defined by the initial state of the system. The ideal FEBS has the largest possible nondecaying upper-state population $U_{00}^2$, and the maximal achievable degree of emitter-field entanglement for any position and transition frequency of the emitter within the PC. One might add here that this maximal degree can be achieved only by specific initial excitation of the reservoir together with the emitter. If only the emitter is excited, the whole system is in the superposition of states (7), and the frozen upper-level population cannot exceed $U_{00}^4$.

To implement the COM for modeling, one usually takes some finite interval of frequencies around the emitter's transition frequency. The extent of this interval required for an accurate simulation can be estimated by considering the set of coefficients $U_{0j}^2$ [12]. In the outer parts of the interval the sum $\Sigma_j U_{0j}^2$ should change slowly with the addition of new terms. After choosing the interval, one usually discretizes it using the appropriate discretization method [12,21,22]. Then one diagonalizes the Hamiltonian matrix (1), and finds the unitary matrix $U$ and eigenvalues $\lambda_j$. Throughout this work we shall use for illustration of our discussion the simple steplike density of states

$$p(w) = D\Theta(w-w_g),$$

where $D$ is the constant describing the value of the density of states; $\Theta(x)$ is the step-function, and $w_g$ is the band-edge frequency. Thus we implement the simplest equidistant discretization scheme, dividing the frequency interval in equal parts and assuming $g_j=g\ \forall\ j$.

In practice one hardly needs to implement any renormalization to calculate the eigenvalues $\lambda_j$. However, one might notice that realistic densities of states behave asymptotically like the density of states of homogeneous structures. Thus, the right-hand side of Eq. (5) is divergent. To tackle this problem one may follow the standard nonrelativistic recipe described in detail in Ref. [6]. In the context of the COM, the effect of the renormalization procedure is to subtract from Hamiltonian (2) the following integral of motion:

$$\lambda_r\sum_{j=0} C_j^\dagger C_j,$$

where the value $\lambda_r$ is provided by a solution of Eq. (5) with the emitter's transition frequency $\omega_0$ set to zero.

Among the most important properties of the coefficients $U_{0j}$ one needs to mention the following one: that for an arbitrary fixed frequency interval $[\omega_g,\omega_h]$ one has

$$U_{0j} \overset{N\to\infty}{\to} 0$$

for $j>0$, where $N$ is the number of discretization points. Then, sufficiently far from the gap $\lambda_j U_{0j}\to g_j$, reflecting the fact that far from the gap the collective operators $C_j$ are very close to the modal operators $b_j$; similarly $U_{jj}\to 1$.

It is useful to note that the discretization procedure implemented here allows one to find a simple approximation for the coefficients $U_{0j}$ and the eigenvalues $\lambda_j$ far away from the band edge. Far from the band edge, the eigenvalues $\lambda_j$ tend to be very close to the corresponding frequencies $\omega_j$. So from Eqs. (4) and (5) it follows that

$$\lambda_j \approx \Delta_j + \frac{g^2}{\Delta_j},$$

$$\lambda_j^2 U_{0j}^2 \approx g^2. \tag{10}$$

As follows from these formulas, far from the band edge the coefficients $U_{0j}$ decrease in inverse proportion to the frequency.

## III. MASTER EQUATION

This section is devoted to the derivation of a master equation able to capture the FEBS. The derivation is based on the fact that one may select from the whole set of reservoir's modes those that play an important role in the interaction process with the emitter from those that do not. One way to proceed to such a separation of the reservoir is pointed out by the COM: those modes that compose the FEBS are the important ones and as such their dynamics should be fully accounted [18].

### A. Separation of the reservoir

To proceed with separation of the reservoir one notices that the "ideal entangled state" $|\psi_0\rangle$ is the zero-eigenvalue eigenstate of the following operator:

$$C|\psi_0\rangle = \left(\sigma^- - \frac{U_{00}}{\sqrt{1-U_{00}^2}}B_0\right)|\psi_0\rangle = 0. \tag{11}$$

Every collective operator can be represented with help of this operator and with the collective bosonic operators $B_j$ (8) as

$$C_j = U_{0j}C + B_j\sqrt{1-U_{0j}^2} + B_0\frac{U_{00}U_{0j}}{\sqrt{1-U_{00}^2}} = U_{0j}C + \acute{B}_j, \tag{12}$$

where all $\acute{B}_{j>0}$ commute with $B_0^\dagger$; so the Hamiltonian $H_{pc}$ can be recast as

$$H_{pc} = -\lambda_0 C^\dagger C + \lambda_0(\sigma^+\sigma^- + B_0^\dagger B_0) + \sum_{j=1}\lambda_j \acute{B}_j^\dagger \acute{B}_j + \sum_{j=1}\lambda_j U_{0j}(C^\dagger \acute{B}_j + \acute{B}_j^\dagger C). \tag{13}$$

From Eq. (13) it is clear that the state $|\psi_0\rangle|\bar{\mathrm{vac}}\rangle$ (where the vector $|\bar{\mathrm{vac}}\rangle$ denotes the vacuum state of the reservoir composed of all modes $\acute{B}_{j>0}$) is the eigenstate of the Hamiltonian $H_{pc}$. Operators $\acute{B}_{j>0}$ introduced by Eq. (12) are not mutually independent. They satisfy the following commutation relations:

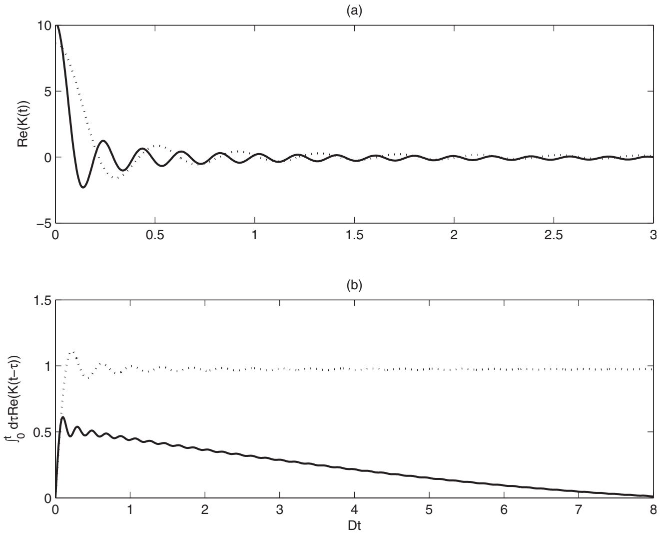


FIG. 2. Examples of reservoir correlation functions (15) (dotted line) and (16) (solid line) (a); the same functions integrated (b). Parameters are the same as in Fig. 1(a).

$$[\acute{B}_{j>0}, \acute{B}_{k>0}^\dagger] = \delta_{jk} - \frac{U_{0j}U_{0k}}{1-U_{00}^2}. \tag{14}$$

Nevertheless, for a large number $N$ of discretization points in the fixed frequency interval, they are close to being almost mutually independent (as it could be seen in the previous section). The Hamiltonian (1) rewritten in the form (13) points directly to the way to obtain a master equation for the problem. Certainly, one needs to average over the reservoir of all modes $\acute{B}_{j>0}$, but the procedure is not trivial. Generally, the Born-Markov approximation is not applicable in this case. It can be seen from the behavior of the correlation function of the reservoir of modes $\acute{B}_{j>0}$,

$$\acute{K}(t) \approx \sum_{j=1}\lambda_j^2 U_{0j}^2 \exp(-i\lambda_j t). \tag{15}$$

It exhibits the same "long-tale" behavior as the correlation function of the reservoir of modes $b_j$,

$$K(t) = \sum_{j=1} g_j^2 \exp(-i\Delta_j t), \tag{16}$$

which is not surprising, as it can be seen from Eq. (10). Examples of correlation functions are illustrated in Fig. 2(a).

However, one can also see in Fig. 2(b) that behaviors of the integrated correlation functions are drastically different. The real part of the integrated function (15) tends to a constant nonzero value

$$\lim_{t\to\infty}\int_0^t d\tau\,\mathrm{Re}[\acute{K}(t-\tau)] = \mathrm{const} \neq 0, \tag{17}$$

whereas the real part of the integrated function (16) tends to zero,

$$\lim_{t\to\infty}\int_0^t d\tau\,\mathrm{Re}\{K(t-\tau)\} = 0. \tag{18}$$

As was shown in Ref. [23], Eq. (18) is a necessary condition to observe the freezing of spontaneous decay. Moreover, the fulfillment of this condition allows one to capture the freezing even with a master equation, obtained directly from the Hamiltonian (1) by applying an averaging over the whole field reservoir and the Born approximation. For example, for the upper-state emitter's population in Ref. [9] (and after a long elaboration in recent work [24]) in the case of spontaneous emission into the vacuum of a reservoir the following equation has been obtained:

$$\frac{d}{dt}\langle\sigma^{\dagger}(t)\sigma^{-}(t)\rangle = -2\int_{0}^{t} d\tau \,\mathrm{Re}\{K(t-\tau)\}\langle\sigma^{\dagger}(\tau)\sigma^{-}(\tau)\rangle. \tag{19}$$

As a consequence of condition (18), Eq. (19) is capable of capturing the freezing of spontaneous decay. Moreover, it is capable of capturing it, even when the Markovian approximation is bluntly performed on it. Indeed, in this case one obtains

$$\frac{d}{dt}\langle\sigma^{\dagger}(t)\sigma^{-}(t)\rangle = 0,$$

which is certainly true for the freezing of decay, i.e., for times much longer than the time required for the FEBS formation. Needless to say, one is unable to obtain reliable results for the frozen population in this way, and also unable to estimate transition times. In our previous work [23] we show that Eq. (19) holds only for times of the FEBS formation too small compared with the inverse rate of decay at the initial stage of the population's dynamics. This rate is defined by the shape of the projected local density of states near the band edge. It should be noted here that such a rate is strongly dependent on the shape of the projected local density of states in the vicinity of the band edge but not on the detuning of the emitter transition from the frequency of the band edge, as illustrated in Fig. 1, in contrast with the time required for FEBS formation. Therefore, Eq.(19) holds only in the limit of a distant band edge, where the effects of the field-emitter entanglement are negligible.

Furthermore, time-non-local master equations derived by the Born approximation are prone to producing nonphysical, non-semi-positive definite results for the density matrix, especially when the initial states of the emitter and of the reservoir are correlated [25]. Even if such a master equation in the absence of emitter's interaction with additional systems leads to a positively semidefinite result, one is not guaranteed to get a meaningful result after switching in, for example, a classical driving of the emitter.

### B. Derivation

Here we proceed with the derivation of the time-local Lindblad master equation on the basis of the system-reservoir separation described by the Hamiltonian (13). To average over the reservoir of modes $\acute{B}_{j>0}$, it is worth taking a look at the "free" evolution of this reservoir. This evolution is governed by the Hamiltonian

$$H_B = \sum_{j=1} \lambda_j \acute{B}_j^{\dagger}\acute{B}_j.$$

From commutation relations (14) one gets the following equations for the free evolution:

$$\frac{d}{dt}\acute{B}_j = -i\lambda_j\acute{B}_j - i\frac{U_{0j}}{1-U_{00}^2}\sum_{k=1}\lambda_k U_{0k}\acute{B}_k. \tag{20}$$

So, in the process of such a free evolution of the reservoir all modes $\acute{B}_{j>0}$ seem coupled to the same collective mode, described by the annihilation operator

$$A = \frac{1}{\sqrt{\nu_A}}\sum_{j=1}\lambda_j U_{0j}\acute{B}_j, \tag{21}$$

where to make it satisfy the usual bosonic commutation relation, the normalization parameter is introduced.

$$\nu_A = [A,A^{\dagger}] = \sum_{j=1}\lambda_j^2 U_{0j}^2 - \frac{\lambda_0^2 U_{00}^4}{1-U_{00}^2}. \tag{22}$$

Also, the operator $C$ describing the FEBS is coupled only to this mode. One can select $A$ out from the reservoir by introducing new reservoir operators independent of $A$ in the following form:

$$\bar{B}_j = \frac{1}{\sqrt{\nu_{jj}}}(\acute{B}_j - \nu_{jA}A) \to [\bar{B}_j, A^{\dagger}] = 0, \tag{23}$$

where

$$\nu_{Aj} = [\acute{B}_j, A^{\dagger}] = \frac{U_{j0}}{\sqrt{\nu_A}}\left(\lambda_j + \frac{\lambda_0 U_{00}^2}{1-U_{00}^2}\right),$$

$$\nu_{jk} = [\acute{B}_j - \nu_{jA}A, \acute{B}_k^{\dagger} - \nu_{kA}A^{\dagger}] = \delta_{jk} - \nu_{Aj}\nu_{Ak} - \frac{U_{0j}U_{0k}}{1-U_{00}^2}. \tag{24}$$

Using operators $\bar{B}_j$ and $A$, Hamiltonian (13) can be represented in the following way:

$$H_{res} = -\lambda_0 C^{\dagger}C + \lambda_0(\sigma^{\dagger}\sigma^{-} + B_0^{\dagger}B_0) + \omega_A A^{\dagger}A + G(C^{\dagger}A + A^{\dagger}C) + \sum_{j=1}\bar{\lambda}_j\bar{B}_j^{\dagger}\bar{B}_j + \sum_{j=1}\bar{g}_j(A^{\dagger}\bar{B}_j + \bar{B}_j^{\dagger}A), \tag{25}$$

where

$$\omega_A = \sum_{l=1}\lambda_l \nu_{Al}^2, \quad \bar{g}_j = \lambda_j\nu_{Aj}\sqrt{\nu_{jj}}, \quad \bar{\lambda}_j = \lambda_j\nu_{jj}, \quad G = \sqrt{\nu_A}. \tag{26}$$

Operators $\bar{B}_j$ are not independent. Similarly to $\acute{B}_j$, one has

$$[\bar{B}_j, \bar{B}_k^{\dagger}] = \frac{\nu_{jk}}{\sqrt{\nu_{jj}\nu_{kk}}} \to \delta_{jk}$$

for $N \to \infty$. Furthermore,

$$\sum_{j=1} U_{0j}\bar{B}_j = 0, \quad \sum_{j=1} \lambda_j U_{0j}\bar{B}_j = 0. \tag{27}$$

Taking into account Eqs. (24) and (27), one can introduce a new set of $N-2$ independent reservoir operators $\tilde{B}_j$ instead of $N$ operators $\bar{B}_j$,

$$\tilde{B}_j = \sum_{k=1}^{N} W_{jk}\bar{B}_k, \tag{28}$$

in such a way that the form of the Hamiltonian (25) is preserved. However, for our subsequent consideration it is not necessary to calculate this transformation explicitly. It is sufficient to know that for wide spectral interval $[\omega_g, \omega_h]$ and large $N$, far away from the band edge the transformation leads to negligible deviations from the interaction constants and frequencies provided by Eq. (26) [18]. It ought to be expected, since far from the band edge the coefficients $U_{jj}$ are very close to unity, and the operators $\bar{B}_k$ are very close to the original reservoir operators $b_k$.

Here we want to emphasize that up to this point no approximations were made. The Hamiltonian (25) is completely equivalent to the original Hamiltonian (1). However, the Hamiltonian (25) now gives a clear indication of how the averaging over the reservoir may be carried out. For an extended density of state (like realistic ones, or the model ones traditionally considered, for example, such as the steplike one, or isotropic or anisotropic ones) the frequency of the collective mode $A$ is far away from the band edge and deep inside the continuum. Also, $\omega_A$ tends to infinity with an increase of $\omega_h$. Indeed, as it was shown in the previous section, far from the band edge one has from Eq. (26), the following relations hold:

$$\bar{g}_j^2 \approx \lambda_j^4 U_{0j}^2/\nu_A \approx \Delta_j^2 g_j^2/\nu_A. \tag{29}$$

Then it becomes clear that the frequency $\omega_A$ is the average detuning of frequencies of the reservoir modes from the atomic transition frequency,

$$\omega_A \approx \frac{1}{\nu_A}\sum_{l=1} \Delta_l g_l^2 \to \frac{1}{\nu_A}\int_{\omega_g}^{\omega_h} d\omega(\omega-\omega_0)p(\omega),$$

where $p(\omega)$ represents the projected localized density of states.

For the steplike density of states one has $\omega_A \approx (\omega_h+\omega_g-2\omega_0)/2$. The quantity $\nu_A$ plays the role of a normalization constant, that is,

$$\nu_A \approx \int_{\omega_g}^{\omega_h} d\omega p(\omega).$$

It follows from Eq. (29) that the spectrum of interaction constants is smooth near the frequency $\omega_A$. As a consequence, the Born-Markovian approximation can be applied when describing the mode $A$ interaction with the reservoir of the modes $\tilde{B}_j$ [18].

Working in the interaction picture with respect to the Hamiltonian

$$H_R = \omega_A A^\dagger A + \sum_{j=1}^{N-2} \bar{\lambda}_j \tilde{B}_j^\dagger \tilde{B}_j, \tag{30}$$

one implements the Born approximation in a standard way, and obtains the following master equation for the density matrix averaged over the reservoir composed by modes $\tilde{B}_j$:

$$\begin{aligned}\frac{d\varrho(t)}{dt} \approx\; & i[\lambda_0 C^\dagger C - \lambda_0(\sigma^\dagger\sigma^- + B_0^\dagger B_0) \\ & - G(C^\dagger A \exp(-i\omega_A t) + \text{H.c.}), \varrho(t)] \\ & - i\langle[V(t),\rho_c(t)]\rangle_R - \int_0^t \langle[V(t),[V(\tau),\rho_c(\tau)]]\rangle_R,\end{aligned} \tag{31}$$

where

$$\varrho(t) = \langle\rho_c(t)\rangle_R,$$

$\rho_c(t)$ denotes the complete system's density matrix, and $\langle\ldots\rangle_R$ denotes the averaging over the reservoir of modes $\bar{B}_j$. The interaction Hamiltonian $V(t)$ in Eq. (31) is given by

$$V(t) \approx \sum_{j=1} \bar{g}_j\{A_j^\dagger \tilde{B}_j \exp[-i(\bar{\lambda}_j - \omega_A)t] + \text{H.c.}\}. \tag{32}$$

It should be noted that the master equation (31) is quite general. It can be applied and holds for arbitrary states of the reservoir.

### C. Initial state of the reservoir

The main purpose of this work is to investigate the influence of the thermal excitation of the reservoir on the dynamics of the FEBS. Generally, to find the initial state of modes $B_0$, $A$, and $\tilde{B}_j$, one needs to calculate the transformation (28), and then to perform a unitary transformation on the initial state of the system. So, even if the initial states of modes $b_j$ were not entangled, states of modes $B_0$, $A$, and $\tilde{B}_j$ might be as a consequence of this transformation. However, if the initial state of the reservoir of modes $b_j$ is not entangled with the state of the emitter, and is a thermal state characterized by the same average number of photons $\bar{n}$ in every mode, the initial states of modes $B_0$, $A$, and $\tilde{B}_j$ are also thermal states with the same average number of photons $\bar{n}$. It is easy to see that by considering the density matrix of the thermal reservoir, which is given by

$$\rho_T(0) = [1-\exp\{-\beta\}]^N \exp\left\{-\beta\sum_{j=1}^{N} b_j^\dagger b_j\right\}, \tag{33}$$

where

$$\exp\{-\beta\} = \frac{\bar{n}}{\bar{n}+1}.$$

The sum over the photon number operators $b_j^\dagger b_j$ is invariant with respect to an arbitrary rotation of these operators. So, we have

$$\sum_{j=1}^{N} b_j^\dagger b_j = B_0^\dagger B_0 + A^\dagger A + \sum_{j=1}^{N-2} \tilde{B}_j^\dagger \tilde{B}_j.$$

In particular, this means that

$$\langle \tilde{B}_j(0)\rangle = 0, \quad \langle \tilde{B}_j(0)\tilde{B}_l(0)\rangle = 0, \quad \langle \tilde{B}_j^\dagger(0)\tilde{B}_l(0)\rangle = \bar{n}\delta_{jl}.$$

Thus, for $\omega_A$ far away from the band edge, by applying the Markovian approximation to Eq. (31), the following Lindblad master equation is obtained:

$$\begin{aligned}\frac{d\varrho(t)}{dt} \approx{}& i[\lambda_0 C^\dagger C - \lambda_0(\sigma^\dagger\sigma^- + B_0^\dagger B_0) - \delta A^\dagger A \\ &- G(C^\dagger A \exp(-i\omega_A t) + \text{H.c.}), \varrho(t)] \\ &+ \gamma(\bar{n}+1)(2A\varrho(t)A^\dagger - A^\dagger A\varrho(t) - \varrho(t)A^\dagger A) \\ &+ \gamma\bar{n}(2A^\dagger\varrho(t)A - AA^\dagger\varrho(t) - \varrho(t)AA^\dagger), \end{aligned} \tag{34}$$

where, according to Eq. (29), the relaxation rate and the frequency shift are given as

$$\gamma \approx \frac{\pi}{\nu_A}\omega_A^2 p(\omega_A + \omega_0), \quad \delta \approx \frac{1}{\nu_A}\mathrm{P}\int_{w_g}^{w_h} dw \frac{(w-\omega_0)^2 p(w)}{w - \omega_0 - \omega_A}, \tag{35}$$

where $p(w)$ denotes the projected local density of states of the original reservoir described by the set of modes $b_j$.

One might mention here that in deriving the master equation (34) we have actually represented the interaction of the emitter-field bound system (in fact, it is the Jaynes-Cummings one) with a non-Markovian reservoir as an interaction of this system with an intermediate system (the mode $A$), which is subjected to a Markovian dissipation. A somewhat similar scheme of two coupled two-level systems, with only one of them coupled to the Markovian reservoir, was considered recently as the simplest model of the non-Markovian reservoir [26].

### D. Elimination of the mode A

From Eq. (34) one can see that the evolution of the mode $A$ under the influence of reservoir $\tilde{B}_j$ occurs on a time scale much shorter than the evolution of $C$. Indeed, from Eq. (34) one has $\delta \sim \omega_A$. Also, for extended densities of states the rate $\gamma$ increases when one takes a larger frequency interval for the simulation. It might exceed by far the inverse time of the FEBS formation (as it happens, for example, for the steplike density of states, where $\gamma \sim \omega_A$). In this case a relaxation of the mode $A$ toward a thermal equilibrium with the reservoir of the modes $\tilde{B}_j$ is practically unaffected by the coupling of the mode $A$ with the emitter and the mode entangled with it. In this case one has

$$\begin{aligned}\langle A^\dagger(t)A(\tau)\rangle &\approx \langle A^\dagger(t)A(t)\rangle \exp\{-\gamma|t-\tau| + i\delta(t-\tau)\} \\ &\approx \bar{n}\exp\{-\gamma|t-\tau| + i\delta(t-\tau)\}.\end{aligned} \tag{36}$$

Thus, the mode $A$ can be adiabatically eliminated from the master equation (34). Finally, in the basis rotating with the frequency $\lambda_0$ one obtains the following equation:

$$\begin{aligned}\frac{ds(t)}{dt} ={}& i[(\lambda_0 + \bar{\delta})C^\dagger C - 2\bar{n}\bar{\delta}\sigma^+\sigma^-, s(t)] \\ &+ \bar{\gamma}(\bar{n}+1)(2Cs(t)C^\dagger - C^\dagger C s(t) - s(t)C^\dagger C) \\ &+ \bar{\gamma}\bar{n}(2C^\dagger s(t)C - CC^\dagger s(t) - s(t)CC^\dagger),\end{aligned} \tag{37}$$

where $s$ is the density matrix $\varrho$ traced over the mode $A$, and

$$\bar{\gamma} = \frac{\nu_A \gamma}{\gamma^2 + (\omega_A + \delta)^2}, \quad \bar{\delta} = \frac{\nu_A(\omega_A + \delta)}{\gamma^2 + (\omega_A + \delta)^2}.$$

For the example of the steplike density of states, in the limit of large $\omega_h$, one has

$$\bar{\gamma} \approx 4D/(4+\pi^2), \quad \bar{\delta} \approx 2\bar{\gamma}/\pi.$$

Master equation (37) is the main result of this work. It is non-Markovian in essence and yet has a time-local Lindblad form. So, it preserves a semipositive definiteness of the density matrix $s(t)$. In the course of its derivation, we have taken into account only those features of the reservoir that support the FEBS and therefore it is suitable to describe those phenomena that take place during times longer than the time required to induce the FEBS. One can extend the region of applicability of this equation to shorter times by accounting for the structure of the $\tilde{B}_j$ reservoir, as was done in Ref. [18]. In particular, for the zero-temperature reservoir [for which Eq. (37) reduces to the one obtained in Ref. [18]] one can capture slowly decaying oscillations accompanying the emitter's population transition to the frozen value. Here, we are interested in the dynamics for times longer than the FEBS formation time. It is useful to point out that such time is comparable to the inverse decay rate in the homogeneous media [as can be seen both for our model density of states (Fig. 1) and for calculated densities of states of realistic PCs [7]]. Also it is worth mentioning that for the particular case of a Lorenzian density of states with a single-point gap and the zero-temperature reservoir, a similar master equation has been derived in Ref. [27] with the help of the pseudomodes method.

## IV. DISCUSSION

Recently, while considering the resonance fluorescence near the band edge, it was noticed that a weak coherent excitation might destroy the FEBS [10,18]. While it seems counterintuitive at first glance, the mechanism of this phenomenon is quite simple. The weak coherent driving causes the system to oscillate between the vacuum and the single-photon subspace, since the driving field is too weak to provide for more than one photon in a field coupled with the emitter. Under the action of a coherent driving the system might undergo a transition from the ground state either to the frozen state $|\psi_0\rangle$ or to the decaying state $|\phi\rangle$, orthogonal to it. The probability of transition to the decaying state is low because the driving is weak. So, the FEBS is destroyed with a rate much smaller than the typical inverse time of the FEBS formation. Finally, the probability to occupy the FEBS reaches the stationary value, which for a weak driving can be much smaller than the initial value.

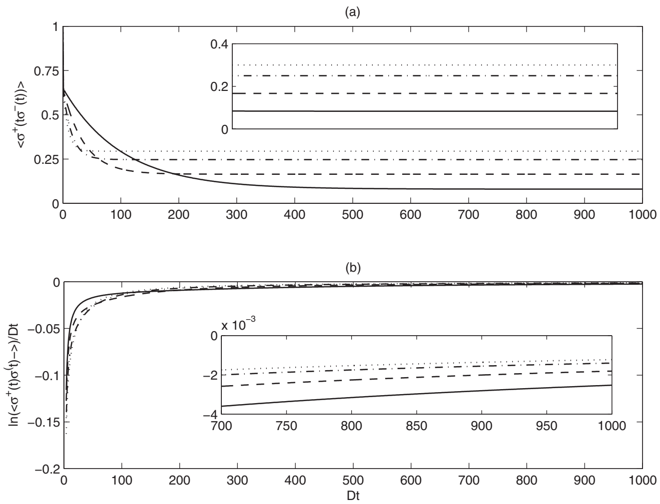


FIG. 3. The destruction of the FEBS by a low-temperature reservoir. In (a) the emitter's upper-state population is depicted. In (b) the rate of the emitter's population decay is plotted. Solid, dashed, dash-dotted, and dotted lines correspond to the following average numbers of photons in the reservoir modes, respectively, $\bar{n}=0.1, 0.25, 0.5, 0.75$. The inset in (a) depicts the stationary values of the emitter's upper-state population for the smooth Markovian reservoir given by Eq. (38); curves correspond to the same values of $\bar{n}$ as before. The inset in (b) depicts long-time values of the decay rates.

It seems that a similar phenomenon occurs also for a weak thermal excitation of the reservoir. In Fig. 3 one can see examples of it.

For small temperatures (corresponding to an average number of photons in the reservoir modes less than unity) the upper-level population of the emitter decreases slowly with a rate much smaller than the inverse time of the FEBS formation. Finally, it reaches a stationary value typical of an emitter interacting with an unstructured reservoir. The inset shows the long-time behavior of the emitter's upper-state population for such a Markovian reservoir. For an emitter initially in the upper state, such behavior is given by the following formulas:

$$\langle\sigma^{\dagger}(t)\sigma^{-}(t)\rangle = \frac{\bar{n}}{2\bar{n}+1} + \frac{\bar{n}+1}{2\bar{n}+1}\exp\{-2\Gamma(2\bar{n}+1)t\}, \tag{38}$$

where we have chosen $\Gamma=\pi D$.

It is quite curious, however, that the decay remains clearly nonexponential during all the time of the evolution [Fig. 3(b)]. The decay rates tend to zero asymptotically. They depend on the temperature, and remain different even for long times when the system's state becomes very close to the stationary one. One might conclude that due to the presence of the band edge, the effect of an entanglement between the emitter and the field keeps on affecting the evolution of the system during times much longer than the FEBS formation time.

In the long-time limit the average number of photons in the mode $B$ becomes $\bar{n}$, which is the average number of

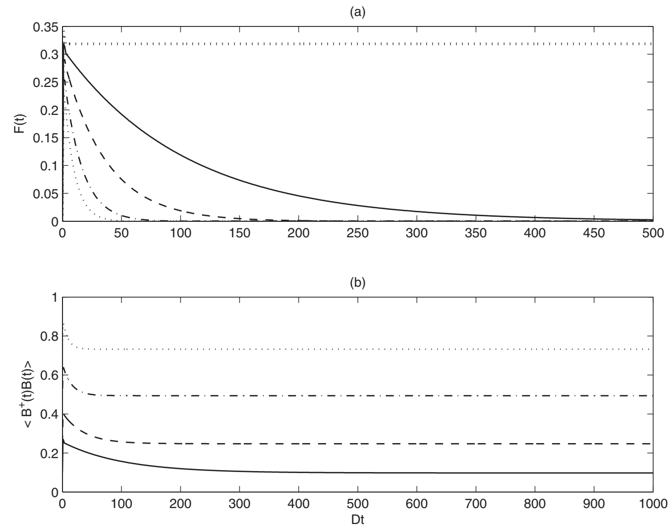


FIG. 4. Examples of the dynamics of the sum of off-diagonal elements (39) (a) and of the average number of photons in the mode $B$ (b). Similarly to the previous figure, solid, dashed, dash-dotted, and dotted lines correspond to the following average numbers of photons in the reservoir modes $\bar{n}=0.1, 0.25, 0.5, 0.75$. The thick dotted line depicts the value of $F(t)$ for the zero-temperature reservoir. The emitter is initially in the upper state.

photons in every mode of the reservoir at a given temperature [see Fig. 4(b)].

In Fig. 5 we give an example of the photon number distribution for the mode $B$,

$$P(n) = \langle n|\mathrm{Tr}\{s(t)\}_e|n\rangle,$$

where $\mathrm{Tr}\{\cdot\}_e$ denotes the trace over the states of the emitter. One sees that at the initial stage of the dynamics, for times close to those of FEBS formation, the photon-number distri-

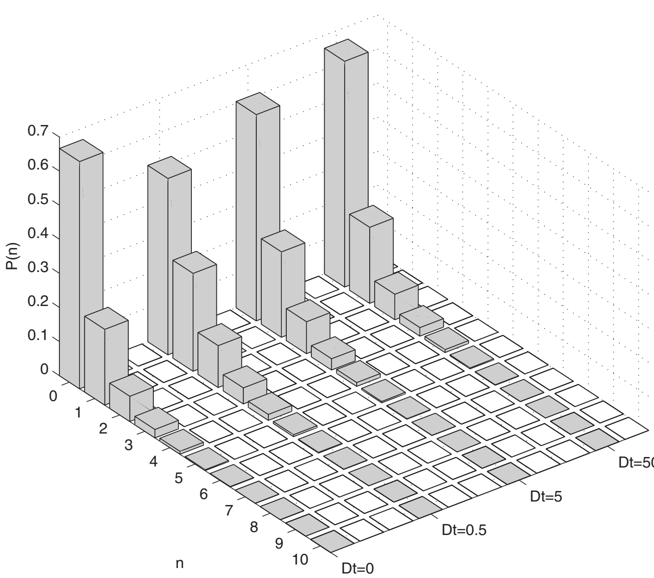


FIG. 5. The photon-number distribution for the state of the mode $B$ in different moments of time. The average number of photons is $\bar{n}=0.5$. The emitter is initially in the upper state.

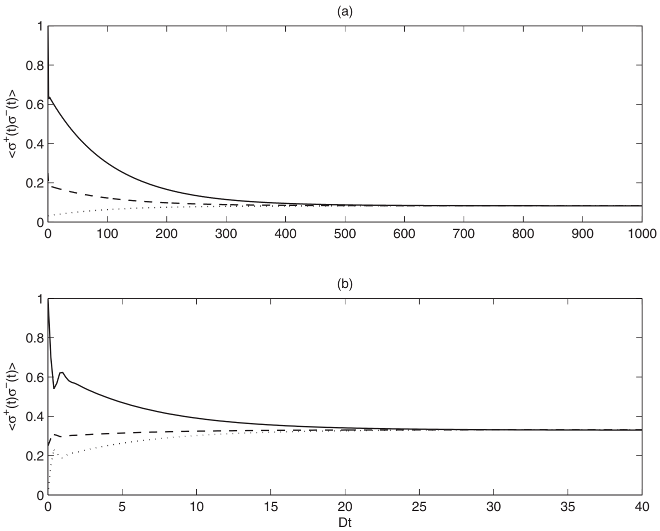


FIG. 6. Emitter's upper-state population for $\bar{n}=0.1$ (a) and for $\bar{n}=1$ (b). Solid, dashed, and dotted lines correspond to initial states of the form (40) with the parameter $\alpha=1,0.5,0$.

bution $P(n)$ deviates from the thermal one. Then, $P(n)$ slowly reaches the thermal distribution with an average number of photons $\bar{n}$ once again.

Thus mode $B$ returns to its initial state and one might conjecture that the correlations between the emitter and mode $B$ have died out. By plotting the sum of the off-diagonal elements of the density matrix, that is,

$$F(t) = \sum_{n=0} |\langle + |\langle n|s(t)|n+1\rangle|-\rangle|, \tag{39}$$

one may verify that this is indeed what is going on [see Fig. 4(a)]. For the zero-temperature reservoir the sum (39) tends to the nonzero stationary value. From Eq. (7) one has

$$F(t\to\infty) = U_{00}^3\sqrt{1-U_{00}^2}.$$

For a finite temperature it invariably tends to zero. The value of the temperature affects only the rate of decrease. Therefore, one comes to the following conclusion: even though a weak thermal excitation destroys the FEBS by a mechanism similar to those occurring for a weak coherent excitation, the final result is completely different. The coherent driving leaves the system in the state preserving the information about the initial state of the emitter [10,18]. Even the rate of transition to the stationary state depends on the initial state. This feature allowed one to suggest the coherently controlled FEBS as a possible candidate for a basis to devise single-emitter optical memory cells. In contrast, an incoherent excitation of the reservoir washes out any information about the initial state of the emitter. The emitter and the mode $B$ arrive to the same state independently of the state they start with. To illustrate this feature, we show in Fig. 6 the dynamics of the emitter's upper-state population for various initial states of the form

$$s(0) = s_e(0)\otimes s_B(0), \quad s_e = |\phi\rangle\langle\phi|, |\phi\rangle = \alpha|+\rangle + \sqrt{1-\alpha^2}|-\rangle, \tag{40}$$

where $s_e(0)$ is the initial density matrix of the emitter, and $s_B(0)$ is the initial thermal state of the mode $B$ of the form (33).

Clearly, such an influence of a finite reservoir's temperature might be quite an obstacle to the observation of any effects associated with the FEBS, for structures with gaps in the long-wavelength region. For example, it is not so difficult to create good-quality PCs with gaps in the microwave region (one might mention as the first example the famous "yablonovite" structure [28]). However, since having an average number of thermal photons about few units already leads to the destruction of the FEBS over times comparable with times required for its formation, it is hardly reasonable to look for the FEBS in the microwave region at room temperature.

It is interesting to note that the detrimental effect of a weak thermal excitation of the reservoir has been noticed some time ago in a work, where the Born approximation has been used directly on the structured reservoir, and a time-nonlocal master equation has been obtained [29]. However, as was discussed in Sec. III A, the Born approximation generally gives wrong times of transitions to new stationary states, and also wrong population values. In Ref. [29] it also led to an erroneous prediction of small deviations from the frozen values of the upper-state population for small temperatures, whereas master equation (38) leads to a physically consistent prediction of the same stationary value of the upper-state population for arbitrary initial states of the emitter. This stationary value depends only on the temperature. As it should be expected, eventually the emitter comes to equilibrium with the thermal bath. When the temperature goes to zero, the time interval required to reach the equilibrium goes to infinity. The spontaneous decay appears to be genuinely frozen.

Thus, the FEBS is destroyed by an incoherent driving. However, it might be enhanced by the incoherent driving as well. It sounds highly counterintuitive. But it is actually a direct consequence of the very fact of an eventual equilibrium with the reservoir. Indeed, it is known already that for the two-level emitter the FEBS might contain no more than a single photon [10,18]. For the zero-temperature reservoir, only a state satisfying condition $Cs_{st}=0$ can be the stationary solution of Eq. (38). Generally, such a state can be represented as

$$s_{st} = \sum_{i,j=1}^{2} s_{ij}|f_i\rangle\langle f_j|, \tag{41}$$

where the basis states are given by

$$|f_1\rangle = |\psi_0\rangle, \quad |f_2\rangle = |-\rangle|0\rangle.$$

Thus, when the incoherent driving is switched off, the system starts to decay. Eventually, it comes to state (41). This state is an entangled one with the concurrence proportional to $s_{11}$ (and, as a consequence, to the upper-state population) [10,18]. An interaction with the structured reservoir creates

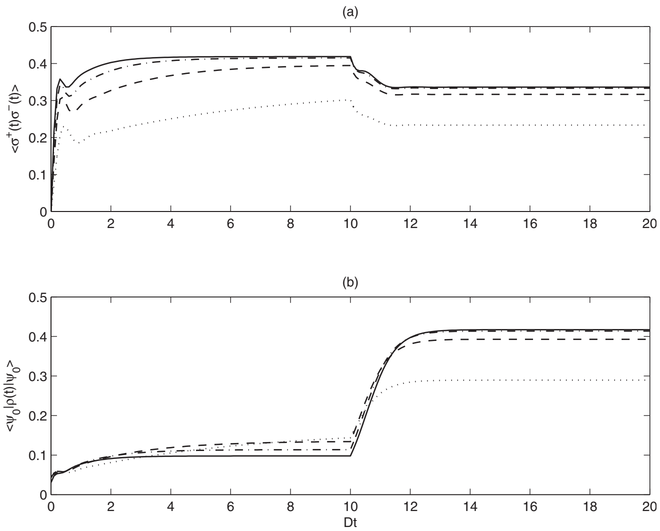


FIG. 7. Illustration of the excitation of the FEBS by the incoherent driving. The dynamics of the emitter's upper-state population is shown in (a); the projection of the system's state on the ideal frozen states is shown in (b). On both figures solid, dashed, dash-dotted, and dotted lines correspond to the following average numbers of photons in the reservoir modes $\bar{n}=4,3,2,1$, respectively. In both figures the driving is switched off at $Dt=10$.

the entanglement between the emitter and the field. As one can see in Fig. 7, the induced entanglement might be quite strong, since the matrix element $s_{11}$ grows strongly after switching off the driving.

Moreover, since in the high-temperature limit the upper-level population of the two-level emitter tends to the value 0.5, one might even, in principle, create by the incoherent excitation even the ideally entangled state (the ideally entangled "ideal frozen state" corresponds to $U_{00}=1/\sqrt{2}$ with the emitters population $U_{00}^4=1/4$). In Fig. 7 one can see that this is quite a realistic expectation.

To conclude, such a possibility to excite the FEBS with an incoherent driving leads to different possibilities of devising lasing devices on the basis of the FEBS. For such incoherent driving one does not need to spoil the structure with defects to produce bands of allowed states inside the gap. It is enough to excite available reservoir modes. Nevertheless, the FEBS is strongly affected by such an excitation. Since the FEBS is a cavity by itself (the field is localized in a volume of a few elementary cells around the emitter [2]), one does not need a localized defect to provide for the intensive coupling between the emitter and the field. The very FEBS exists due to its presence. The emitter might be located just in the extended waveguidelike defect designed to transfer the emitted radiation away from the PC.

## V. CONCLUSIONS

In this work we have derived the non-Markovian Lindblad time-local master equation for the interaction of a two-level emitter with a structured reservoir of field modes. We have considered reservoirs with a gap in the density of photonic states. For reservoirs at zero temperature the obtained equation is able to capture the freezing of the emitter's population decay. It is valid for thermal reservoirs at arbitrary temperatures. It predicts that for a finite temperature the FEBS is eventually destroyed. The emitter reaches the equilibrium with its surroundings and becomes decoupled from the field completely. However, by decreasing the temperature the time interval required for such a transition extends to infinity. We have found that for a few thermal photons in every reservoir's mode, the time of the transition to the thermal equilibrium is comparable to the time for FEBS formation. Thus, it might cause problems when one attempts to look for effects connected to the FEBS in PCs with band gaps opened in regions of the spectra corresponding longer wavelength, for example, in the microwave region at room temperature. But the process of the FEBS decay retains the non-Markovian, nonexponential character throughout the evolution.

We have also found that the FEBS might be effectively excited by an incoherent driving. After the driving is switched off, the system state decays to a mixture of the lowest state with the ideally frozen state. That resulting mixture can be an entangled state with high values of concurrence for the emitter-field entanglement. This feature opens new perspectives for devising active optical devices on the basis of the FEBS. It is to be emphasized that the FEBS is a "cavity" by itself. The field entangled with the emitter is localized in its vicinity. Furthermore, to excite the FEBS incoherently, one does not need to create bands of allowed states inside the gap.

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