Electric-field-induced Feshbach resonances in ultracold alkali-metal mixtures

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We present a detailed analysis of Feshbach resonances in ultracold collisions of Li and Cs atoms in the presence of superimposed electric and magnetic fields. We show that electric fields induce resonances through couplings between the *s*- and *p*-wave scattering channels and modify the scattering length to a great extent. Electric-field-induced resonances lead to the anisotropy of ultracold scattering and provide the diagnostics for magnetic *p*-wave resonances in ultracold gases. We show that the electric field couplings may shift the positions of *s*-wave magnetic resonances, thereby making the electric field control of ultracold atoms possible even far away from *p*-wave resonances. Finally, we demonstrate that electric fields may rotate and spin up the collision complex of ultracold atoms at substantial rates.

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I. INTRODUCTION

The creation of ultracold atoms and molecules has generated an upheaval in atomic, molecular, and optical physics [1–8] and may lead to groundbreaking discoveries in condensed-matter physics [9–23], nuclear physics [24–27], quantum computation [28–30], and chemical physics [31–35]. The development of experimental techniques for the production of dense ultracold gases is therefore pursued by many researchers (see [7] and references therein). Ultracold gases offer the possibility of controlling atomic and molecular systems at the single-quantum level, which is exploited in studies of quantum information processing [28–30] and coherent matter-light interactions [11]. External control of atomic interactions can be achieved by applying static magnetic or resonant laser fields to induce Feshbach scattering resonances [36–39]. Ultracold atoms and molecules are usually confined in magnetic or magneto-optical traps, and magnetic Feshbach resonances are particularly important for studies of ultracold gases. They have been used to create molecular Bose-Einstein condensates (BEC's) and explore the dynamics of ultracold correlated systems in the BEC-BCS crossover regime [13,16]. Collision cross sections of ultracold atoms and molecules change dramatically as the magnetic field is varied through a resonance. Feshbach resonances thus provide a mechanism for controlling atomic and molecular collisions with magnetic fields.

In the present paper, we explore the possibility of inducing Feshbach resonances in ultracold atomic gases with dc electric fields. Electric fields can be tuned much faster than magnetic fields, so electric field control of interatomic interactions may prove to be more versatile for quantum information processing than magnetic Feshbach resonances. Using electric fields for inducing scattering resonances may also be important for experiments with gases in deep magnetic traps where large field gradients complicate the dynamics of magnetic resonances or when magnetic resonances cannot be tuned in the available interval of magnetic fields. Magnetic field control of interatomic interactions is limited to paramagnetic species, so the possibility of inducing scattering resonances with electric fields may expand the scope of studies of correlation phenomena in ultracold gases.

Marinescu and You [40] and Melezhik and Hu [41] proposed to control interactions in ultracold atomic gases by polarizing atoms with strong electric fields. The polarization changes the long-range form of the atom-atom interaction potential and modifies the scattering cross section in the limit of zero collision energy. The interaction between an atom and an electric field is, however, extremely weak, and fields of as much as 250–700 kV/cm were required to alter the elastic scattering cross section of ultracold atoms in these calculations. We have recently proposed an alternative mechanism for electric field control of ultracold atom interactions [42] and demonstrated that collisions and interactions in binary mixtures of ultracold atoms can be effectively manipulated by electric fields below 100 kV/cm. The mechanism of electric field control is based on the interaction of the instantaneous dipole moment of the collision pair with external electric fields. The duration of an ultracold collision is so long that this interaction, while insignificant in thermal gases, may dramatically change the dynamics of atomic collisions at temperatures near absolute zero.

This is an extension of our preceding Letter [42]. To elucidate the possibility of inducing Feshbach resonances with electric fields, we study the collision dynamics in a binary mixture of ultracold Li and Cs atoms. Ultracold mixtures of Li and Cs gases have been recently created by Mudrich *et al.* [43,44] in order to produce ultracold polar LiCs molecules [45], and accurate interaction potentials for the LiCs molecule have been derived from high-precision spectroscopy measurements [46]. We use the interaction potentials of Staanum *et al.* [46] to calculate the positions and widths of magnetic Feshbach resonances that may be used to link ultracold atoms together for the creation of ultracold molecules either directly [43–45] or by enhancing the probability of photoassociation [47]. We then show that applying superimposed magnetic and electric fields may result in three-state Feshbach resonances tunable by electric fields. We explore the main features of such resonances. In particular, we show that the positions of the electric-field-induced resonances depend on the strength of the electric field and demonstrate that electric fields may modify the magnetic Feshbach reso-

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TABLE I. Definition of quantum numbers used in this article.

l	nuclear orbital angular momentum of the diatomic system
m_l	projection of l on the space-fixed quantization axis
S	total electronic spin angular momentum of the diatomic system
M_S	projection of S on the space-fixed quantization axis
I_{Li}	nuclear spin angular momentum of Li
$M_{I_{\mathrm{Li}}}$	projection of I_{Li} on the space-fixed quantization axis
I_{Cs}	nuclear spin angular momentum of Cs
$M_{I_{\mathrm{Cs}}}$	projection of I_{Cs} on the space-fixed quantization axis
S_{Li}	electronic spin angular momentum of Li
$M_{S_{\mathrm{Li}}}$	projection of S_{Li} on the space-fixed quantization axis
S_{Cs}	electronic spin angular momentum of Cs
$M_{S_{\mathrm{Cs}}}$	projection of S_{Cs} on the space-fixed quantization axis

nances. Finally, we show that electric fields may spin up the collision complex of ultracold atoms at substantial rates and induce the anisotropy of the differential scattering cross sections at ultracold temperatures.

II. THEORY

The dynamics of Li-Cs collisions in the presence of superimposed electric and magnetic fields is described by the Hamiltonian

$$\hat{H} = -\frac{1}{2\mu R}\frac{\partial^2}{\partial R^2}R + \frac{\hat{l}^2(\theta,\varphi)}{2\mu R^2} + \hat{V}(R) + \hat{V}_{\mathrm{E}}(R) + \hat{V}_{\mathrm{B}} + \hat{V}_{\mathrm{hf}}, \tag{1}$$

where μ is the reduced mass of the colliding atoms, R is the interatomic distance, $\hat{l}$ is the rotational angular momentum of the collision complex, and the angles θ and φ specify the orientation of the interatomic axis in the space-fixed coordinate frame. The electronic interaction potential $\hat{V}(R)$ can be represented as

$$\hat{V}(R) = \sum_S \sum_{M_S} |SM_S\rangle V_S(R)\langle SM_S|, \tag{2}$$

where S is the total electronic spin of the diatomic molecule, M_S is the projection of S on the z axis, and $V_S(R)$ represents the adiabatic interaction potentials of the molecule in the spin state S. The atomic and molecular quantum numbers used in this article are defined in Table I.

The operator $\hat{V}_{\mathrm{E}}(R)$ describes the interaction of the collision complex LiCs with an external electric field. It can be written in the form

$$\hat{V}_{\mathrm{E}}(R) = -E\cos\theta \sum_S \sum_{M_S} |SM_S\rangle d_S(R)\langle SM_S|, \tag{3}$$

with d_S denoting the dipole moment functions of LiCs in the different spin states and E the electric field magnitude. The interaction potentials $V_S(R)$ and the dipole moment functions $d_S(R)$ for the LiCs molecule in the $^1\Sigma$ and $^3\Sigma$ states used for this work are shown in Fig. 1. The dipole moment functions are represented by the expressions

$$d_s(R) = D\exp[-\alpha(R-R_e)^2], \tag{4}$$

with the parameters R_e=7.7 bohrs, α=0.1 bohr^{-2}, and D =6 D for the singlet state and D=0.5 D for the triplet state. These analytical expressions approximate the numerical data for the dipole moment functions computed by Aymar and Dulieu [48].

The interaction of the atoms with an external magnetic field B is described by

$$\hat{V}_{\mathrm{B}} = 2\mu_0 B(\hat{S}_{Z_{\mathrm{Li}}} + \hat{S}_{Z_{\mathrm{Cs}}}) - B\left(\frac{\mu_{\mathrm{Li}}}{I_{\mathrm{Li}}}\hat{I}_{Z_{\mathrm{Li}}} + \frac{\mu_{\mathrm{Cs}}}{I_{\mathrm{Cs}}}\hat{I}_{Z_{\mathrm{Cs}}}\right), \tag{5}$$

where B is the magnetic field strength, μ_0 is the Bohr magneton, and μ_{Li} and μ_{Cs} denote the nuclear magnetic moments of Li and Cs. We assume that the electric and magnetic fields are both directed along the z axis. The hyperfine interaction $\hat{V}_{\mathrm{hf}}$ can be represented as

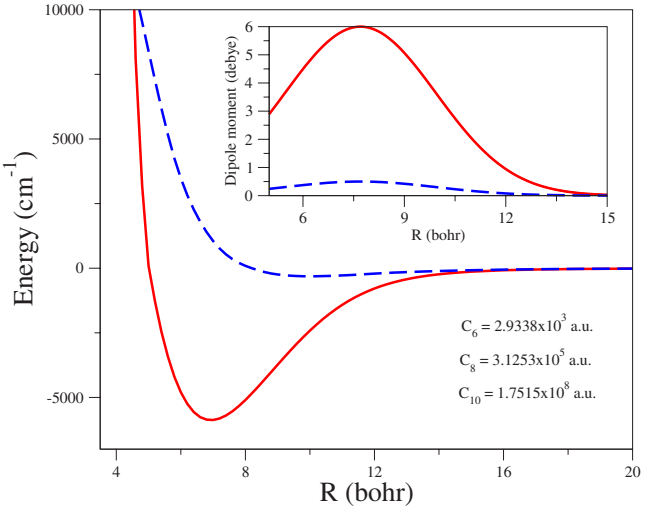

FIG. 1. (Color online) The interaction potentials and dipole moment functions (inset) of the LiCs molecule in the ground $^1\Sigma$ (solid lines) and excited $^3\Sigma$ (dashed lines) states. The interaction potentials were taken from Ref. [46] and the dipole moment functions approximate the data of Ref. [48].

$$\hat{V}_{\rm hf} = \gamma_{\rm Li}\hat{I}_{\rm Li}\cdot\hat{S}_{\rm Li} + \gamma_{\rm Cs}\hat{I}_{\rm Cs}\cdot\hat{S}_{\rm Cs}, \tag{6}$$

where $\gamma_{\rm Li}$ and $\gamma_{\rm Cs}$ are the atomic hyperfine interaction constants: $\gamma_{\rm Li}$=402.00 MHz and $\gamma_{\rm Cs}$=2298.25 MHz.

We expand the total wave function of the diatomic system in terms of the eigenfunctions of $\hat{l}^2$, $\hat{l}_Z$, $\hat{I}_{\rm Li}^2$, $\hat{I}_{\rm Cs}^2$, $\hat{I}_{Z_{\rm Li}}$, and $\hat{I}_{Z_{\rm Cs}}$ as follows:

$$\psi = \frac{1}{R}\sum_{\alpha}\sum_{l}\sum_{m_l} F_{\alpha l m_l}(R)|\alpha l m_l\rangle, \tag{7}$$

where

$$|\alpha\rangle = |I_{\rm Li}M_{I_{\rm Li}}\rangle|S_{\rm Li}M_{S_{\rm Li}}\rangle|I_{\rm Cs}M_{I_{\rm Cs}}\rangle|S_{\rm Cs}M_{S_{\rm Cs}}\rangle. \tag{8}$$

The substitution of this expansion into the Schrödinger equation with the Hamiltonian (1) results in a system of coupled differential equations

$$\left[\frac{d^2}{dR^2} - \frac{l(l+1)}{R^2} + 2\mu\epsilon\right]F_{\alpha l m_l}(R) = 2\mu\sum_{\alpha'}\sum_{l'}\sum_{m_l'}\langle\alpha l m_l|\hat{V}(R) + \hat{V}_{\rm E}(R) + \hat{V}_{\rm B} + \hat{V}_{\rm hf}|\alpha' l' m_l'\rangle F_{\alpha' l' m_l'}(R), \tag{9}$$

which we solve at fixed values of total energy ϵ.

To evaluate the matrix elements of the interaction potential $\hat{V}(R)$, we write the product states $|I_{\rm Li}M_{I_{\rm Li}}\rangle|S_{\rm Li}M_{S_{\rm Li}}\rangle|I_{\rm Cs}M_{I_{\rm Cs}}\rangle|S_{\rm Cs}M_{S_{\rm Cs}}\rangle$ as

$$|I_{\rm Li}M_{I_{\rm Li}}\rangle|S_{\rm Li}M_{S_{\rm Li}}\rangle|I_{\rm Cs}M_{I_{\rm Cs}}\rangle|S_{\rm Cs}M_{S_{\rm Cs}}\rangle = \sum_{S}\sum_{M_S}(-1)^{M_S}(2S+1)^{1/2}\times\begin{pmatrix} S_{\rm Li} & S_{\rm Cs} & S \\ M_{S_{\rm Li}} & M_{S_{\rm Cs}} & -M_S\end{pmatrix}|I_{\rm Li}M_{I_{\rm Li}}\rangle|I_{\rm Cs}M_{I_{\rm Cs}}\rangle|SM_S\rangle \tag{10}$$

and note that

$$\langle SM_S|\hat{V}(R)|S'M_S'\rangle = V_S(R)\,\delta_{SS'}\,\delta_{M_SM_S'}. \tag{11}$$

The parentheses in Eq. (10) denote a $3j$ symbol. The operator $\hat{V}(R)$ is independent of the nuclear spin states and l and m_l quantum numbers.

The matrix elements of $\hat{V}_{\rm E}(R)$ are evaluated using the expressions

$$\langle lm_l|\cos\theta|l'm_l'\rangle = \delta_{m_lm_l'}(-1)^{m_l}\begin{pmatrix} l & 1 & l' \\ -m_l & 0 & m_l\end{pmatrix}\begin{pmatrix} l & 1 & l' \\ 0 & 0 & 0\end{pmatrix}\times[(2l+1)(2l'+1)]^{1/2} \tag{12}$$

and

$$\langle SM_S|\left(\sum_{S''}\sum_{M_S''}|S''M_S''\rangle d_{S''}\langle S''M_S''|\right)|S'M_S'\rangle = d_S\delta_{SS'}\delta_{M_SM_S'}. \tag{13}$$

The terms describing the interaction of the atoms with magnetic fields are diagonal in the representation $|I_{\rm Li}M_{I_{\rm Li}}\rangle|S_{\rm Li}M_{S_{\rm Li}}\rangle|I_{\rm Cs}M_{I_{\rm Cs}}\rangle|S_{\rm Cs}M_{S_{\rm Cs}}\rangle$. The matrix elements of the hyperfine interaction operators can be readily evaluated using the relations

$$\hat{I}_{\rm Li}\cdot\hat{S}_{\rm Li} = \hat{I}_{Z_{\rm Li}}\hat{S}_{Z_{\rm Li}} + \frac{1}{2}(\hat{I}_{\rm Li+}\hat{S}_{\rm Li-} + \hat{I}_{\rm Li-}\hat{S}_{\rm Li+}) \tag{14}$$

and

$$\hat{I}_{\rm Cs}\cdot\hat{S}_{\rm Cs} = \hat{I}_{Z_{\rm Cs}}\hat{S}_{Z_{\rm Cs}} + \frac{1}{2}(\hat{I}_{\rm Cs+}\hat{S}_{\rm Cs-} + \hat{I}_{\rm Cs-}\hat{S}_{\rm Cs+}), \tag{15}$$

where $\hat{I}_{\pm}$ and $\hat{S}_{\pm}$ are the raising and lowering operators. We note that the matrix of the Hamiltonian in the basis $|\alpha l m_l\rangle$ does not become diagonal as $R\to\infty$. Therefore, the boundary conditions cannot be properly applied to obtain the solutions to Eq. (9). Before constructing the scattering S matrix from the solutions of Eq. (9), we apply an additional transformation that diagonalizes the matrix of $\hat{V}_{\rm E}+\hat{V}_{\rm B}+\hat{V}_{\rm hf}$. This procedure has been described in Ref. [49]. The scattering matrix thus obtained yields the probabilities of elastic and inelastic scattering of Li and Cs in the presence of electric and magnetic fields.

III. RESULTS

Ultracold mixtures of Li and Cs gases have been recently created in the laboratory of Weidemüller [43,44] for the formation of ultracold polar LiCs molecules through photoassociation [45]. An alternative method of producing ultracold molecules is based on linking ultracold atoms together with magnetic-field-induced Feshbach resonances. Feshbach resonances may also enhance the probability for photoassociation [47] and provide detailed information for the analysis of interatomic interaction potentials [50]. Experimental measurements of the positions and widths of magnetic Feshbach resonances are therefore very important for dynamical studies of ultracold gases. To guide future experiments in the search of Feshbach resonances, we present in Table II the positions and widths of purely magnetic s-wave resonances calculated with the potentials of Staanum *et al.* [46]. The data were obtained by fitting the scattering length to the following equation:

$$a = a_{\rm bg}\left(1 - \frac{\Gamma}{B - B_{\rm res}}\right), \tag{16}$$

where $a_{\rm bg}$ is the background scattering length, B is the magnitude of the magnetic field, Γ is the width of the resonance, and $B_{\rm res}$ is the position of the resonance. To analyze the resonances in detail, we have also computed the eigenphase sum. The first derivative of the eigenphase sum peaks at slightly different points from $B_{\rm res}$ in Eq. (16). Table III lists the positions of the resonances as determined by Eq. (16).

In the absence of electric fields, different partial wave states $|lm_l\rangle$ of the Li-Cs collision complex are uncoupled and s-wave scattering entirely determines the collision cross sections at ultralow kinetic energies. The interaction with elec-

TABLE II. The positions (B_{res}) and widths (Γ) of s-wave magnetic resonances for Li-Cs at magnetic fields below 500 G.

Atomic states	B_{res} (G)	Γ (G)
$M_{Li}=1$, $M_{Cs}=-2$	2.03	>2.00
$M_{Li}=1$, $M_{Cs}=-3$	1.49	>2.00
	21.50	>2.00
	387.81	>2.00
$M_{Li}=0$, $M_{Cs}=-3$	4.92	>2.00
$M_{Li}=-2$, $M_{Cs}=4$	20.05	0.70
$M_{Li}=-2$, $M_{Cs}=3$	0.86	0.03
	2.27	0.16
	7.06	1.68
$M_{Li}=-2$, $M_{Cs}=2$	1.02	0.06
	2.64	0.34
	7.20	2.00
$M_{Li}=-2$, $M_{Cs}=1$	0.47	0.02
	1.25	0.12
	3.15	0.42
	6.65	1.50
$M_{Li}=-2$, $M_{Cs}=0$	0.59	0.04
	1.65	0.22
	4.47	1.06
$M_{Li}=-2$, $M_{Cs}=-1$	0.63	0.10
	2.49	0.56

tric fields (3) induces couplings between different angular momentum states and may thus affect the scattering length. Figures 2, 3, and 4 display the magnetic field dependence of the s-wave and p-wave scattering cross sections calculated for various Zeeman states of Li and Cs at zero electric field and at $E=100$ kV/cm. The resonance structure of the s-wave cross sections is dramatically modified by the electric field.

The examination of Figs. 2–4 leads to two important observations: (i) p-wave resonances result in resonant enhancement of the s-wave cross section (or equivalently the scattering length) in the presence of an electric field, and (ii) the interaction with electric fields shifts the positions of both the s-wave and p-wave resonances. We refer to the former as electric-field-induced resonances. Figure 5 presents two examples of the electric-field-induced resonances and shows that even relatively weak electric fields (~20–30 kV/cm) may modify the s-wave Li-Cs scattering to a great extent. P-wave magnetic resonances are essential for the electric-field-induced resonances in ultracold atomic collisions. To guide future experiments in the search of such resonances, we present in Table III the positions of p-wave magnetic Feshbach resonances calculated at zero electric field.

TABLE III. The positions (B_{res}) of p-wave magnetic Feshbach resonances for Li-Cs at magnetic fields below 1000 G.

Atomic states	B_{res} (G)
$M_{Li}=-1$, $M_{Cs}=3$	953.54
$M_{Li}=0$, $M_{Cs}=3$	862.74, 907.55, 965.61
$M_{Li}=1$, $M_{Cs}=2$	998.79
$M_{Li}=1$, $M_{Cs}=3$	785.57, 862.47

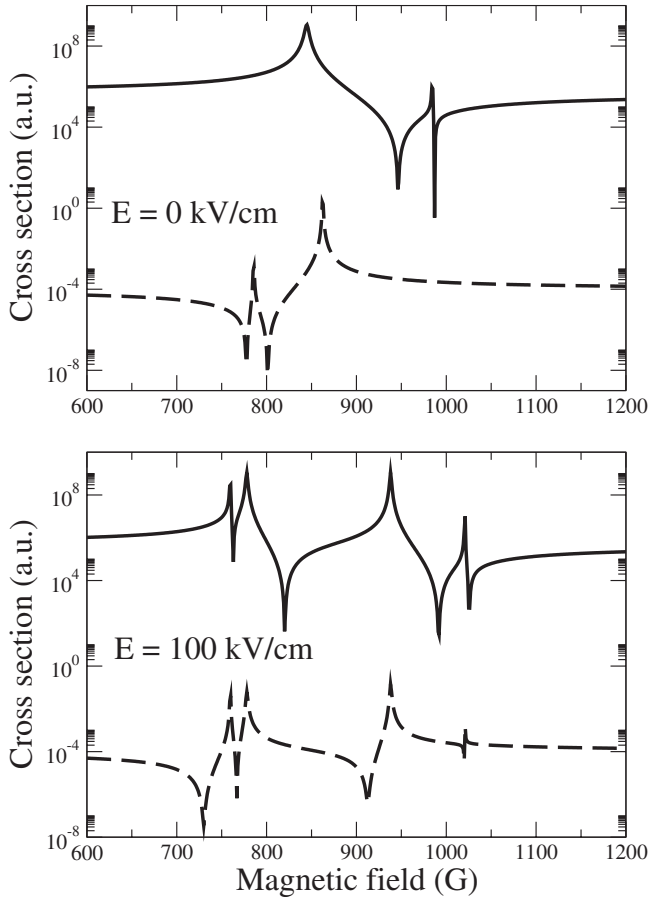

FIG. 2. Cross sections for elastic s-wave (solid curves) and p-wave (dashed curves) scattering of Li and Cs in the states $M_{Li}=1$ and $M_{Cs}=3$ computed at zero electric field (upper panel) and an electric field strength of 100 kV/cm (lower panel). The collision energy is 10^{-7} cm^{-1}.

Figure 6 is an expanded view of two s-wave resonances in Fig. 3. As shown in the third panel of the figure, the s-to-p couplings induced by electric fields are significant *both* in the presence of p-wave resonances *and* near s-wave resonances. Electric field control of ultracold Li-Cs collisions may therefore be possible even in the absence of p-wave resonances. The s-wave resonances may shift as shown in Fig. 6, which can lead to dramatic changes of the scattering length: consider, for example, the variation of the scattering length with increasing electric field at the magnetic field value 1071 G shown in Fig. 7. Not all s-wave resonances are sensitive to electric fields. For example, we found that the resonances at low magnetic fields listed in Table II shift by less than 1 G in an electric field of 100 kV/cm, so no variation of these resonances with electric fields should be expected. Apparently, the effect of electric fields on s-wave resonances depends on the width of the resonances and the strength of the background p-wave scattering. The background p-wave scattering at magnetic fields of about 1071 G in Fig. 3 is enhanced by the wings of the p-wave resonances at lower magnetic fields.

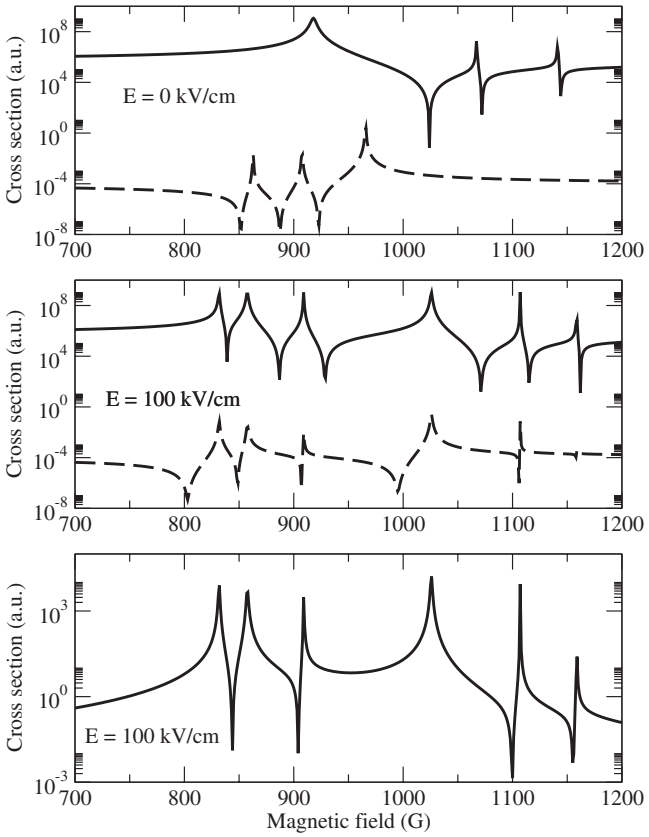

FIG. 3. Cross sections for elastic s-wave (solid curves) and p-wave (dashed curves) scattering of Li and Cs in the states $M_{\mathrm{Li}}=0$ and $M_{\mathrm{Cs}}=3$ computed at zero electric field (upper panel) and an electric field strength of 100 kV/cm (middle panel). The lower panel presents the cross section for the $s\rightarrow p$ transition. The collision energy is 10^{-7} cm^{-1}.

Ultracold s-wave scattering is isotropic: the probability to find the atoms after s-wave collisions does not depend on the scattering angle. The interaction with electric fields (3), however, couples the spherically symmetric s waves to anisotropic p waves, so electric fields may induce the anisotropy of ultracold scattering. The differential scattering cross section in the presence of external fields is defined as

$$\frac{d\sigma_{\beta\rightarrow\beta'}}{d\hat{R}_i d\hat{R}} = \frac{4\pi^2}{k_\beta^2}\sum_{l_1}\sum_{m_{l_1}}\sum_{l_2}\sum_{m_{l_2}}\sum_{l_1'}\sum_{m_{l_1}'}\sum_{l_2'}\sum_{m_{l_2}'} i^{l_1'-l_1+l_2-l_2'} Y_{l_1 m_{l_1}}(\hat{R}_i) \times Y^*_{l_2 m_{l_2}}(\hat{R}_i) Y^*_{l_1' m_{l_1}'}(\hat{R}) Y_{l_2' m_{l_2}'}(\hat{R}) \times T^*_{\beta l_1 m_{l_1}\rightarrow\beta' l_1' m_{l_1}'} T_{\beta l_2 m_{l_2}\rightarrow\beta' l_2' m_{l_2}'}, \quad (17)$$

where β and β' label the initial and final scattering states, k_β is the collision wave number, and $\hat{R}_i$ and $\hat{R}$ specify the direction of the initial and final collision fluxes. Assuming that the initial collision flux is directed along the field axis, we can write the differential cross section (17) for elastic scattering in terms of the s-wave and s-to-p wave elements of the T matrix in the form

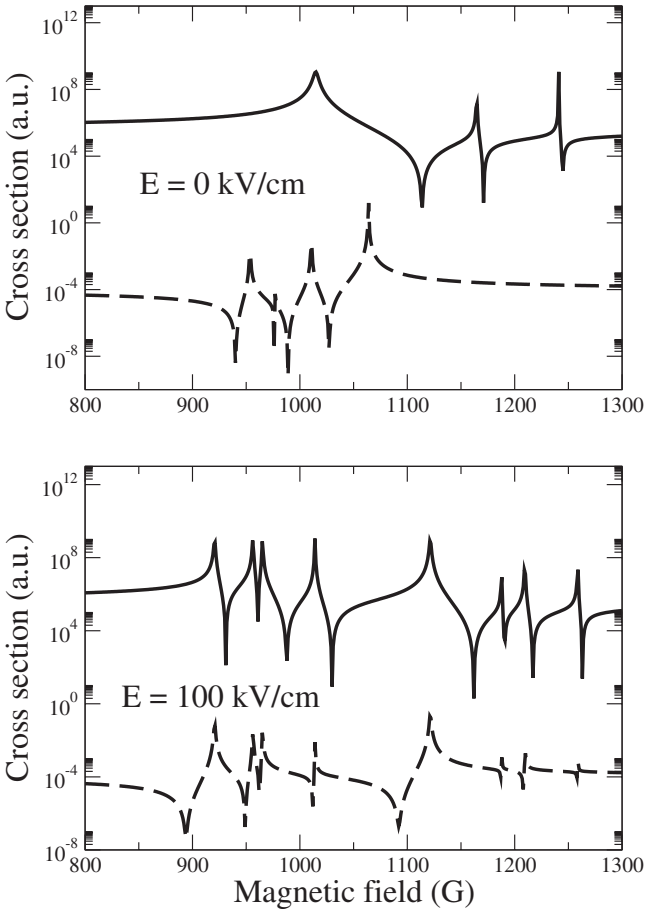

FIG. 4. Cross sections for elastic s-wave (solid curves) and p-wave (dashed curves) scattering of Li and Cs in the states $M_{\mathrm{Li}}=-1$ and $M_{\mathrm{Cs}}=3$ computed at zero electric field (upper panel) and an electric field strength of 100 kV/cm (lower panel). The collision energy is 10^{-7} cm^{-1}.

$$\sigma_d = \frac{d\sigma}{d\hat{R}} = \frac{\pi}{4k_\alpha^2}\{|T_{l=0\rightarrow l'=0}|^2 + 3\cos^2\theta|T_{l=0\rightarrow l'=1}|^2 + 2\sqrt{3}\cos\theta[\mathrm{Re}(T_{l=0\rightarrow l'=0})\mathrm{Im}(T_{l=0\rightarrow l'=1}) - \mathrm{Im}(T_{l=0\rightarrow l'=0})\mathrm{Re}(T_{l=0\rightarrow l'=1})]\}. \quad (18)$$

The first term is independent of the scattering angle, and it usually dominates at ultralow collision energies. Figure 5, however, shows that at certain values of the magnetic and electric fields, the s-wave cross section becomes very small. At these points, the differential scattering may be determined by the third term in Eq. (18), which leads to an angular dependence of the scattering cross sections (see Fig. 8). The contribution of elastic p-wave scattering at the collision energies below 10^{-5} is negligible.

Collisions of ultracold atoms are determined by s-wave interactions; however, electric fields rotate and spin the collision complex up, leading to the $s\rightarrow p$ transition. Figure 3 demonstrates that the probability of the $s\rightarrow p$ transition near s-wave and p-wave threshold resonances is sensitive to the magnitude of the electric field. The s- and p-wave scattering channels are degenerate at the infinite interatomic separation,

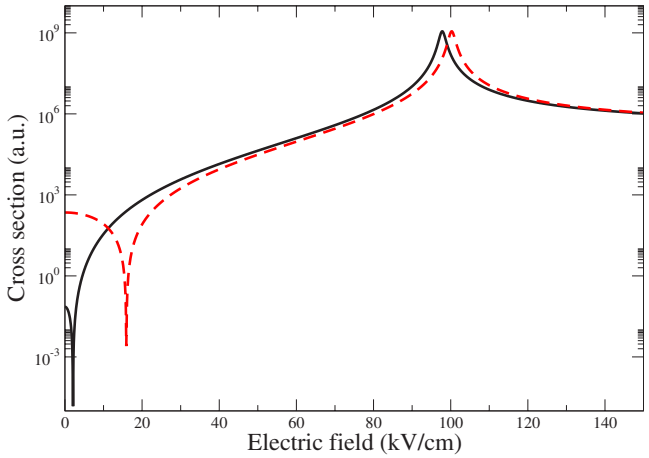

FIG. 5. (Color online) Electric-field-induced resonances: variation of the cross sections for s-wave collisions of Li and Cs in the states $M_{\mathrm{Li}}=0$ and $M_{\mathrm{Cs}}=3$ with the electric field strength. The magnetic field is fixed at 1024 G (solid curve) and 1026 G (dashed curve). The collision energy is 10^{-7} cm^{-1}.

and the $s \rightarrow p$ transition must be suppressed by the centrifugal barrier in the p state. The rate constant for this transition therefore vanishes in the limit of zero temperature, and it varies with temperature as [51]

$$R_{s\rightarrow p}(T) = A\left(\frac{8}{\pi\mu}\right)^{1/2}(k_B T)^{3/2} 2!, \tag{19}$$

where k_B is the Boltzmann constant and A is a proportionality constant given by the ratio of the cross section and the collision energy. Using the value of the cross section in Fig. 3 at $B=1027$ G and $E=100$ kV/cm, we estimate the rate constant for the $s \rightarrow p$ excitation due to the electric field to be about 3×10^{-14} cm^3 s^{-1} at 10 nK and at 1 μK it is on the order of 3×10^{-11} cm^3 s^{-1}.

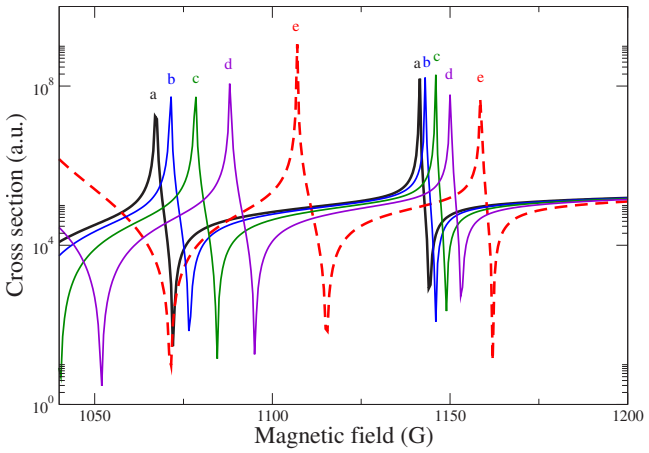

FIG. 6. (Color online) Cross sections for elastic s-wave collisions of Li and Cs in the $M_{\mathrm{Li}}=0$ and $M_{\mathrm{Cs}}=3$ states computed at zero electric field (curve labeled a) and electric field strengths of 30 kV/cm (curve labeled b), 50 kV/cm (curve labeled c), 70 kV/cm (curve labeled d), and 100 kV/cm (dashed curve labeled e). The collision energy is 10^{-7} cm^{-1}.

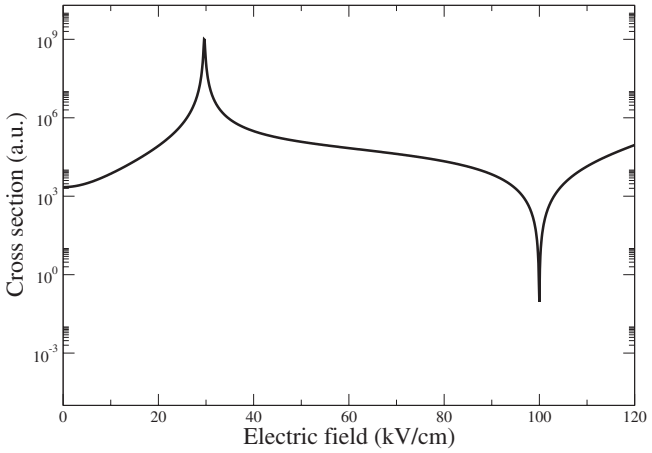

FIG. 7. Electric field dependence of the s-wave scattering cross section for collisions of Li and Cs in the $M_{\mathrm{Li}}=0$ and $M_{\mathrm{Cs}}=3$ states at the magnetic field strength 1071 G. The variation of the cross sections is due to shifts of the s-wave resonances shown in Fig. 6.

IV. CONCLUSIONS

We have presented a detailed analysis of Feshbach resonances in ultracold collisions of Li and Cs atoms in the presence of superimposed electric and magnetic fields. Our calculations show that electric fields below 100 kV/cm may significantly modify the collision dynamics in binary mixtures of ultracold gases by inducing couplings between s-wave and p-wave collision channels. These couplings are dramatically enhanced near s-wave or p-wave scattering resonances. Electric fields can thus be used for tuning the scattering length like in the experiments with magnetic Feshbach resonances [7]. We have identified two mechanisms for electric-field-control of ultracold atom mixtures: (i) shifts of the s-wave resonance positions and (ii) electric-field-induced resonances. The shifts of the positions of s-wave resonances can be induced by static fields on the order of

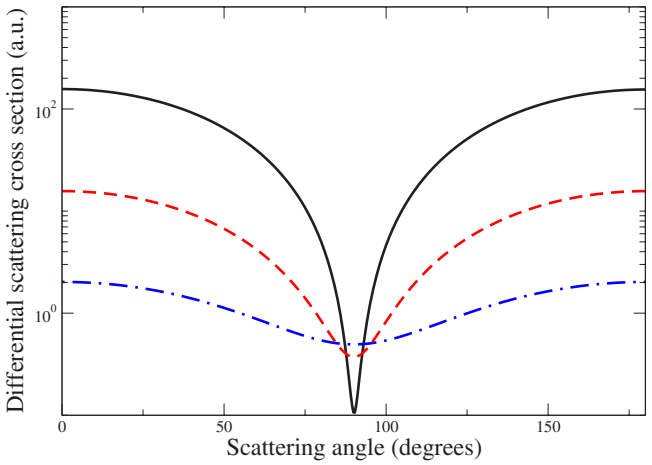

FIG. 8. (Color online) Differential scattering cross sections for ultracold collisions of Li and Cs in the $M_{\mathrm{Li}}=-1$ and $M_{\mathrm{Cs}}=3$ states computed at an electric field strength of 100 kV/cm. The collision energy is 10^{-5} cm^{-1} (solid curve), 10^{-6} cm^{-1} (dashed curve), and 10^{-7} cm^{-1} (dotted curve). The magnetic field is fixed at 1162 G.

30 kV/cm. This effect appears to depend on the width of the *s*-wave resonances and the strength of the background *p*-wave scattering, and it may result in rapid variations of the *s*-wave cross sections even hundreds of gauss away from *p*-wave resonances. Electric-field-induced resonances occur near *p*-wave scattering resonances. The *p*-wave enhancement of the cross section induces the resonant variation of the *s*-wave scattering even at ultralow energies when the probability of *p*-wave scattering is negligibly small.

Electric-field-induced resonances discussed here are three-state resonances involving the scattering *s*- and *p*-wave channels and a bound state of the molecules with the rotational angular momentum 1. The coupling between the *p*-wave scattering channel and the molecular state can be induced by magnetic fields as in the experiments of Ticknor *et al.* [52]. The coupling between the *s*- and *p*-wave scattering channels can be induced by electric fields as shown in this paper. Electric-field-induced Feshbach resonances may thus allow for two-dimensional control of interatomic interactions with both magnetic and electric fields. Electric-field-induced resonances may also be used in the search for *p*-wave resonances at ultracold temperatures. In the absence of electric fields, the *s*- and *p*-wave channels are uncoupled and ultracold collisions are dominated by *s*-wave scattering. It is therefore difficult, if not impossible, to detect *p*-wave resonances in ultracold gases of binary mixtures directly. Applying an electric field may thus be an important tool for spectroscopic studies of ultracold atoms. The measurements of *p*-wave resonances may provide important information for the analysis of interatomic interaction potentials, especially for systems with anisotropic long-range interactions [53].

Interactions between molecules will generally be characterized by a significant dipole moment function, and the resonances described in this paper will similarly occur in ultracold collisions of molecules. Electric fields induce couplings between different total angular momenta of the collision complex of molecules. Feshbach resonances of higher total angular momenta may thus affect ultracold molecular collisions through electric-field-induced couplings. The density of Feshbach resonances in molecule-molecule collisions is quite large [54], and we expect that the effects of electric fields on ultracold collisions of molecules will be even more pronounced than observed here. We have shown that electric fields induce the anisotropy of differential scattering at ultracold temperatures. Controllable angle-dependent scattering may modify the properties of ultracold gases such as the expansion of Bose-Einstein condensates released from the trap [55]. Measurements of the differential scattering cross sections may probe the anisotropy of interatomic interactions [56] and provide detailed information on molecular structure.

Finally, we would like to point out that the mechanism of electric field control described here does not perturb the separated atoms. The atoms interact with electric fields only when in a molecular collision complex, so the wave function of the isolated atoms may be more immune to decoherence than in varying magnetic or optical fields. Coupled with the possibility of tuning the electric fields very fast, this makes electric-field-induced resonances a useful tool for the development of quantum computation with ultracold atoms and molecules.

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