Thermal entanglement and teleportation in a two-qubit Heisenberg chain with Dzyaloshinski-Moriya anisotropic antisymmetric interaction

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(Received 22 January 2007; revised manuscript received 28 February 2007; published 22 March 2007)

Thermal entanglement of a two-qubit Heisenberg chain in the presence of the Dzyaloshinski-Moriya (DM) anisotropic antisymmetric interaction and entanglement teleportation when using two independent Heisenberg chains as the quantum channel are investigated. It is found that the DM interaction can excite entanglement and teleportation fidelity. The output entanglement increases linearly with increasing value of the input; its dependences on the temperature, DM interaction, and spin coupling constant are given in detail. Entanglement teleportation will be better realized via an antiferromagnetic spin chain when the DM interaction is turned off and the temperature is low. However, the introduction of the DM interaction can cause the ferromagnetic spin chain to be a better quantum channel for teleportation. A minimal entanglement of the thermal state in the model is needed to realize the entanglement teleportation regardless of whether the spin chains are antiferromagnetic or ferromagnetic.

DOI: 10.1103/PhysRevA.75.034304 PACS number(s): 03.67.Hk, 03.65.Ud, 75.10.Jm

I. INTRODUCTION

Entanglement is one of the most fascinating features of quantum mechanics and plays a central role in quantum-information processing. In recent years, there has been an ongoing effort to characterize qualitatively and quantitatively the entanglement properties of condensed matter systems and apply them in quantum information. The quantum entanglement in solid state systems such as spin chains is an important emerging field [1–8]. Spin chains are natural candidates for the realization of entanglement compared with other physics systems. The Heisenberg chain, the simplest spin chain, has been used to construct a quantum computer [9]. By suitable coding, the Heisenberg interaction alone can be used for quantum computation [10–12]. In addition, quantum teleportation has been extensively investigated both experimentally and theoretically. Since decoherence from the environment always impacts on the degree of entanglement, the resource of maximally entangled states is hard to prepare in a real experiment. Certainly, a mixed entangled state as the resource is approximately near to the real circumstances. As an important source of entanglement, thermal entanglement has been widely investigated in many previous studies. Also entanglement teleportation via thermal entangled states of a two-qubit Heisenberg *XX* chain has been reported [13]. Yeo *et al.* [14] studied the influence of anisotropy and magnetic field on quantum teleportation via a Heisenberg *XY* chain. But only the spin-spin interaction was considered in those studies; the effects of spin-orbit coupling on the entanglement and teleportation are rarely included. These are the motivations of this Brief Report.

Here, we investigate the influence of spin-orbit coupling on thermal entanglement. The information transmission by a pair of thermal mixed states in a two-qubit Heisenberg chain in the presence of the Dzyaloshinski-Moriya (DM) anisotropic antisymmetric interaction is investigated. A minimal entanglement in the quantum channel is needed to transfer entanglement information. Thermal entanglement will be investigated in Sec. II. The entanglement teleportation of two-qubit pure states and its fidelity are derived in Secs. III and IV. In Sec. V a discussion concludes the paper.

II. THE EFFECT OF DM INTERACTION ON THERMAL ENTANGLEMENT

In this paper, we consider the Heisenberg model with DM interaction, which can be described by

$$H_{DM} = \frac{J}{2}[(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \sigma_{1z}\sigma_{2z}) + \vec{D}\cdot(\vec{\sigma}_1 \times \vec{\sigma}_2)]; \tag{1}$$

here J is the real coupling coefficient and $\vec{D}$ is the DM vector coupling. The DM anisotropic antisymmetric interaction arises from spin-orbit coupling [15,16]. The coupling constant $J>0$ corresponds to the antiferromagnetic case and

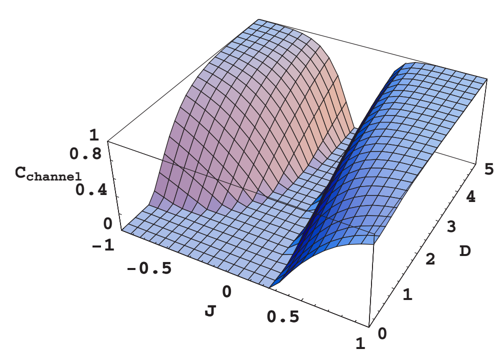

FIG. 1. (Color online) Thermal concurrence for the spin channel when $T=0.5$. T is plotted in units of the Boltzmann constant k. We work in units where D and J are dimensionless.

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$J<0$ to the ferromagnetic case. For simplicity, we choose $\vec{D}=D\vec{z}$. Then the Hamiltonian H_{DM} becomes

$$\begin{aligned} H_{DM} &= \frac{J}{2}[\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \sigma_{1z}\sigma_{2z} + D(\sigma_{1x}\sigma_{2y} - \sigma_{1y}\sigma_{2x})] \\ &= J[(1+iD)\sigma_{1+}\sigma_{2-} + (1-iD)\sigma_{1-}\sigma_{2+}]. \end{aligned} \tag{2}$$

Without loss of generality, we define $|0\rangle$ ($|1\rangle$) as the ground (excited) state of a two-level particle. The eigenvalues and eigenvectors of H_{DM} are given by

$$H_{DM}|00\rangle = \frac{J}{2}|00\rangle,$$

$$H_{DM}|11\rangle = \frac{J}{2}|11\rangle,$$

$$H_{DM}|+\rangle = \left(J\sqrt{1+D^2} - \frac{J}{2}\right)|+\rangle,$$

$$H_{DM}|-\rangle = \left(-J\sqrt{1+D^2} - \frac{J}{2}\right)|-\rangle, \tag{3}$$

where $|\pm\rangle=(1/\sqrt{2})(|01\rangle \pm e^{i\theta}|10\rangle)$ and $\theta=\arctan D$.

As thermal fluctuation is introduced into the system, the state of a typical solid state system at thermal equilibrium (temperature T) is $\rho(T)=(1/Z)e^{-\beta H}$, where H is the Hamiltonian and $Z=\mathrm{tr}\, e^{-\beta H}$ is the partition function. In the standard basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$, the density matrix $\rho(T)$ can be expressed as

$$\rho(T) = \frac{1}{Z}\begin{pmatrix} e^{-\beta J/2} & 0 & 0 & 0 \\ 0 & \frac{1}{2}e^{\beta(J-\delta)/2}(1+e^{\beta\delta}) & \frac{1}{2}e^{i\theta}e^{\beta(J-\delta)/2}(1-e^{\beta\delta}) & 0 \\ 0 & \frac{1}{2}e^{-i\theta}e^{\beta(J-\delta)/2}(1-e^{\beta\delta}) & \frac{1}{2}e^{\beta(J-\delta)/2}(1+e^{\beta\delta}) & 0 \\ 0 & 0 & 0 & e^{-\beta J/2} \end{pmatrix}, \tag{4}$$

where $Z=2e^{-\beta J/2}[1+e^{\beta J}\cosh(\beta\delta/2)]$, $\beta=1/kT$, and $\delta=2J\sqrt{1+D^2}$. In the following calculation, we will write the Boltzmann constant $k=1$. The entanglement of two qubits can be measured by the concurrence C, which is defined as $C=\max[0, 2\max(\lambda_i) - \Sigma_i^4 \lambda_i]$ [17], where λ_i are the square roots of the eigenvalues of the matrix $R=\rho S \rho^* S$, ρ is the density matrix, $S=\sigma_{1y}\otimes\sigma_{2y}$, and the asterisk stands for the complex conjugate. The concurrence is available no matter whether ρ is pure or mixed. Note that we are working in units where D and J are dimensionless.

Based on the definition of concurrence, we can obtain the concurrence at finite temperature:

$$C_{channel} \equiv C[\rho(T)] = \frac{2}{Z}\max\left(\frac{1}{2}|e^{\beta(J-\delta)/2}(1-e^{\beta\delta})| - e^{-\beta J/2}, 0\right). \tag{5}$$

The concurrence $C=0$ indicates vanishing entanglement. The critical temperature T_c above which the concurrence is zero is determined by the nonlinear equation

$$e^{J/T}\sinh\frac{\delta}{2T} = -1 \quad \text{if } J<0,$$

$$e^{J/T}\sinh\frac{\delta}{2T} = 1 \quad \text{if } J>0, \tag{6}$$

which can be solved numerically. When $D=0$, i.e., $\delta=2J$, it is found that $T_c=2J/\ln 3$ for $J>0$, but there is no entanglement at any temperature for $J<0$. These accord with the conclusions in Ref. [18]. Figure 1 demonstrates the dependence of thermal entanglement on J and D at $T=0.5$. Although there is no entanglement for the ferromagnetic case when $D=0$, when D increases, entanglement will be inspired and the area of J for which $C=0$ will decrease. The entanglement can reach a maximum value by adjusting the DM interaction constant for the two cases.

III. THERMAL ENTANGLEMENT TELEPORTATION

For the entanglement teleportation of a whole two-qubit system of a thermal mixed state in a Heisenberg spin chain, the standard teleportation through mixed states can be regarded as a general depolarizing channel [19,20]. Similar to standard teleportation, entanglement teleportation for the mixed channel of an input entangled state is destroyed and its replica state appears at the remote place after applying a local measurement in the form of linear operators. We consider as input a qubit in an arbitrary pure state $|\psi\rangle_{in}=\cos(\theta/2)|10\rangle + e^{i\phi}\sin(\theta/2)|01\rangle$ $(0\leq\theta\leq\pi, 0\leq\phi\leq 2\pi)$. Here different values of θ describe all states with different amplitudes, and ϕ stands for the phase of these states. The output state is then given by [21]

$$\rho_{out} = \sum_{ij} p_{ij}(\sigma_i\otimes\sigma_j)\rho_{in}(\sigma_i\otimes\sigma_j), \tag{7}$$

where σ_i $(i=0,x,y,z)$ signify the unit matrix I and three components of the Pauli matrix $\vec{\sigma}$, respectively, $p_{ij} = \mathrm{tr}[E^i\rho(T)]\mathrm{tr}[E^j\rho(T)]$, $\Sigma_{ij}p_{ij}=1$, and $\rho_{in}=|\psi\rangle_{in}\langle\psi|$. Here $E^0 = |\Psi^-\rangle\langle\Psi^-|$, $E^1=|\Phi^-\rangle\langle\Phi^-|$, $E^2=|\Phi^+\rangle\langle\Phi^+|$, $E^3=|\Psi^+\rangle\langle\Psi^+|$, in which $|\Psi^\pm\rangle=(1/\sqrt{2})(|01\rangle \pm |10\rangle)$, $|\Phi^\pm\rangle=(1/\sqrt{2})(|00\rangle \pm |11\rangle)$.

It follows that the concurrence of the initial state $|\psi\rangle_{in}$ is $C_{in}=2|\sin(\theta/2)\cos(\theta/2)e^{i\phi}|$. We calculate the measure of entanglement for the teleported state ρ_{out} to be

$$C[\rho_{out}] \equiv C_{out} = \max\left(\frac{2\{C_{in}e^{\beta J}[\sinh(\beta\delta/2)]^2 - 2(1+D^2)\cosh(\beta\delta/2)\}}{Z^2(1+D^2)},0\right). \tag{8}$$

From Eq. (8), it can be seen that C_{out} increases linearly with increasing C_{in}. The result can also be seen from Figs. 2(a) and 3.

The quantity C_{out} as a function of C_{in} is plotted in Fig. 2 when the DM interaction D, the temperature T, and the coupling coefficient J are changed. Figure 2(a) is a plot of C_{out} as a function of C_{in} and T when $D=0$ and $J=1$, for which the critical temperature of the channel concurrence $T_c=2J/\ln 3 = 2/\ln 3 \approx 1.82$. From Fig. 2(a), we know that C_{out} remains zero when $T>1$, so a minimal entanglement of the thermal mixed state must be provided in this quantum channel in order to realize entanglement teleportation. When the initial state is in a maximum entangled state, which corresponds to Fig. 2(b), C_{out} exists regardless of the sign of J. For $J<0$, first the output entanglement increases with increasing D from zero to a certain value that is much smaller than C_{in} and then it begins to fall to zero. However, C_{out} decreases monotonically with increasing D for $J>0$. It may be advantageous for increasing C_{out} and the channel entanglement $C_{channel}$ to introduce the DM interaction for $J<0$; however, when $J>0$ the DM interaction can only cause $C_{channel}$ to increase. As the DM interaction increases, C_{out} will decrease to zero when D is large for both $J>0$ and $J<0$. This is due to the fact that $C_{out}=\max[-4\cosh(\beta\delta/2)/Z^2,0]=0$ when $D\to\infty$. The maximum value of C_{out} is much smaller than that of the channel entanglement. Under general circumstances, the output entanglement of a two-qubit state $|\psi\rangle_{in}$ will decrease via the quantum channel. These results can be found by comparing Fig. 2 with Fig. 1 [22].

Figure 3 shows the dependence of C_{out} on C_{in} and spin coupling J for a given DM interaction and temperature. As the channel concurrence shows, C_{out} behaves obviously differently for $J>0$ and $J<0$. For $J<0$, only when $|J|>0.5$ is C_{out} nonvanishing when the initial state is a maximum entangled state. If $0<C_{in}<1$, $|J|$ must be larger in order to realize entanglement teleportation. We can see that $C_{channel} \approx 0.597$ for $J=-0.5$ in the same conditions in Fig. 3; these results show again that a minimum entanglement must be provided. The same conclusion can be obtained for $J>0$.

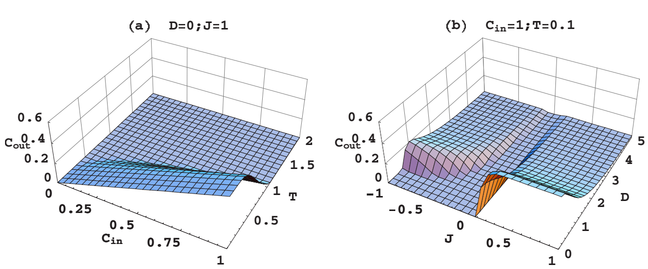

FIG. 2. (Color online) Teleported thermal concurrence C_{out} as a function of the input concurrence C_{in}, DM interaction D, spin coupling J, and temperature T. T is plotted in units of the Boltzmann constant k. We work in units where D and J are dimensionless.

IV. THE FIDELITY OF ENTANGLEMENT TELEPORTATION

To characterize the quality of the teleported state ρ_{out}, it is often quite useful to look at the fidelity between ρ_{out} and ρ_{in} defined by [23]

$$F(\rho_{in},\rho_{out}) = \{\mathrm{tr}[\sqrt{(\rho_{in})^{1/2}\rho_{out}(\rho_{in})^{1/2}}]\}^2. \tag{9}$$

The concept of fidelity has been a useful indicator of the teleportation performance of a quantum channel when the input state is a pure state. The average fidelity F_A of teleportation can be formulated as

$$F_A = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} F \sin\theta\, d\theta}{4\pi}. \tag{10}$$

If our model is used as the quantum channel, F_A can be expressed as

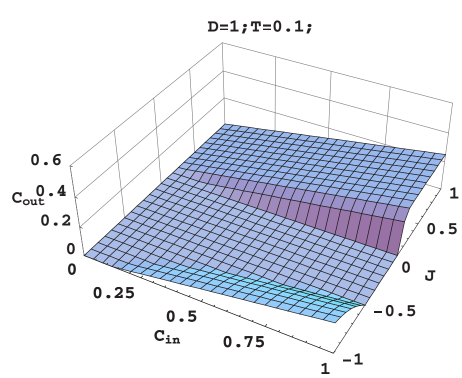

FIG. 3. (Color online) Teleported thermal concurrence C_{out} as a function of the input concurrence C_{in} and spin coupling J when temperature $T=0.1$ and DM interaction $D=1$. T is plotted in units of the Boltzmann constant k. We work in units where D and J are dimensionless.

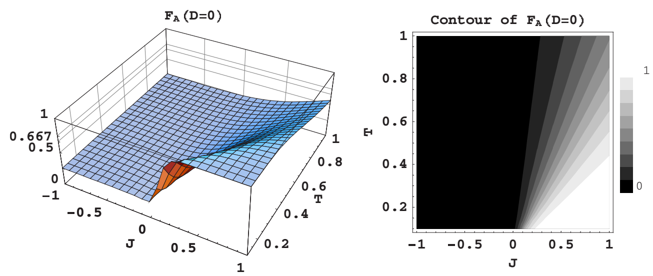

FIG. 4. (Color online) Average fidelity F_A as a function of spin coupling J and temperature T when $D=0$. T is plotted in units of the Boltzmann constant k. We work in units where D and J are dimensionless.

$$F_A = \frac{2(1+D^2) + e^{2\beta J}[1 + 2D^2 + (3+2D^2)\cosh \beta\delta]}{6(1+D^2)[1 + e^{\beta J}\cosh(\beta\delta/2)]^2}. \tag{11}$$

This is the maximal fidelity achievable from $\rho(T)$. In order to transmit $|\psi\rangle_{in}$ with better fidelity than any classical communication protocol, we require Eq. (11) to be strictly greater than 2/3 ($\approx$0.667). When $D=0$, this requirement becomes $e^{2\beta J}>11$.

The average fidelity F_A is plotted as a function of spin coupling J and temperature T when $D=0$ in Fig. 4. When $D=0$, F_A is larger than 2/3 if $0<T<2J/\ln 11$. So F_A is always smaller than 2/3 for $J<0$ at any temperature. However, for $J>0$, F_A can become 1 at near zero temperature and begins to fall to 2/3 at the point $T=2J/\ln 11$. This means that the entanglement teleportation of the mixed channel is inferior to the classical communication when $J<0$ without DM interaction. Figure 5 gives the dependence of F_A on the DM interaction D and spin coupling J. By introducing the DM interaction, F_A can be larger than 2/3 for $J<0$ (for example, $T=0.1$, $J\in[-0.5,-1]$). For $J>0$, F_A decreases monotonically with increasing D. When the DM interaction is very strong, F_A approaches infinitely close to the value of 2/3 for both cases. These results show that we must strengthen the DM interaction to be a certain value in order to use ferromagnetic spin chain as a quantum channel for entanglement teleportation, which is contrary to antiferromagnetic spin chain.

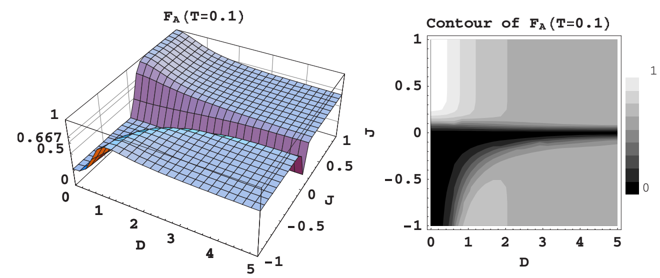

FIG. 5. (Color online) Average fidelity F_A as a function of J and D for a given temperature. T is plotted in units of the Boltzmann constant k. We work in units where D and J are dimensionless.

V. CONCLUSIONS

We have investigated the thermal entanglement of a two-qubit spin chain with DM anisotropic antisymmetric interaction and entanglement teleportation via the model. The entanglement can reach a maximum value by adjusting the DM interaction constant for the ferromagnetic and antiferromagnetic cases. By introducing the DM interaction, the output entanglement and fidelity can be increased for the ferromagnetic case, contrary to the antiferromagnetic case. When the DM interaction is very strong, the average fidelity of entanglement teleportation will approach a fixed value that is the maximal one for classical communication. A minimal entanglement of the thermal state in the model is needed to realize the entanglement teleportation.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Grant No. 10604053).

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