Efficient and high-fidelity generation of atomic cluster states with cavity QED and linear optics

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(Received 9 January 2007; published 28 March 2007)

We propose a scheme to generate cluster states of atomic qubits by using cavity quantum electrodynamics (QED) and linear optics, in which each atom is confined in a resonant optical cavity with two orthogonally polarized modes. Our scheme is robust to imperfect factors such as dissipation, photon loss, and detector inefficiency. We discuss the experimental feasibility of our scheme.

DOI: 10.1103/PhysRevA.75.034308 PACS number(s): 03.67.Mn, 03.67.Lx, 42.50.Pq

Recently, much attention has been paid to entangled states for testing quantum nonlocality [1–3] and for achieving quantum-information processing with, for example, cavity QED [4–6], trapped ions [7], and free photons [8].

The focus of this work is on the generation of a cluster state [9], a special multipartite entangled state essential to one-way quantum computing [10]. It is considered that this quantum computing idea opens up a new paradigm for constructing reliable quantum computers by measurements [11–16]. We have noticed a recent experiment realizing cluster states using photonic qubits [16]. For one-way quantum computing, however, moving qubits are not good candidates in view of the problem of accurate manipulation of quantum states. The same problem also exists when using moving atoms [13]. Anyway, the technique to manipulate individual photonic polarization is much ahead of that for atoms. Due to this fact, we may consider static atoms combined with moving photons to carry out quantum-information processing. Based on this idea, Cho and Lee [14] have proposed a scheme to generate atomic cluster states through the cavity input-output process, in which the atoms as static qubits are trapped in cavities and a single moving photon is the medium. Considering the practical aspects of current single-photon techniques, the success probability of the scheme would be quite small due to uncontrollable imperfections, and the cascade characteristics may also limit the length of the cluster state. In particular, additional single-photon resources are required in that scheme, which is also an experimental challenge. In another paper [17], an efficient strategy using entangling operation to build cluster states was proposed. However, for each of the entangling operations, a double-heralded single-photon detection and two rotations of static qubits are required, which complicates the implementation and prolongs the operational time. This is not good, particularly for the case of low success rate of that scheme. In addition, the generation of four-atom cluster states proposed in [18] is strongly sensitive to quantum noise and therefore is difficult to extend to many-atom cases.

In this Brief Report, we propose an alternative scheme to generate cluster states using cavity decay and considering the symmetric use of polarizing beam splitters (PBSs) and photon detectors. The photons, emitted by the atoms, leaking out of the cavities and passing through the PBSs become entangled, and then detectors could be used to map the entanglement from the photons to the atoms. The favorable features of our scheme include the following. (1) The static qubits are in fully controllable atoms, and postselection by detectors makes the scheme robust to photon loss and other sources of error such as spontaneous emission, mismatch of cavity parameters and detector inefficiency. Although the dissipative factors reduce the success rate of our scheme, the fidelity of the generated cluster state is not affected. (2) The scheme is easily restarted. So, in the absence of dark counts of the detectors, we may achieve many-atom cluster states with simpler and faster operations than in previous proposals.

For convenience of our description, we will focus on four-qubit cluster states in most of the paper. Consider an atom with three degenerate excited states and three degenerate ground states as shown in Fig. 1, where we consider $(1,-1)=|g\rangle$, $(1,1)=|e\rangle$ in the ground states to be qubit states and $(1,0)=|\alpha\rangle$ in the excited states to be an ancillary state. The transitions $|\alpha\rangle\rightarrow|g\rangle$ and $|\alpha\rangle\rightarrow|e\rangle$ could be coupled by left- and right-polarized radiation, respectively. Suppose that we have four such atoms, with each confined in an optical cavity, as shown in Fig. 2. The cavities are identical and each of them has two orthogonally polarized modes to resonantly couple $|\alpha\rangle$ to $|g\rangle$ and to $|e\rangle$, respectively. As each cavity is

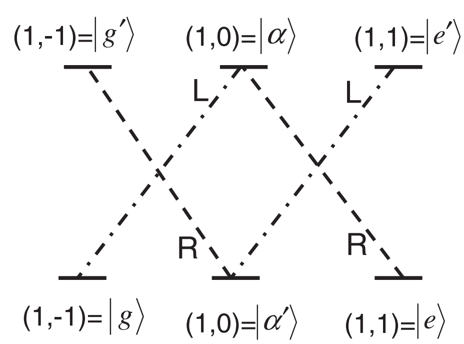

FIG. 1. Level configuration of the atoms. The dot-dashed and dashed lines denote the coupling by left- and right-polarized cavity fields, respectively.

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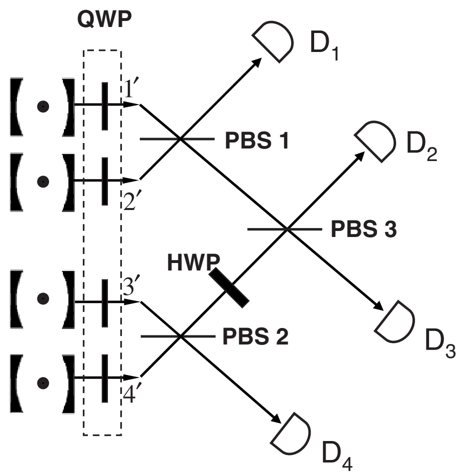

FIG. 2. Experimental setup for generation of a four-qubit cluster state. The bold lines in the dashed box are four quarter-wave plates (QWPs), which transform left- and right-polarized photons to be horizontally and vertically polarized, respectively. HWP is a half-wave plate working as a Hadamard gate, i.e., $H\rightarrow(1/\sqrt{2})(H+V)$ and $V\rightarrow(1/\sqrt{2})(H-V)$. PBS is a polarized beam splitter which transmits the state $|H\rangle$ and reflects the state $|V\rangle$. D_i $(i=1,2,3,4)$ are single-photon detectors.

one sided, the photons leaking away will reach the PBSs as we show in Fig. 2. We assume the cavities to be initially in the vacuum state and each atom to be initially in the state $|\alpha\rangle$, the latter of which could be achieved by pumping from the ground state $|\alpha'\rangle$ by a resonant π-polarized laser pulse before the scheme gets started. So the initial state of the whole system could be written as $|\psi(0)\rangle=\Pi_{k=1}^{4}|\alpha,0_L,0_R\rangle_k$, where $|\cdots\rangle_k$ denotes the atomic state and the left- and right-polarized modes of the optical cavity k, respectively. In the interaction picture and under the rotating-wave approximation, the Hamiltonian in the kth $(k=1,2,3,4)$ cavity is (assuming $\hbar=1$) [19]

$$H_k=\frac{h_k}{2}[a_{k,L}^{\dagger}(|g\rangle_{kk}\langle\alpha|+|a'\rangle_{kk}\langle e'|)+a_{kR}^{\dagger}(|e\rangle_{kk}\langle\alpha|+|a'\rangle_{kk}\langle g'|)+\text{H.c.}], \tag{1}$$

where $a_{kL}^{\dagger}$ and $a_{k,R}^{\dagger}$ create left- and right-polarized photons, respectively, in the kth cavity. For simplicity, we have assumed that the coupling strengths between the cavity modes and their corresponding trapped atoms (i.e., for transitions $|\alpha\rangle\rightarrow|g\rangle$, $|\alpha\rangle\rightarrow|e\rangle$ and $|g'\rangle\rightarrow|\alpha'\rangle$, $|e'\rangle\rightarrow|\alpha'\rangle$) have the constant value h_k. This can be reached by setting the relevant Clebsch-Gordan coefficients to be $C_{\alpha,g}=C_{\alpha,e}=C_{g',\alpha'}=C_{e',\alpha'}=1/\sqrt{2}$. Considering weak cavity decay and weak spontaneous emission from the excited states under the condition that no dissipation actually occurs during our implementation of the scheme, we may describe the system governed by a non-Hermitian Hamiltonian as follows (assuming $\hbar=1$):

$$H_k=\frac{h_k}{2}[a_{k,L}^{\dagger}(|g\rangle_{kk}\langle\alpha|+|a'\rangle_{kk}\langle e'|)+a_{kR}^{\dagger}(|e\rangle_{kk}\langle\alpha|+|a'\rangle_{kk}\langle g'|)+\text{H.c.}]-i\frac{\gamma}{2}(|a\rangle_{kk}\langle a|+|g'\rangle_{kk}\langle g'|+|e'\rangle_{kk}\langle e'|)-i\kappa(a_{kL}^{\dagger}a_{kL}+a_{kR}^{\dagger}a_{kR}), \tag{2}$$

where γ is represents the spontaneous emission from the excited states and 2κ accounts for the one-sided decay rate of the kth cavity. For simplicity, we have assumed the same rates regarding spontaneous emissions from different excited levels and the same decay rate for each mode of the cavities. This assumption is for reaching maximum implementation efficiency of our scheme discussed below, because our scheme would be affected by differently shaped wave packets in the case of different γ and κ for different atoms and cavities. After an evolution time t from the initial state $|\psi(0)\rangle=\Pi_{k=1}^{4}|\alpha,0_L,0_R\rangle_k$, the system evolves to an entangled state

$$|\psi(t)\rangle=\prod_{k=1}^{4}\exp\left(-\frac{\kappa+\gamma/2}{2}t\right)\left[\left(\frac{(\kappa-\gamma/2)}{\beta}\frac{e^{\beta t}-e^{-\beta t}}{2}+\frac{e^{\beta t}+e^{-\beta t}}{2}\right)|\alpha,0_L,0_R\rangle_k-i\frac{h_k}{\beta}\frac{e^{\beta t}-e^{-\beta t}}{2}(|g,0_L,1_R\rangle_k+|e,1_L,0_R\rangle_k)\right], \tag{3}$$

with

$$\beta=\frac{1}{2}\sqrt{\left(\kappa+\frac{\gamma}{2}\right)^2-2(\gamma\kappa+h_k^2)}.$$

So, due to dissipative factors, a left-polarized or right-polarized photon is created with the success probability

$$P_k=\exp\left[-\left(\kappa+\frac{\gamma}{2}\right)t\right]\left(h_k\frac{e^{\beta t}-e^{-\beta t}}{2\sqrt{2}\beta}\right)^2. \tag{4}$$

Once the deexcitation actually happens, before reaching the PBSs 1 and 2 as shown in Fig. 2, each of the photons has to pass through a quarter-wave plate (QWP), which transforms left- and right-polarized photons to be horizontally (H) and vertically (V) polarized, respectively. Thus the whole system reaches the state

$$|\psi_1\rangle=\frac{1}{4}\bigotimes_{k=1'}^{4'}(|g\rangle_k|H\rangle_k+|e\rangle_k|V\rangle_k),$$

where the subscripts $1'$, $2'$, $3'$, and $4'$ label positions after the action of QWP as shown in Fig. 2. We have dropped time in the above equation because the expression can be written formally without including time. As the PBS plays the role of a parity check on the input photons, the detection of a photon at each output port projects the state $|\psi_1\rangle$ into an entangled state of the four atoms, which, including the action by a half-wave plate, is

$$|\psi_2\rangle = \frac{1}{2}(|gggg\rangle_{1234} + |eegg\rangle_{1234} + |ggee\rangle_{1234} - |eeee\rangle_{1234}). \tag{5}$$

This means that, once we have a click in each of the detectors D_1, D_2, D_3, and D_4, we have generated the cluster state of the four atoms. Please note that the state in Eq. (5) is actually equivalent, under a Hadamard transformation $H = (1/\sqrt{2})(\sigma_x+\sigma_z)$ on the first and last atoms, to the cluster state defined in [9] with $N=4$, i.e., $|\phi_4\rangle = \frac{1}{4}4\otimes_{a=1}^{4}(|e\rangle_a\sigma_z^{(a+1)} + |g\rangle_a)$.

If we do not have a click in each of the four detectors during a waiting time, e.g., $3/k$, which means failure of our implementation, we have to restart the scheme. This could be done by the following steps as follows. (1) Excite $|g\rangle$ to $|g'\rangle$ or $|e\rangle$ to $|e'\rangle$ by a π-polarized laser pulse. (2) The dissipation induced by the cavity modes changes both $|g'\rangle$ and $|e'\rangle$ to $|\alpha'\rangle$. (3) Excite $|\alpha'\rangle$ to $|\alpha\rangle$ by a π-polarized laser pulse. Then our scheme is ready to be done again. Actually, the above steps are also useful for fusing two cluster states into a bigger one. Consider two independent four-qubit cluster states as described above located in dot-dashed boxes, respectively, in Fig. 3. We first perform the transformation $|g\rangle \to |g'\rangle$ and $|e\rangle \to |e'\rangle$ on the last qubit of one of the cluster states (e.g., in block I labeled in Fig. 3) and on the first qubit in another (e.g., in block II in Fig. 3). Before both the detectors D_{I} and D_{II} click, the total state of the system is

$$|\tilde{\psi}_1\rangle_{\mathrm{I,II}} = \frac{1}{4}(|ggg\alpha'\rangle_{\mathrm{I}}|V\rangle_{\mathrm{I}} + |eeg\alpha'\rangle_{\mathrm{I}}|V\rangle_{\mathrm{I}} + |gge\alpha'\rangle_{\mathrm{I}}|H\rangle_{\mathrm{I}} - |eee\alpha'\rangle_{\mathrm{I}}|H\rangle_{\mathrm{I}}) \otimes (|\alpha' ggg\rangle_{\mathrm{II}}|V\rangle_{\mathrm{II}} + |\alpha' egg\rangle_{\mathrm{II}}|H\rangle_{\mathrm{II}} + |\alpha' gee\rangle_{\mathrm{II}}|V\rangle_{\mathrm{II}} - |\alpha' eee\rangle_{\mathrm{II}}|H\rangle_{\mathrm{II}}), \tag{6}$$

where we have considered the action of the QWP. After both detectors have fired, we reach a six-atom entangled state,

$$|\tilde{\psi}_2\rangle_{\mathrm{I,II}} = \frac{1}{2\sqrt{2}}(|gggggg\rangle + |eegggg\rangle + |ggggee\rangle + |eeggee\rangle + |ggeegg\rangle - |eeeegg\rangle - |ggeeee\rangle + |eeeeee\rangle), \tag{7}$$

where we have omitted the product states $|\alpha'\rangle$ of the last atom of the cluster state I and the first atom of the cluster state II. So the entangled state exists in only six atoms. By carrying out a Hadamard transformation on the first and last of the six atoms, respectively, we get to a cluster state with six atoms. In this way we can generate a many-qubit cluster state, for example, of length $N+M-2$ from two cluster states of respective lengths N and M.

We now give a brief discussion about the experimental implementation of our scheme. The level configuration under our consideration in Fig. 1 can be found in ^{87}Rb or ^{171}Yb$^+$; for example, the level with $F=1$ (e.g., $5\,^2S_{1/2}$ of ^{87}Rb or $6\,^2S_{1/2}$ of ^{171}Yb$^+$) acts as the ground state and the excited state could be $5\,^2P_{3/2}$ of ^{87}Rb or $6\,^2P_{1/2}$ of ^{171}Yb$^+$. We consider ^{87}Rb confined in an optical cavity such as Cs in [20]. Although the atom is moving, as it moves very slowly with respect to the photon and is well controllable, the atom can be considered as a good carrier of static qubits. Alternatively,

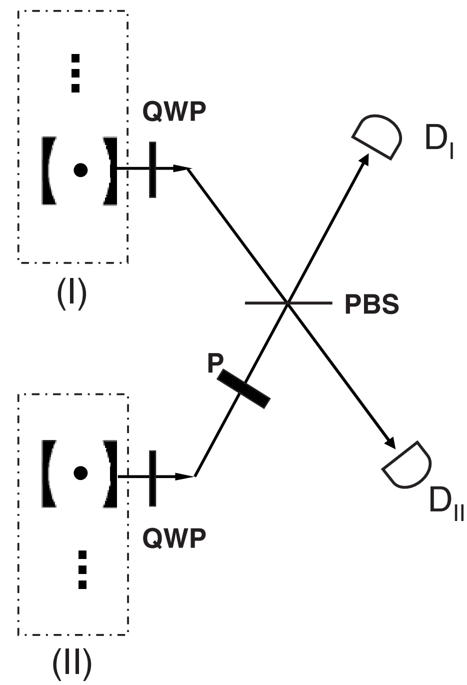

FIG. 3. Schematic for fusing two cluster states into a larger one. In each of the dot-dashed boxes there is a four-qubit cluster state. P represents a unitary rotator for the operation $\sigma_x H \sigma_x = (1/\sqrt{2})(\sigma_x - \sigma_z)$.

we may suppose the atom ^{87}Rb to be fixed by an optical lattice embedded in a cavity, as was done in [21] for an ensemble of ^{87}Rb. Using the numbers in [21] with the coupling strength $h_k = 2\pi \times 27$ MHz and the cavity decay rate $\kappa = 2\pi \times 2.4$ MHz, and supposing the atomic decay rate $\gamma = 2\pi \times 6$ MHz, we have the success probability 0.208 for all four cavities to emit photons simultaneously. Moreover, we may employ ^{171}Yb$^+$ in an ion trap which is embedded in an optical cavity. This case has been demonstrated experimentally using Ca$^+$ [22]. If we assume the coupling strength to be $h_k \sim 30$ MHz and the cavity decay rate κ to be about ten times smaller than the coupling strength [23], we have a success probability of our scheme larger than 0.16 in the case of the atomic decay rate $\gamma < 10$ MHz. As ^{171}Yb$^+$ could be well localized in individual ion traps for a long time, such a system is very suitable for our scheme. We expect in future experiments higher-Q cavities, lower spontaneous emission rate, and larger cavity-atom coupling to increase the success rate of our scheme.

Since the quantum logical operation with photons is basically probabilistic, when the photons go through each PBS, the success probability will decrease by one-half. As a result, a cluster state of four atoms is obtained in an ideal implementation of our scheme only with the success rate $1/8$, which is the same as in [17]. However, compared to [17] with additional operations necessary on the atoms, our scheme is much simpler in the many-atom case. Moreover, we may compare with [14], a previous scheme to generate cluster states with sequential single-photon interactions with different cavities. The success probability of that scheme is proportional to $(1-\eta)^{2n}$, with η the loss rate of the single photon and n the number of components of the cluster state. In contrast, in the case of large loss rate η, e.g., large inefficiency of the fiber, our scheme is more efficient with the success probability proportional to $(1-\eta)^n$. Moreover, the implementation of our scheme is faster because the photons leave the cavities in parallel. An additional improvement over [18] is that our scheme can be directly extended to the preparation of cluster states of any size, or two- or three-dimensional cluster states, which are prerequisites for meaningful quantum computation. However, to achieve a scalable scheme the effect of dark counts of the detectors should be seriously considered. For a normal dark count rate of 100 Hz, the dark count probability for single photon in our scheme is estimated to be 10^{-5}, which would be significant in the case of thousands of detections. We hope future technical advances will overcome this difficulty.

In summary, we have presented a scheme to generate atomic cluster states by using cavity QED, linear optical elements, and photon detection. The distinct advantages of our scheme are that the fidelity of the generated state is insensitive to quantum noise and detection inefficiency, and the scheme is easily restarted and in principle scalable. Therefore, despite the imperfect factors, the relaxation of experimental requirements makes our scheme achievable with current techniques.

This work is supported in part by National Natural Science Foundation of China under Grants No. 10474118 and No. 60490280, by Hubei Provincial Funding for Distinguished Young Scholars, and by the National Fundamental Research Program of China under Grants No. 2005CB724502 and No. 2006CB921203.

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