Spectral anomalies and stability of chirped-pulse oscillators

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Comprehensive numerical analysis of chirped-solitary-pulse stability in the positive-dispersion regime is presented. It is found that such a regime allowing generation of femtosecond pulses with energy above the microjoule level directly from a mode-locked oscillator is unstable against long-period pulsations. Pulsations are caused by the internal modes of solitary pulse and result in both symmetrical and asymmetrical perturbations of the pulse spectrum. This reduces a region of pulse stability and its coherence.

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I. INTRODUCTION

In the last decade, femtosecond pulse technology has evolved rapidly and allowed the achievement of few-optical-cycle pulse generation directly from an oscillator [1]. Applications of such pulses range from medicine and micromachining to the fundamental physics of light-matter interaction at unprecedented intensity and time levels [2–4]. To achieve these levels, pulse energies about or above the microjoule level are required. Such energy frontiers have become achievable due to chirped-pulse amplification outside an oscillator [1,2]. However, this technology is complex and expensive. Therefore, it is desirable to find a road to direct over-μJ femtosecond pulse generation without need of extra amplification.

Standard femtosecond pulse oscillators allow pulses with energy of merely a few nano-joules. A prospective recipe for increasing the pulse energy is to substantially increase the oscillator length [5,6]. In this case the pulse energy E is scalable as $E=P_{av}T_{cav}$, where T_{cav} is the oscillator cavity period, and P_{av} is the average power, which is limited by the pump power (e.g., $\approx 1-4$ W for a Ti:sapphire oscillator). However, such a long-cavity oscillator operating in the solitonic regime [7] suffers from strong instabilities [8]. They result from scalability of the pulse peak power with energy, when the pulse width is kept at the femtosecond level. This scalability enhances the nonlinear effects within an oscillator, with subsequent pulse destruction.

It was found that pulse stretching (up to several picoseconds) can provide substantial pulse energy growth [9–11]. This stretching results from a large pulse chirp, which develops in an oscillator [a so-called chirped-pulse oscillator (CPO)] operating in the positive-dispersion regime (PDR). As a consequence of the large chirp, the pulse has a high energy (more than 100 nJ), but its peak power is reduced below the instability threshold. This leads to stable operation of the CPO, whereas the broad spectrum ($\approx$100 nm for a Ti:sapphire oscillator) allows compressing the pulse down to below 30 fs.

Analysis has demonstrated that the chirped pulse developed in the PDR can be described as the stationary (solitary) solution [so-called chirped solitary pulse (CSP)] of the cubic-quintic nonlinear complex Ginzburg-Landau equation (CGLE) [12,13]. It is known that this equation is a direct generalization of the master mode-locking equation [7,14–16] and provides an adequate description of mode-locked oscillators (both fiber and solid state). Therefore, an investigation of the CSP properties is of interest not only from the theoretical but also from the practical point of view.

In this work we analyze the CSP stability and its spectral characteristics numerically and demonstrate that the CSP has an instability causing pulse spectrum perturbations and peak-power pulsations. We associate this instability with the internal modes of the CSP and the spontaneous breaking of its symmetry. The obtained results are in good agreement with the experimental data presented in Refs. [10,13].

II. CSP SOLUTION OF CGLE

A mode-locked oscillator operates under the influence of five main factors: (i) saturable gain and linear loss, (ii) spectral filtering, (iii) group-delay dispersion (GDD), (iv) self-phase-modulation (SPM), and (v) self-amplitude-modulation (SAM). The last factor is decisive for pulse formation and can be provided by an intracavity passive modulator (e.g., a semiconductor saturable absorber), imperfect overlapping between the pump and oscillator modes inside an active medium [Kerr-lens mode locking (KLM)], or some other mechanism (for an overview see [7]). If the contribution of the higher-order dispersions is negligible and the pulse does not change substantially during one cavity round trip, it is possible to describe the mode-locking dynamics on the basis of the cubic-quintic nonlinear CGLE [13,16]:

$$\frac{\partial A(z,t)}{\partial z} = \left(\sigma + (\alpha + i\beta)\frac{\partial^2}{\partial t^2} + (\kappa - i\gamma)P(z,t) - \kappa\varsigma P(z,t)^2\right)A(z,t), \quad (1)$$

where A is the slowly varying (in comparison with the light-wave period) field amplitude, $P \equiv |A|^2$ is the power, z is the

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cavity round-trip number, and t is the local time. The parameter α describes the contribution of the finite spectral width of a gain band (e.g., for a Ti:sapphire oscillator $\alpha \approx 1.1$ fs^2 [13]), and β is the net GDD coefficient. The parameter γ describes SPM inside an active medium (for a solid-state oscillator), a fiber (fiber oscillator), or air (solid-state oscillator with a thin disk active medium). For a solid-state oscillator the maximum value of this parameter is $\gamma_{max} = (2\pi\lambda_0)^2 n_2 n_0$ (λ_0 is the oscillator central wavelength; n and n_2 are the linear and nonlinear refraction coefficients, respectively) [13]. Both κ and ζ are parameters of SAM. The parameter κ describes an effective gain growth with P. For KLM oscillators such a growth results from a better overlapping between the pump and oscillator beams due to self-focusing inside an active medium. In this case, the κ parameter amounts to a few percent of the γ value. The parameter ζ describes the SAM saturation and confines the maximum peak power in an oscillator. A rough estimation for its minimum value gives 0.16γ [13].

The parameter σ is the net gain, which is the difference between the saturated gain and the linear net loss. As a result of the gain saturation, this parameter depends on the energy E. Since $|\sigma(E)| \ll 1$ [13], it is possible to expand it around 0: $\sigma(E) \approx (d\sigma/dE)|_{E=E^*} E^* (E/E^* - 1)$, where E^* corresponds to the energy stored inside an oscillator in the continuous-wave (cw) regime. The parameter $\delta \equiv (d\sigma/dE)|_{E=E^*} E^*$ depends on only the gain and loss coefficients [13].

As was found in [12,13], Eq. (1) has a CSP solution when the GDD is positive. If $\alpha/\beta \ll 1$ and $\kappa/\gamma \ll 1$, the spectral power profile of CSP is

$$p(\omega) \simeq \frac{6\pi\gamma}{\zeta\kappa} \frac{\theta(\Delta^2 - \omega^2)}{\omega^2 + \Omega_L^2}, \qquad (2)$$

where ω is the frequency measured from the gain band centrum, $\theta(x)$ is the Heaviside function, $\Delta^2 = \gamma P_0/\beta$, $\Omega_L^2 = \gamma(1+c)/\zeta\beta - 5\gamma P_0/3\beta$, $c = \alpha\gamma/\beta\kappa$, and P_0 is the CSP peak power. As one can see from Eq. (2), the CSP spectrum has a Lorentz profile truncated at $\pm\Delta$.

The obvious stability criterion for a CSP is the vacuum stability of Eq. (1), that is, $\sigma < 0$, and, thereby, the cw is suppressed. Such a criterion defines the minimum GDD required for CPO stabilization, which agrees with the numerical simulations and the experimental observations [13]. However, the experiment [10] demonstrates that the CPO stability region is substantially narrower than that predicted theoretically. Hence, there exists an instability of the CSP besides the obvious vacuum one.

III. CSP STABILITY

To analyze the CSP stability, Eq. (1) was solved by means of both symmetrized split-step Fourier and finite-difference methods. The local time and propagation steps were 1 fs (2^{15} mesh points) and 10^{-3} of the cavity round trip, respectively. Convergence was controlled by variation of both the time mesh and propagation step. The minimum propagation distance z exceeded 3×10^4 cavity round trips. In the calculations we have used the parameters of the Ti:sapphire oscillator operating in PDR (see, for example, [13]): $\alpha = 1.1$ fs^2, $\kappa = 0.04\gamma$, $\zeta = 0.6\gamma$, $\gamma = 4.55$ MW^{-1}, $\delta = 0.04$.

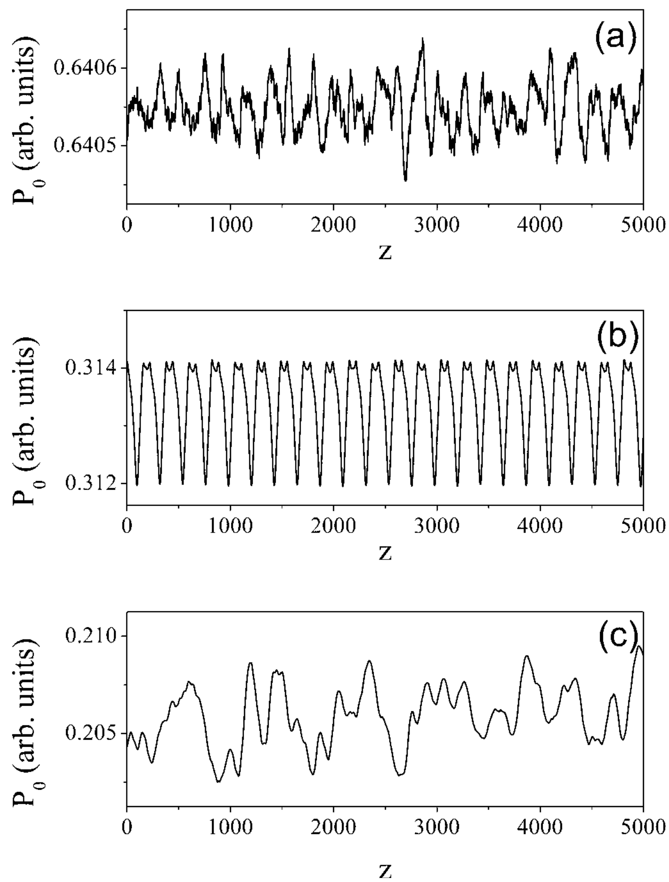

FIG. 1. Evolution of the CSP peak power for $\beta = 50$ (a), 100 (b), and 150 (c) fs^2. $E^* = 180$ nJ, $\kappa = 0.04\gamma$, $\zeta = 0.6\gamma$. The building-up stage is excluded.

First, let us consider the CSP stability on the condition that it is stable against a cw (i.e., $\sigma < 0$). Such a stabilization is possible when the GDD is above some threshold value [13].

The simulations demonstrate that the CSP is unstable in the absence of gain saturation, i.e., when $\sigma < 0$ but the σ parameter does not decrease with E growth. Such an instability has the form of a pulse decay or of its collapselike growth (self-similar collapse; see Ref. [17]). The last is confined by only SAM saturation [the quintic nonlinear term in Eq. (1)].

The analytical solution (2) is obviously temporally and spectrally symmetrical. However, this solution was found to be unstable even in the presence of gain saturation. To explore the instability mechanism, first one can suppress the asymmetrical perturbations by means of imposed symmetrization at every z: the symmetrized $A(z,t)$ equals $[A(z,t) + A(z,-t)]/2$ ($t=0$ corresponds to the CSP maximum).

After starting from a small Gaussian pulse seed, the solution converges to the analytical one. Then, the long-period (exceeding hundreds of cavity round trips) pulsations appear. The amplitude of the pulsations increases with the GDD (Fig. 1; note that the building-up stage is excluded in the figures). Since the experiment [10] demonstrates CSP desta-

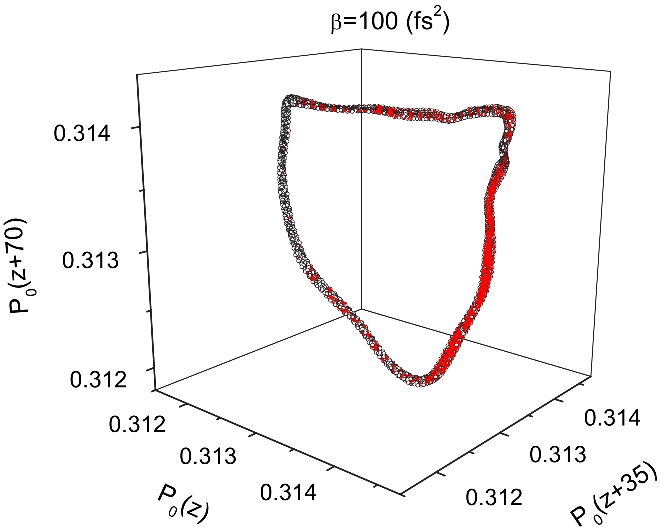

FIG. 2. (Color online) Phase-space reconstruction from the CSP peak-power set (5000 points) for $\beta=100$ fs^2. $Z=35$ cavity round trips. Other parameters correspond to Fig. 1.

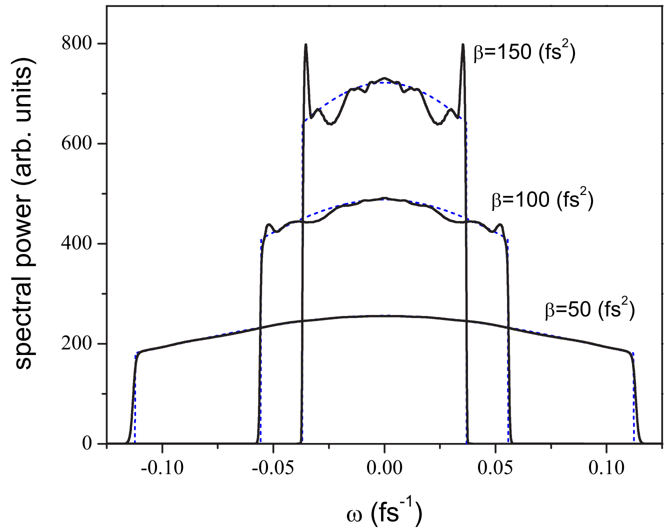

FIG. 3. (Color online) Numerical (solid curves) and analytical (dashed curves) spectra. $z=3\times10^4$; parameters correspond to Fig. 1.

bilization with GDD growth, one can suppose that this destabilization results from the numerically observed growth of the pulse pulsation. Pulsating solitons exist in the negative-dispersion regime [18–22], but their counterparts have not been analyzed in the PDR, to date. In contrast to the negative-dispersion regime, pulsations in the PDR do not entail a change of the pulse shape.

The pulsations undergo a set of bifurcations with GDD growth: a nonregular pulsation [Fig. 1(a)] changes into a regular one [Fig. 1(b)] and then into quasiregular pulsation [Fig. 1(c)]. To analyze the type of bifurcation, one can use the technique of phase-space reconstruction based on the z-delay embedding of the peak-power series. Such a reconstruction can be made in the coordinates $[P_0(z),P_0(z+Z),P_0(z+2Z)]$, where Z is the propagation lag defined from the first minimum of the autocorrelation function of the $P_0(z)$ set [23]. The resulting three-dimensional embedding for $\beta=100$ fs^2 is shown in Fig. 2. This figure demonstrates the existence of the Hopf instability, resembling a beating of the bright solitary-wave solution of the cubic-quintic nonlinear CGLE near the nonlinear Schrödinger equation limit [24,25]. Thus, the source of instability considered can be identified with the edge bifurcation, when the pulsation arises from a cw perturbation. Such an instability can explain the experimental fact that the CSP exists only within a limited dispersion range [10,12,13]. The lower GDD stability threshold results from the vacuum instability (cw amplification) whereas the upper one arises from the edge bifurcation, i.e., from the pulse pulsation. Variation of the ζ parameter from 0.16γ to γ does not effect noticeably the pulsation amplitude or the bifurcation character.

Growth of the κ parameter stabilizes the CSP. In practice, this growth can be provided by a semiconductor saturable absorber [10,11,13]. It should be noted that the regime with extremely strong SAM, low pulse energy, and narrow gain band has been considered in Ref. [26]. Such a regime results in pulsations with a disturbance of the pulse shape. As the analysis demonstrates, our regime corresponding to the high-energy CSP cannot be connected continuously with that of Ref. [26] only by variation of the SAM parameters, as one has simultaneously to decrease the GDD and suppress the gain saturation. Also, the analysis demonstrates a substantial growth of pulsation due to a higher-order GDD contribution (i.e., when the β parameter is frequency dependent).

The most interesting characteristic of the CSP instability is the pulse spectrum distortion. As was found in the experiment [10,13], the spectrum profile in PDR is modulated and this modulation increases with the GDD. The simulations demonstrate that the CSP pulsation entails its spectrum profile modulation. Figure 3 shows the numerically obtained spectra (solid curves) in comparison with the analytical ones (dashed curves). Growth of the spectrum profile modulation with the GDD is clearly visible. Such a modulation is extremely long lived and cannot be attributed to numerical artifacts. One can suppose that the pulse peak-power pulsation in combination with the spectrum profile modulation results from an excitation of the internal modes of a solitary wave [27,28]. Such modes have been predicted for the negative-dispersion regime but have not been observed, to date. The internal mode is not some compound like a higher-order or interacting soliton (e.g., see [29]), but a perturbation localized (bound) inside the pulse without an outgoing radiation (unlike the dispersive wave [16]).

IV. SPONTANEOUS SYMMETRY BREAKING

In the previous section, the symmetry of the complex amplitude A relative to $t=0$ was assumed. This leads to a symmetrical spectrum in agreement with Eq. (2). However, an evenly symmetrized CSP is unstable against edge bifurcation. In the general case, the CGLE allows symmetry breaking with an abrupt pulse destabilization [30]. Therefore let us consider CSP evolution without imposed symmetrization.

Figure 4 shows the peak-power evolution in this case. The transitional stage is very long ($>2\times10^4$ round trips) and

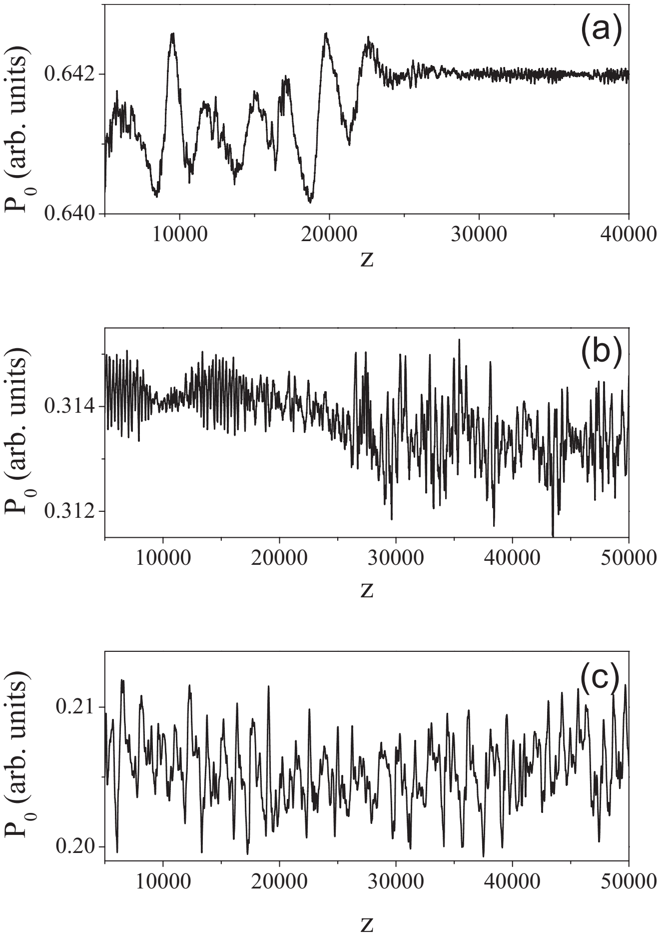

FIG. 4. Evolution of the CSP peak power for $\beta=50$ (a), 100 (b), and 150 (c) fs^2. $E^*=180$ nJ, $\kappa=0.04\gamma$, $\zeta=0.6\gamma$. No imposed symmetrization.

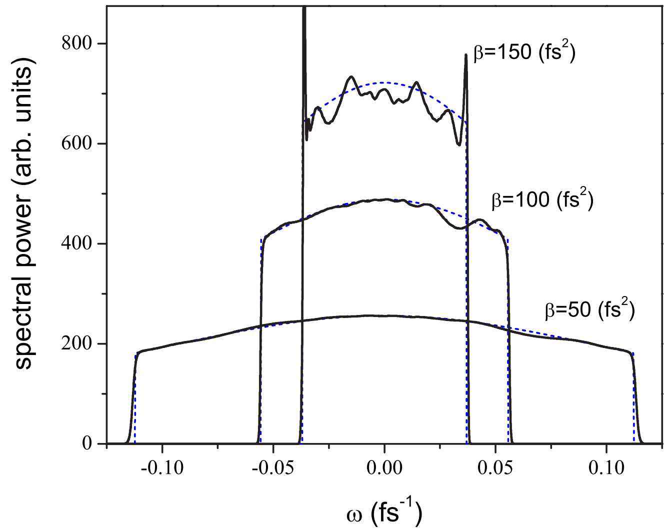

FIG. 5. (Color online) Numerical (solid curves) and analytical (dashed curves) spectra. $z=5\times10^4$. No imposed symmetrization.

exceeds essentially that for the negative-dispersion solitonic regime. As in the case with symmetrization, there are only pulsating solutions and the amplitude excursion grows with the GDD. The regularity of the pulsations disappears and their frequency decreases approximately twice. The temporal profile of P_0 (pulse shape) is not distorted as before.

A most interesting property of the regime is the symmetry breaking in the spectral domain that is shown in Fig. 5. Earlier (see Ref. [11]), it was supposed that the spectral asymmetry results from only the frequency dependence of the GDD. However, the simulations demonstrate that this phenomenon can be dynamic in nature.

Thus, the pulsation can be interpreted as beating between two asymmetrical internal modes of the CSP. The symmetry breaking and the beating are clearly visible in the spectral profile (Fig. 6). But again one has to note that the pulse shape has no asymmetry.

One can suppose that the revealed instability has a phase nature and thereby not only confines a region of the CSP existence but also reduces its spectral coherence. The last is very important for applications and needs further analysis.

V. CONCLUSION

The stability of the chirped-solitary-pulse solutions of the CGLE has been investigated numerically. It was found that the CSP is unstable in the absence of gain saturation, i.e., when the source term in the CGLE is energy independent. Even if the gain saturation contributes to the pulse dynamics, the CSP suffers from edge bifurcation. This results in (i) long-period pulsations (both regular and irregular) of the CPO peak power and (ii) modulation of the CSP spectrum profile. The instability, which can be attributed to an excitation of internal modes of the solitary pulse, grows with the GDD and is clearly visible in the experimental spectra. One can suppose that this instability defines the maximum GDD providing CPO stabilization so that the CPO operates within a confined range of the GDD.

Also, it was found that the CSP is unstable against symmetry breaking so that the dynamics of the CPO in the PDR is the beating between asymmetrical internal modes of the CSP solution of the CGLE. This effect reduces not only the region of CSP existence but also its spectral coherence. The proposed way to suppress this instability is similar to that for suppression of cw amplification [11,13]: the self-amplitude-modulation has to be intensified, which is possible using a semiconductor saturable absorber.

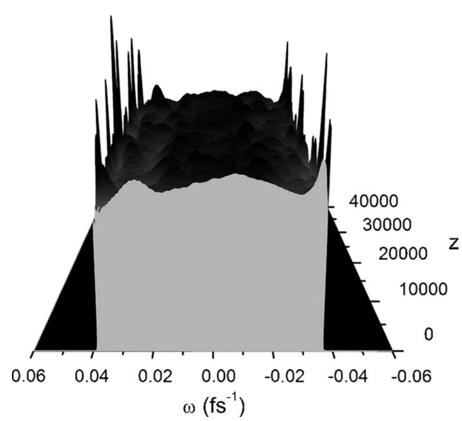

FIG. 6. Spectrum profile evolution for $\beta=150$ fs^2.

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