Sequential generation of matrix-product states in cavity QED

C. Schön,[1] K. Hammerer,[2] M. M. Wolf,[3] J. I. Cirac,[3] and E. Solano[4,5]

[1]*Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2BZ, United Kingdom*

[2]*Institute for Theoretical Physics, University of Innsbruck, 6020 Innsbruck, Austria*

[3]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany*

[4]*Physics Department, ASC, and CeNS, Ludwig-Maximilians-Universität, Theresienstrasse 37, 80333 Munich, Germany*

[5]*Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado Postal 1761, Lima, Peru*

(Received 13 December 2006; published 8 March 2007)

We study the sequential generation of entangled photonic and atomic multiqubit states in the realm of cavity QED. We extend the work of C. Schön *et al.* [Phys. Rev. Lett. **95**, 110503 (2005)], where it was shown that all states generated in a sequential manner can be classified efficiently in terms of matrix-product states. In particular, we consider two scenarios: photonic multiqubit states sequentially generated at the cavity output of a single-photon source and atomic multiqubit states generated by their sequential interaction with the same cavity mode.

DOI: 10.1103/PhysRevA.75.032311 PACS number(s): 03.67.Mn, 42.50.Dv, 42.50.Pq

I. INTRODUCTION

Triggered single-photon sources [1–3] have important applications in quantum communication and quantum computation and are subject of intense experimental investigation [4–17]. Substantial progress is being made along various routes using single atoms [4–8], ions [9,10], molecules [11], quantum dots [12–15], or color centers [16,17]. The basic operating principle common to these approaches is that a triggered single quantum emitter excites the mode of a cavity or a photonic crystal with a single-photon, which coherently leaks out into a well defined field mode. If the source is run in a cyclic fashion: sequentially initialized, loaded, and fired, the result is ideally a train of identical single-photon wave packets.

Given the impressive experimental achievements along these lines, it is interesting to go one step further and address the following scenario. Assume that the source is not initialized after each step, but stays in some quantum state, which can in turn be correlated to the field state generated so far. What kind of multipartite quantum states can then in principle be created with such a sequential generation scheme? We answered this question in Ref. [18] and proved that the class of sequentially generated states is exactly identical to a class of so-called matrix-product states (MPSs) [19–21]. These states play an important role in a completely different context, namely in the theory of one-dimensional spin chains [22], where they constitute the set of variational states over which density matrix renormalization group techniques are carried out [23–25]. This classification is significant in at least three respects. (i) It does not only apply to all the various types of single-photon sources [1–17], but literally to any system where a multipartite quantum state is generated by sequential interaction with a source or ancilla system [26]. (ii) It provides a constructive protocol for the generation of any desired quantum state. It allows one to decide whether or not a given state can be generated with a given setup and, if so, what sequential operations one has to apply. (iii) It stresses the importance of the MPS formalism as it demonstrates that they naturally occur in vastly different physical contexts.

In this work, we elaborate on the results of Ref. [18], providing more details and further applications. In Sec. II, we first illustrate the basic idea of sequential generation of quantum states by means of a cavity QED (CQED) single-photon source [4,5,9]. Then, we give a detailed derivation and extension of the main results in Ref. [18] without reference to any specific setup. In Sec. III, we show how to create important entangled multiqubit photonic states encoded in photonic time bins and in polarization states. In Sec. IV, we consider a microwave cavity interacting with a sequence of atoms flying through it, such as in Refs. [28,29], generating entangled multiqubit atomic states.

II. SEQUENTIAL GENERATION OF ENTANGLED MULTIQUBIT STATES

In this section, we consider first the CQED single-photon source [4,5,9] and show, with an elementary example, how the structure of MPS arises quite naturally in this setup. We then treat the sequential generation scenario without referring to any particular physical system. First, we assume that an arbitrary source-qubit interaction is available in each step of the sequential generation. In the second part, we restrict the interaction in a way which resembles the situation in current cavity QED setups for the generation of single-photon pulses. Finally, we discuss a more abstract scenario, a register of qubits, to which nearest neighbor gates are applied sequentially. This situation is again covered conveniently by the MPS formalism, even if the sequence of gates is applied repeatedly.

A. Basic idea

The CQED single-photon source utilizes the possibility to excite the cavity mode via an atom, which is trapped inside the cavity [4,5,9]. The photon is then coherently emitted through the cavity mirror. A photonic qubit may be defined either by different polarization states or by the absence and the presence of the photon. If we allow for specific operations inside the source before each photon emission, we will

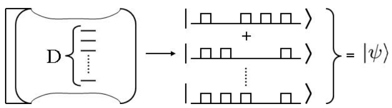

FIG. 1. A trapped D-level atom is coupled to a cavity qubit, determined by the energy eigenstates $|0\rangle$ and $|1\rangle$. After bipartite source-qubit operations, photonic time bins are sequentially and coherently emitted at the cavity output, creating a desired entangled multiqubit stream.

be able to create different multiqubit states at the output (see Fig. 1).

Typically, a CQED single-photon source employs an effective two-level atom with hyperfine ground states $|a\rangle$ and $|b\rangle$ coupled via a Raman transition driven by the cavity mode and an external laser. The latter controls the population transfer

$$|a,0\rangle \to \cos(\phi_1)|a,0\rangle + \sin(\phi_1)e^{i\varphi_1}|b,1\rangle, \tag{1}$$

where $|0\rangle$ and $|1\rangle$ denote the absence and the presence of a photon in the cavity mode. This so-called time-bin qubit is then coherently emitted as depicted in Fig. 1.

We define $c_i = \cos(\phi_i)$ and $s_i = \sin(\phi_i)e^{i\varphi_i}$ for step i and repeat the procedure n times. We end up with

$$\begin{aligned}|a\rangle &\to c_1|a,0\rangle + s_1|b,1\rangle\\ &\to c_1c_2|a,0,0\rangle + |b\rangle(c_1s_2|1,0\rangle + s_1|0,1\rangle)\\ &\to \cdots\\ &\to |b\rangle[c_1\cdots c_{n-1}|1,0,\ldots,0\rangle + c_1\cdots c_{n-2}s_{n-1}|0,1,0,\ldots,0\rangle\\ &\quad + \cdots + c_1s_2|0,\ldots,0,1,0\rangle + s_1|0,\ldots,0,1\rangle],\end{aligned} \tag{2}$$

where we chose $c_n = 0$ and $s_n = 1$. Then, the atom decouples and the resulting photonic state is a W-type n-qubit state.

This is just a particular example of a more general sequential generation scheme: an ancillary system $\mathcal{A}$ (the atom) with Hilbert space $\mathcal{H}_{\mathcal{A}} \simeq \mathbb{C}^D$ (D=2) couples sequentially to initially uncorrelated qubits $\mathcal{B}_i$ (the time-bin qubits) with Hilbert spaces $\mathcal{H}_{\mathcal{B}} \simeq \mathbb{C}^2$. In every step we have a unitary time evolution of the joint system $\mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$. Since each qubit is initially in the state $|0\rangle$, we disregard the qubits at the input and write the evolution in the form of an isometry $V: \mathcal{H}_{\mathcal{A}} \to \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$. The photon-generating process discussed above is given by

$$V_{[i]} = c_i|0\rangle_i|a\rangle\langle a| + s_i|1\rangle_i|b\rangle\langle a| + |0\rangle_i|b\rangle\langle b|, \tag{3}$$

for step i and fulfills the isometry condition $V_{[i]}^\dagger V_{[i]} = \mathbb{1}_2$. The final state can then be written as

$$|\Psi\rangle = V_{[n]}\cdots V_{[1]}|\varphi_I\rangle, \tag{4}$$

where $|\varphi_I\rangle = |a\rangle$ is the initial state of the atom. Since the atom decouples in the last step, we may also write

$$|\Psi\rangle = |\varphi_F\rangle\langle\varphi_F|V_{[n]}\cdots V_{[1]}|\varphi_I\rangle = |\varphi_F\rangle|\psi\rangle, \tag{5}$$

where $|\varphi_F\rangle = |b\rangle$ is the final state of the atom and $|\psi\rangle$ the n-qubit photonic state. The goal in the following sections is to classify all achievable states $|\psi\rangle$ in terms of the required resources such as number of ancilla levels D and possible operations on the atom-cavity system $V_{[i]}$.

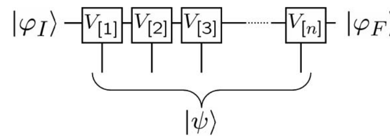

FIG. 2. Sequential generation of a multiqubit state $|\psi\rangle$. In each step the D-dimensional ancilla produces one qubit. This process is described by a $(2\mathrm{D}\times\mathrm{D})$-dimensional isometry $V_{[i]}$. The achievable multiqubit states are instances of matrix-product states with D-dimensional bonds and open boundary conditions specified by $|\varphi_I\rangle$ and $|\varphi_F\rangle$.

B. Arbitrary source-qubit interaction

Now, we want to look at the problem from a more general perspective. We assume that the operators $V_{[i]}$ are arbitrary isometries and that the ancilla decouples in the last step. We express the isometries in a given basis

$$V = \sum_{i,\alpha,\beta} V^i_{\alpha,\beta}|\alpha,i\rangle\langle\beta|, \tag{6}$$

where each V^i is a $D\times D$ matrix and $\{|\alpha\rangle,|\beta\rangle\}$ are any of the D ancillary levels. This is the generalization of Eq. (3) and the isometry condition then reads

$$V^\dagger V = \sum_{i=0}^{1} V^{i\dagger}V^i = \mathbb{1}_D. \tag{7}$$

The resulting n-qubit state is then given by

$$|\psi\rangle = \sum_{i_1\cdots i_n=0}^{1} \langle\varphi_F|V^{i_n}_{[n]}\cdots V^{i_1}_{[1]}|\varphi_I\rangle|i_n,\ldots,i_1\rangle. \tag{8}$$

This is a matrix-product state (MPS) [20,21] with D-dimensional bonds and open boundary conditions, which are specified by the initial ancilla state $|\varphi_I\rangle$ and the final ancilla state $|\varphi_F\rangle$. In Fig. 2, we illustrate the sequential generation process schematically.

In practice, the question whether a given state $|\tilde{\psi}\rangle$ can be generated with certain resources is more important. Therefore we have to show that every MPS of the form

$$|\tilde{\psi}\rangle = \langle\tilde{\varphi}_F|\tilde{V}_{[n]}\cdots\tilde{V}_{[1]}|\tilde{\varphi}_I\rangle, \tag{9}$$

with arbitrary maps $\tilde{V}_{[k]}: \mathcal{H}_{\mathcal{A}} \to \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$, can be generated by isometries of the same dimension such that the ancilla decouples in the last step. Since the idea of the proof is an explicit construction of all involved isometries, it will provide a general recipe for the sequential generation of $|\tilde{\psi}\rangle$. Note that every state has a MPS representation [19,30].

We start by writing

$$(\langle\tilde{\varphi}_F|\otimes\mathbb{1}_2)\tilde{V}_{[n]} = V'_{[n]}M_{[n]}, \tag{10}$$

where the 2×2 matrix $V'_{[n]}$ is the left unitary in the singular value decompositions (SVD) of the left hand side and $M_{[n]}$ is the remaining part.

The recipe for constructing the isometries is the induction

$$(M_{[k]} \otimes \mathbb{1}_2)\widetilde{V}_{[k-1]} = V'_{[k-1]}M_{[k-1]}, \tag{11}$$

where the isometry $V'_{[k-1]}$ is constructed from the SVD of the left hand side, and $M_{[k-1]}$ is always chosen to be the remaining part.

After n applications of Eq. (11) in Eq. (9), from left to right, we set $|\varphi_I\rangle = M_{[1]}|\widetilde{\varphi}_I\rangle$, producing

$$|\widetilde{\psi}\rangle = V'_{[n]} \cdots V'_{[1]}|\varphi_I\rangle. \tag{12}$$

Simple rank considerations show that $V'_{[n-k]}$ has dimension $2\min[D,2^k] \times \min[D,2^{k+1}]$. The dimension of the left unitary grows exponentially, i.e., $V'_{[n-k+1]}$ has dimension $2^k \times 2^k$, as long as $2^k < D$. For $2^{k+1} > D$, superfluous columns appear in $V'_{[n-k]}$ since the original matrix $(M_{[n-k+1]} \otimes \mathbb{1}_2)\widetilde{V}_{[n-k]}$ has at most D singular values. Truncation leads to a $2^{k+1} \times D$ dimensional isometry $V'_{[n-k]}$.

Now $M_{[n-k]}$ has dimension $D \times D$ and all subsequent left isometries have dimension $2D \times D$. Therefore every $V'_{[k]}$ can be embedded into an isometry $V_{[k]}$ of dimension $2D \times D$. Physically, this means that we have redundant ancillary levels which we do not use. Finally, decoupling the ancilla in the last step is guaranteed by the fact that, after the application of $V_{[n-1]}$, merely two levels of $\mathcal{H}_{\mathcal{A}}$ are yet occupied, and can be mapped entirely onto the system $\mathcal{H}_{\mathcal{B}}$. This is precisely the action of $V_{[n]}$ through its embedded unitary $V'_{[n]}$.

Together with Eqs. (8) and (9), this proves the equivalence of three sets of n-qubit states.

(1) MPS with D-dimensional bonds and open boundary conditions.

(2) States which are generated sequentially and isometrically by a D-dimensional ancillary system which decouples in the last step. That is, the generation is deterministic.

(3) States which can be generated sequentially by a D-dimensional ancillary system in a probabilistic manner. That is, the preparation may only be successful with some probability and include measurements and conditional operations.

In Secs. II C and II D, we will show two other equivalent classes. We emphasize that this result holds as well for higher dimensional systems (beyond qubits) and that its constructive proof provides a recipe for the sequential generation of any state with minimal resources, namely a D-dimensional source or ancilla for a D-dimensional MPS.

Note that in order to obtain MPS with periodic boundary conditions we require an interaction between the first and the last qubit. For systems, where an ancilla is available, one may store the first qubit within the ancilla. Then, in order to produce MPS with D-dimensional bonds, we would require a $(2D)$-dimensional system to store the additional qubit. On the other hand many interesting states belong to the class of MPS with two-dimensional bonds and in this case two additional atomic levels would suffice.

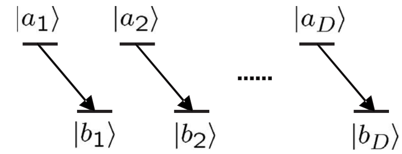

FIG. 3. Restricted interaction between ancilla and qubit. Each atomic transition from $|a_i\rangle$ to its respective $|b_i\rangle$ is accompanied by the generation of a photon in a certain time bin.

C. The standard map

The situation considered above assumes that arbitrary isometries can be achieved in the generation of a qubit, which amounts to have complete control over the source-qubit interaction. This does not quite correspond to what is the case in current cavity QED single-photon sources [4,5,9], where only atomic degrees of freedom can be manipulated easily, while the isometry describing the generation of a photon (qubit) is fixed. In the following we will show that also in this restricted scenario it is possible to generate arbitrary MPS with D-dimensional bonds if a $2D$-level atom is used as a source.

We consider an atomic system with D states $|a_i\rangle$ and D states $|b_i\rangle$, as depicted in Fig. 3, so that $\mathcal{H}_{\mathcal{A}} = \mathcal{H}_a \oplus \mathcal{H}_b \simeq \mathbb{C}^D \otimes \mathbb{C}^2$. That is, we will write $|\varphi\rangle|1\rangle$ for a superposition of $|a_i\rangle$ states, whereas $|\varphi\rangle|0\rangle$ denotes a superposition of $|b_i\rangle$ states. Since the last qubit marks whether the atomic level belongs to the $|a_i\rangle$ or to the $|b_i\rangle$ subspace, we will refer to it as the tag-qubit and write $\mathcal{H}_{\mathcal{A}} = \mathcal{H}_{\mathcal{A}'} \otimes \mathcal{H}_{\mathcal{T}}$.

Now assume atomic transitions from each $|a_i\rangle$ state to its respective $|b_i\rangle$ state are accompanied by the generation of a photon in a certain time bin. This is described by a unitary evolution, from now on called "D-standard map," of the form

$$T:|\varphi\rangle_{\mathcal{A}'}|1\rangle_{\mathcal{T}}|0\rangle_{\mathcal{B}} \mapsto |\varphi\rangle_{\mathcal{A}'}|0\rangle_{\mathcal{T}}|1\rangle_{\mathcal{B}},$$

$$|\varphi\rangle_{\mathcal{A}'}|0\rangle_{\mathcal{T}}|0\rangle_{\mathcal{B}} \mapsto |\varphi\rangle_{\mathcal{A}'}|0\rangle_{\mathcal{T}}|0\rangle_{\mathcal{B}}. \tag{13}$$

Hence, T effectively interchanges the tag-qubit with the time-bin qubit. If, additionally, arbitrary atomic unitaries $U_{\mathcal{A}}$ are allowed at any time, we can exploit the swap caused by T in order to generate the operation

$$V|\varphi\rangle = \langle 0|_{\mathcal{T}} T(U_{\mathcal{A}}(|\varphi\rangle_{\mathcal{A}'}|0\rangle_{\mathcal{T}})|0\rangle_{\mathcal{B}}), \tag{14}$$

which is the most general isometry $V:\mathcal{H}_{\mathcal{A}'} \to \mathcal{H}_{\mathcal{A}'} \otimes \mathcal{H}_{\mathcal{B}}$. Equation (14) is also illustrated in Fig. 4.

Therefore, the so generated n-qubit states include all possible states arising from subsequent applications of $(2D \times D)$-dimensional isometries. On the other hand, they are a subset of the MPS in Eq. (9) with arbitrary $(2D \times D)$-dimensional maps, assuming that the atom decouples at the end. Hence, this set is again equivalent to the three mentioned above.

D. Qubit-qubit interaction without ancilla

In Fig. 5(a) we depicted a system of N initially uncorrelated qubits, which interact sequentially with their nearest

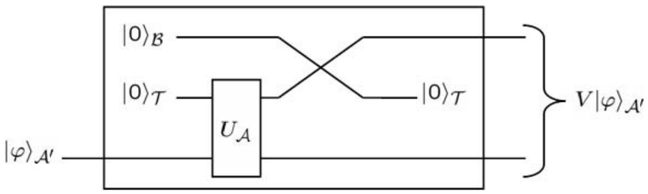

FIG. 4. The box represents a single step in the sequential generation scheme. Since tag- and time-bin qubit are always in state $|0\rangle$ after the preceding step, they can be ignored at the input. After an arbitrary unitary operation $U_{\mathcal{A}}$ on the atom, tag- and time-bin qubit are interchanged by the standard map. Ignoring the tag qubit also at the output (since it is always in $|0\rangle$), the process can be described by an arbitrary isometry V applied on the effective ancillary system $\mathcal{A}'$.

neighbor qubit. This situation is in fact identical to the one considered so far, as one can imagine the operation $V_{[k]}$ being performed not between qubit k and $k+1$ directly but between qubit k and a two-dimensional ancilla, which is then swapped on qubit $k+1$ [see Fig. 5(b)]. In the last step, the swap ensures that the ancilla decouples from the desired multiqubit state $|\psi\rangle$. Thus, also for direct qubit-qubit interaction, the class of achievable states $|\psi\rangle$ is equivalent to the class of MPS with two-dimensional bonds (see Ref. [19] for a more formal argument). In Sec. IV, we will come back to this scenario and consider an experimental setup, where always two neighbor atoms of a chain interact via a common cavity mode.

If direct two-qubit interaction between neighbors is possible, there exists at least in principle no reason why one should not apply them more than once. If this is done m times, as illustrated in Fig. 6, still in a sequential manner, i.e., after an operation between the first and the last qubit one starts again with qubits 1 and 2, the achievable class of states

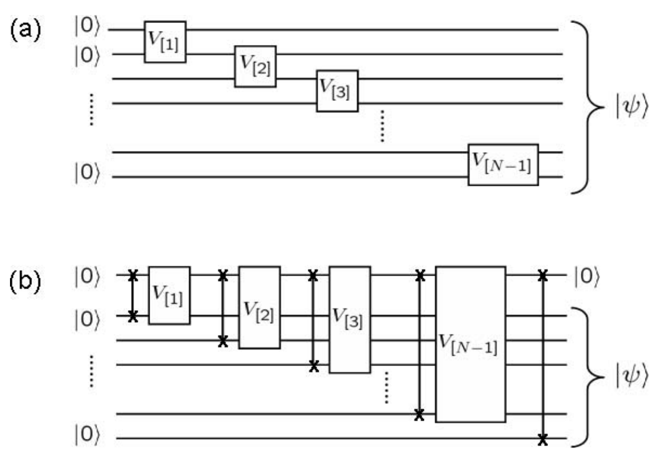

FIG. 5. (a) Two-qubit gates $V_{[i]}$ are sequentially applied between nearest neighbor qubits. This situation can be simulated by a two-dimensional ancilla as demonstrated in (b). Instead of applying the gate between neighboring qubits k and $k+1$, we use qubit k and the ancilla and swap the ancilla state afterwards with the qubit $k+1$. Since we can merge the swap operations and the arbitrary unitary operations $V_{[i]}$ to arbitrary unitary operations, we know that the class of achievable states $|\psi\rangle$ is equivalent to the class of MPS with open boundary conditions and two-dimensional bonds.

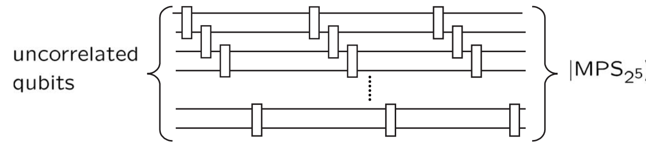

FIG. 6. Repeating the procedure m (here $m=3$) times defines a class of states described by MPS with (2^{2m-1})-dimensional bonds. Note that in between the three sequences we also allow an arbitrary interaction between the first and the last qubit.

will be described by MPS with bonds of dimension d^{2m-1}, that is for qubits 2^{2m-1}. However, in this case the two sets will no longer be equivalent, i.e., there are MPS with $D=2^{2m-1}$ for whose preparation we need more than m such layers of two-qubit gates.

III. SEQUENTIAL GENERATION OF PHOTONIC STATES

There are two possible ways of encoding quantum information in photon pulses: orthogonal polarization states and energy eigenstates. While the latter is more straightforward in terms of the required resources, polarization encoding [31,32] avoids the trouble caused by a failure of the source for the generation of multiqubit entangled states since a missing photon is not being mistaken as an empty time bin, i.e., the success of the encoding step is heralded by the observation of a photon. The generation of multiphoton entangled states using several distant single photon sources was explored in Ref. [33].

A. Time-bin entanglement

We will first consider a situation where cavity decay is negligible on the time scale of the operations performed on the atom-cavity system. After the cavity qubit has left, we start with the next step and repeating the process leads to a multiqubit entangled photon state at the output as sketched in Fig. 1.

1. Arbitrary source-qubit operation

In the following, we demonstrate how an arbitrary unitary operation on the $(2D)$-dimensional Hilbert space of the combined atom-cavity system can be realized. Therefore, we view the D-level atom as a set of M qubits with $D\leqslant 2^M$ as depicted in Fig. 7. Then, we have to show that one can perform arbitrary two-qubit operations between each pair of qubits, i.e., universal quantum computing. Since the atomic levels can be manipulated at will using Raman laser systems

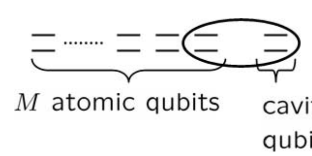

FIG. 7. The D-level atom can be viewed as a set of M qubits with $D\leqslant 2^M$. For an arbitrary operation we need local unitaries for the atomic qubits and a universal two-qubit gate between the cavity qubit and one specific atomic qubit.

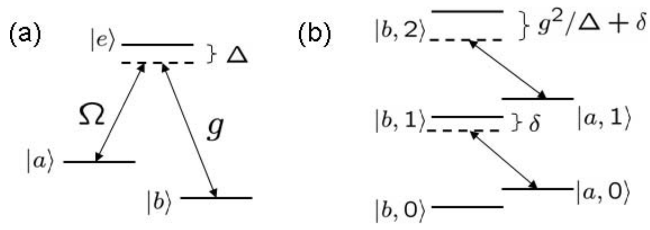

FIG. 8. (a) Atomic level structure: levels $|a\rangle$ ($|b\rangle$) and $|e\rangle$ are coupled by a laser (cavity mode) off resonance. (b) After adiabatic elimination of the upper state $|e\rangle$, we are left with a Jaynes-Cummings type of Hamiltonian, where states $|a,n\rangle$ and $|b,n+1\rangle$ are coupled. Both, the energy difference of those levels and the corresponding Rabi frequency depends on n. The reason for the first is the ac-Stark shift, whereas the second is due to the Jaynes-Cummings coupling.

[34,35], it remains to propose an arbitrary two-qubit gate between one specific atomic qubit and the cavity qubit.

Therefore, we consider a typical three-level Λ configuration (see Fig. 8), where the hyperfine ground states $|a\rangle$ and $|b\rangle$ are coupled to the excited level $|e\rangle$ off-resonantly through a laser with Rabi frequency Ω and detuning $\Delta+\delta$, and the cavity mode a with coupling strength g and detuning δ.

Furthermore, we assume that the cavity decay rate κ is smaller than any other frequency in the problem, so that we can ignore cavity damping during the atom-cavity manipulations. In an appropriate interaction picture, the Hamiltonian of the system is then given by

$$H = -\Delta(\sigma_{aa} + a^\dagger a) + g(\sigma_{eb}a + a^\dagger\sigma_{be}) + \frac{\Omega}{2}(e^{-i\delta t}\sigma_{ea} + e^{i\delta t}\sigma_{ae}), \tag{15}$$

with $\sigma_{kl}=|k\rangle\langle l|$, $\{k,l\}=\{a,b,e\}$ and $|\Delta| \gg g, \Omega \gg \delta$. After adiabatically eliminating the excited state $|e\rangle$, the Hamiltonian for the effective $D=2$ atomic system plus cavity mode, in an interaction picture with respect to $-\Delta(\sigma_{aa}+a^\dagger a)$, is given by

$$H_{\text{ad}} = \frac{\Omega^2}{4\Delta}\sigma_{aa} + \frac{g^2}{\Delta}a^\dagger a\sigma_{bb} + \frac{g\Omega}{2\Delta}(e^{-i\delta t}\sigma_{ab}a + e^{i\delta t}a^\dagger\sigma_{ba}). \tag{16}$$

It describes an effective Jaynes-Cummings coupling between the cavity mode and the atomic $|a\rangle \to |b\rangle$ transition with Rabi frequency $g\Omega/2\Delta$. The other terms correspond to ac-Stark shifts. In an interaction picture with respect to the latter the Hamiltonian is given by

$$H_{\text{ad}}^I = \sum_{n=0}^{\infty} \frac{\sqrt{n}g\Omega}{2\Delta}(e^{-i(\delta-\Omega^2/4\Delta+ng^2/\Delta)t}|a,n-1\rangle\langle b,n| + \text{H.c.}). \tag{17}$$

Then, we choose the laser frequency such that

$$\delta = \frac{\Omega^2}{4\Delta} - \frac{g^2}{\Delta}. \tag{18}$$

For $g^2/\Delta \gg g\Omega/2\Delta$, the effective interaction in all subspaces with $n \neq 0$ is then dispersive and the selective Hamiltonian [36] is given by

$$\begin{aligned} H_{\text{sel}} &= \frac{g\Omega}{2\Delta}(|a,0\rangle\langle b,1| + |b,1\rangle\langle a,0|) \\ &= \frac{g\Omega}{4\Delta}(\sigma_x^{\mathcal{A}} \otimes \sigma_x^{\mathcal{B}} + \sigma_y^{\mathcal{A}} \otimes \sigma_y^{\mathcal{B}}), \end{aligned} \tag{19}$$

where $\sigma_i^{\mathcal{A}}$ and $\sigma_i^{\mathcal{B}}$ denote the Pauli matrices acting on the atomic and the photonic qubit, respectively. Using a laser pulse of an appropriate duration, we obtain the entangling two-qubit gate

$$\sqrt{i\text{SWAP}} = \exp[i\pi(|a,0\rangle\langle b,1| + |b,1\rangle\langle a,0|)/4]. \tag{20}$$

Together with local operations in both qubits [37], this suffices to generate an arbitrary two-qubit operation.

2. Adiabatic passage

In current CQED single-photon sources [4–6,9], an adiabatic passage is employed to realize the one-standard map introduced above. Using one additional level, we will show how to generate familiar multiqubit states such as W [38], GHZ [39], and cluster states [40], which are all MPS with $D=2$ [41].

For this purpose, we consider an atom with three effective levels $\{|a\rangle, |b_1\rangle, |b_2\rangle\}$ trapped inside an optical cavity. With the help of a laser beam, state $|a\rangle$ is mapped to the internal state $|b_1\rangle$, and a photon is generated, whereas the other states remain unchanged. This physical process is described by the map

$$\begin{aligned} M_{\mathcal{AB}}: |a\rangle &\mapsto |b_1\rangle|1\rangle, \\ |b_1\rangle &\mapsto |b_1\rangle|0\rangle, \\ |b_2\rangle &\mapsto |b_2\rangle|0\rangle, \end{aligned} \tag{21}$$

and can be realized with the techniques used in Refs. [4,5,9]. After the application of this process, an arbitrary operation is applied to the atom, which can be performed with Raman transitions. The photonic states that are generated after several applications are those MPS with isometries $V_{[i]} = M_{\mathcal{AB}}U_{\mathcal{A}}^{[i]}$, with $i=1,\ldots,n$, $U_{\mathcal{A}}^{[i]}$ being arbitrary unitary atomic operators.

For example, to generate a W-type state of the form

$$\begin{aligned} |\psi_{\text{W}}\rangle &= e^{i\Phi_1}\sin\Theta_1|0\cdots 01\rangle + \cos\Theta_1 e^{i\Phi_2}\sin\Theta_2|0\cdots 010\rangle \\ &\quad + \cdots + \cos\Theta_1\cdots\cos\Theta_{n-2}e^{i\Phi_{n-1}}\sin\Theta_{n-1}|010\cdots 0\rangle \\ &\quad + \cos\Theta_1\cdots\cos\Theta_{n-1}|10\cdots 0\rangle, \end{aligned} \tag{22}$$

we choose the initial atomic state $|\varphi_I\rangle = |b_2\rangle$ and operations $U_{\mathcal{A}}^{[i]} = U_{ab_2}^{b_1}(\Phi_i,\Theta_i)$, with $i=1,\ldots,n-1$, where

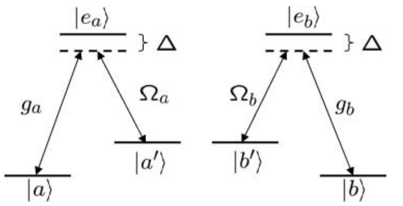

FIG. 9. Levels $|a'\rangle$ and $|b'\rangle$ ($|a\rangle$ and $|b\rangle$) are coupled off-resonance to $|e_a\rangle$ and $|e_b\rangle$ by two lasers (cavity modes).

$$U_{kl}^m(\Phi_i,\Theta_i) = \cos\Theta_i|k\rangle\langle k| + \cos\Theta_i|l\rangle\langle l| + e^{i\Phi_i}\sin\Theta_i|k\rangle\langle l| - e^{-i\Phi_i}\sin\Theta_i|l\rangle\langle k| + |m\rangle\langle m|, \quad (23)$$

and $\{k,l,m\}=\{a,b_1,b_2\}$. To decouple the atom from the photon state, we choose the last atomic operation $U_{\mathcal{A}}^{[n]}=U_{ab_2}^{b_1}(0,\pi/2)$ and, after the last map $M_{\mathcal{AB}}$, the decoupled atom will be in state $|b_1\rangle$. To produce a GHZ-type state in similar way, we choose $|\varphi_I\rangle=|a\rangle$, $U_{\mathcal{A}}^{[1]}=U_{ab_2}^{b_1}(\Phi_1,\Theta_1)$, $U_{\mathcal{A}}^{[i]}=U_{ab_1}^{b_2}(0,\pi/2)$, with $i=2,\ldots,n-1$, and $U_{\mathcal{A}}^{[n]}=U_{b_1b_2}^{a}(0,\pi/2)U_{ab_1}^{b_2}(0,\pi/2)$.

For generating cluster states, we choose $|\varphi_I\rangle=|b_2\rangle$, $U_{\mathcal{A}}^{[i]}=U_{ab_2}^{b_1}(\Phi_i,\Theta_i)U_{ab_1}^{b_2}(0,\pi/2)$, with $i=1,\ldots,n-1$, and $U_{\mathcal{A}}^{[n]}=U_{ab_1}^{b_2}(\Phi_n,\Theta_n)U_{b_1b_2}^{a}(0,\pi/2)U_{ab_1}^{b_2}(0,\pi/2)$, obtaining

$$|\psi\rangle = \bigotimes_{i=1}^{n}(O_{i-1}^0|0\rangle_i + O_{i-1}^1|1\rangle_i), \quad (24)$$

where $O_{i-1}^0=\cos\Theta_i|0\rangle_{i-1}\langle 0|-e^{-i\Phi_i}\sin\Theta_i|1\rangle_{i-1}\langle 1|$ and $O_{i-1}^1=e^{i\Phi_i}\sin\Theta_i|0\rangle_{i-1}\langle 0|+\cos\Theta_i|1\rangle_{i-1}\langle 1|$, with $i=2,\ldots,n-1$. Operators O_{i-1}^0 and O_{i-1}^1 act on the nearest neighbor qubit $i-1$ under the assumption $O_0^0\equiv\cos\Theta_1$ and $O_0^1\equiv e^{i\Phi_1}\sin\Theta_1$. If one chooses $\Phi_i=0$ and $\Theta_i=\pi/4$, this leads to the cluster states defined by

$$|\psi_{\rm cl}\rangle = \frac{1}{2^{n/2}}\bigotimes_{i=1}^{n}(\sigma_{i-1}^z|0\rangle_i + |1\rangle_i), \quad \text{with } \sigma_0^z\equiv 1. \quad (25)$$

B. Polarization entanglement

Here, the cavity qubit is defined by single excitations in two modes a and b, which have equal frequencies but orthogonal polarizations. As above for the time-bin qubits, we will first show how to realize arbitrary source-qubit operations and then focus on the less demanding scenario, where the photonic qubits are generated by a standard map, i.e., an adiabatic passage.

1. Arbitrary source-qubit operation

In this section, we will suggest how to implement an arbitrary operation on the atom-cavity system based on present cavity QED experiments [4,5,9]. Therefore, one has to show how an arbitrary unitary operation on one specific atomic qubit $\{|a\rangle,|b\rangle\}$ and the cavity qubit $\{|1_a\rangle,|1_b\rangle\}$ can be realized. We consider a double-Λ-type atomic level configuration as illustrated in Fig. 9, which couples to the two cavity modes via two Raman transitions.

Two external laser fields drive the transitions from level $|a'\rangle$ to the excited level $|e_a\rangle$ and from level $|b'\rangle$ to the excited level $|e_b\rangle$ with Rabi frequencies Ω_a and Ω_b, respectively. The cavity modes a and b couple to the transitions between $|e_a\rangle$ and level $|a\rangle$ with coupling strength g_a and $|e_b\rangle$ and level $|b\rangle$ with coupling strength g_b.

For large detunings

$$|\Delta_a|,|\Delta_b| \gg g_a,g_b,\Omega_a,\Omega_b \quad (26)$$

we can adiabatically eliminate the excited levels $|e_a\rangle$ and $|e_b\rangle$ and end up with a Jaynes-Cummings type of Hamiltonian, which is block separable in the subspaces spanned by $\{|a',n_a\rangle,|a,n_a+1_a\rangle\}$ and $\{|b',n_b\rangle,|b,n_b+1_b\rangle\}$, where n_a and n_b denote the number of photons in mode a and b. As above, the ac-Stark shifts for $n_a=n_b=0$ can be compensated by choosing the frequency of the laser fields appropriately. Under the condition

$$\frac{n_a g_a^2}{\Delta_a} \gg \frac{g_a\Omega_a}{2\Delta_a}, \quad \frac{n_b g_b^2}{\Delta_b} \gg \frac{g_b\Omega_b}{2\Delta_b}, \quad (27)$$

we obtain the selective Hamiltonian

$$H_{\rm sel}^p = \frac{g_a\Omega_a}{2\Delta_a}|a',0_a\rangle\langle a,1_a| + \frac{g_b\Omega_b}{2\Delta_b}|b',0_b\rangle\langle b,1_b| + \text{H.c.}, \quad (28)$$

where $|0_a\rangle$ and $|0_b\rangle$ denote the empty cavity modes a and b. For appropriate $\Omega_a(t)$ and $\Omega_b(t)$, the evolution operator reads

$$U_{\rm sel}^p = \exp[i\pi(|a',0_a\rangle\langle a,1_a| + |b',0_b\rangle\langle b,1_b| + \text{H.c.})/2]. \quad (29)$$

After the kth photonic qubit leaked out of the cavity, the state of the system is given by

$$|\Psi_k\rangle = \alpha|a\rangle|\psi_k^a\rangle + \beta|b\rangle|\psi_k^b\rangle, \quad (30)$$

where $|\psi_k^a\rangle$ and $|\psi_k^b\rangle$ are k-qubit photonic states. Now we have to initialize the system for the next step, i.e., provide the polarization qubit

$$|\Psi_k\rangle \to -i(\alpha|a',0_a\rangle|\psi_k^a\rangle + \beta|b',0_b\rangle|\psi_k^b\rangle) \to \alpha|a,1_a\rangle|\psi_k^a\rangle + \beta|b,1_b\rangle|\psi_k^b\rangle, \quad (31)$$

where we applied $U_{\rm sel}^p$ in the second line. In order to realize an arbitrary two-qubit gate, we combine local operations with a $\sqrt{i\text{SWAP}}$ two-qubit gate between the atomic and the photonic qubit. The latter can be achieved in three steps,

$$\sqrt{i\text{SWAP}} = (U_{\rm sel}^p)^{-1}e^{i\pi(|a'\rangle\langle b'|+|b'\rangle\langle a'|)/4}U_{\rm sel}^p. \quad (32)$$

It remains to show how to decouple the atom from the generated multiphoton state in the final step n. Therefore, one has to map the atomic state on the last photonic qubit: after the transformation of the atomic levels $|a\rangle\to -i|a'\rangle$ and $|b\rangle\to -i|b'\rangle$, driving only the $|a'\rangle|0\rangle\to|a\rangle|1_a\rangle$ transition by choosing $\Omega_b=0$ in $U_{\rm sel}^p$ leads to

$$|\Psi_{n-1}\rangle \to \alpha|a,1_a\rangle|\psi_a\rangle - i\beta|b',0_b\rangle|\psi_b\rangle, \quad (33)$$

where the cavity mode b remains empty and level $|b\rangle$ is not yet populated. Now we transform $|a\rangle\to|b\rangle$, then apply $U_{\rm sel}^p$

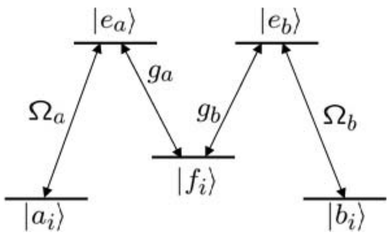

FIG. 10. Atomic level structure: level $|a_i\rangle$ ($|b_i\rangle$) are coupled to $|f_i\rangle$ via an adiabatic passage driven by a laser with Rabi frequency Ω_a (Ω_b) and the cavity mode a (b) with coupling strength g_a (g_b).

again and, since the photon in the mode a cannot be absorbed, end up with

$$|\Psi_n\rangle = |b\rangle \otimes (\alpha|1_a\rangle|\psi_a\rangle + \beta|1_b\rangle|\psi_b\rangle), \tag{34}$$

where the atom decouples in its final state $|b\rangle$.

2. Adiabatic passage

We consider an effective three-level system, with atomic ground states $|a_i\rangle$, $|b_i\rangle$, and $|f_i\rangle$, which couples to the two cavity modes a and b through two independent adiabatic passages controlled by two external lasers with Rabi frequencies Ω_a and Ω_b as depicted in Fig. 10. In each generation step the standard map is achieved,

$$|a_i\rangle \rightarrow |f_i\rangle|1_a\rangle \rightarrow |b_i\rangle|1_a\rangle,$$

$$|b_i\rangle \rightarrow |f_i\rangle|1_b\rangle \rightarrow |b_i\rangle|1_b\rangle, \tag{35}$$

where we first applied the adiabatic passages and then a unitary operation $|f_i\rangle \rightarrow |b_i\rangle$. In general, we have $i=1,\ldots,D$. Instead of an effective $2D$-level atomic system, as for the generation of time-bin entangled MPS_D, we require a $3D$-level atomic system. Note that the results for the generation of time-bin qubits apply: for W-type, GHZ and cluster states we require only $D=1$ and one additional level, i.e., an effective four-level atom.

IV. SEQUENTIAL GENERATION OF ATOMIC STATES

For the generation of atomic multi-qubit states the cavity mode and the atom interchange their function. Now, the cavity field is employed as the ancillary system to interact with initially uncorrelated atoms which sequentially pass through the cavity. Therefore, we require a very stable cavity field which is provided typically by microwave cavities [28,29]. In the second part of this section, we consider another scenario where the atomic qubits interact directly via a common cavity mode [42].

A. Cavity acts as ancillary system

In the first scenario the atoms pass through the cavity in such a manner that only one atom couples to a single cavity mode at a time, as depicted in Fig. 11. We employ the same atomic level configuration as in Sec. III A and define the ancilla qubit as the cavity mode Fock states $|0\rangle$ and $|1\rangle$. The atom-cavity system is then described by the selective Hamiltonian from Eq. (19). Choosing the field frequency and pulse length appropriately leads to an iSWAP gate between the photonic and the atomic qubit. In the basis $\{|b,0\rangle,|b,1\rangle,|a,0\rangle,|a,1\rangle\}$,

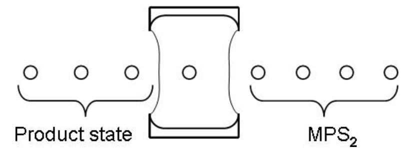

FIG. 11. A stream of uncorrelated atoms crosses a cavity. The atomic qubits couple sequentially to the cavity mode, which acts as a two-dimensional ancillary system. If an arbitrary unitary operation can be realized between the cavity qubit and the atomic qubits, the class of entangled multiqubit atomic states at the output are equivalent to the class of MPS_2.

$$i\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{36}$$

Three applications of the iSWAP gate accompanied by local unitary operations suffice for an arbitrary two-qubit gate [27], though they could only be applied in setups where the position of the atoms is fully controlled [6,7,9]. In fact, it would not only allow the generation of all MPS with two-dimensional bonds, but also all other states since the atoms could be moved back and forth, allowing for universal quantum computing. One of those setups use optical cavities and, unfortunately, the cavity decay is not negligible on the time scale in which the atoms are moved in and out of the cavity.

For setups with high-finesse microwave cavities [28,29], the atoms pass once the cavity and local rotations can be implemented via suitably placed Ramsey zones. So the natural question arises: which states can be generated if only one iSWAP gate and local unitaries on the atomic qubits are available?

If all atoms are prepared in state $|a\rangle$ and the cavity mode is initially empty, sequential application of the $\sqrt{i\text{SWAP}}$ gate leads to

$$\begin{aligned}
|0\rangle &\rightarrow \frac{1}{\sqrt{2}}|0\rangle|a_1\rangle + \frac{i}{\sqrt{2}}|1\rangle|b_1\rangle \\
&\rightarrow \frac{1}{2}|0\rangle|a_2,a_1\rangle + \frac{i}{2}|1\rangle|b_2,a_1\rangle + \frac{i}{\sqrt{2}}|1\rangle|a_2,b_1\rangle \\
&\rightarrow \cdots \\
&\rightarrow \frac{1}{\sqrt{2^{n-1}}}|0\rangle|a_{n-1}\cdots a_1\rangle + \frac{i}{\sqrt{2^{n-1}}}|1\rangle|b_{n-1},a_{n-2},\ldots,a_1\rangle \\
&\quad + \cdots + \frac{i}{\sqrt{2}}|1\rangle|a_{n-1},\ldots,a_2,b_1\rangle.
\end{aligned} \tag{37}$$

This W-type state is still entangled with the cavity qubit. One way of solving this problem is to measure the cavity qubit in the basis $(|0\rangle \pm |1\rangle)/\sqrt{2}$.

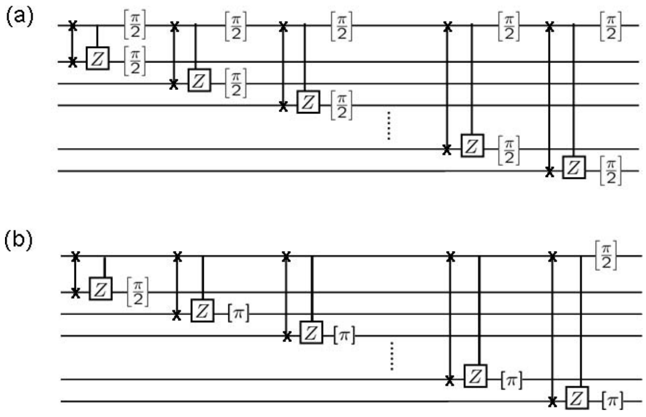

FIG. 12. Cluster state generation. (a) An iSWAP gate is performed sequentially between the atomic qubits and the cavity mode. (b) One can as well assume that the local unitaries are performed on the atomic qubit k after the kth SWAP gate. Therefore they can be compensated in a final step.

Let us concentrate on the generation of the one-dimensional cluster state given in Eq. (25). The required control-Z (CZ) gate between neighboring qubits can be realized through a CZ gate between the ancilla and the first qubit followed by a SWAP gate operation on the ancilla and the second qubit. Therefore, we decompose

$$i\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{38}$$

where the first matrix represents the SWAP gate operation and the second matrix is equivalent to the CZ gate up to local operations. As has been depicted in Fig. 12(a), the iSWAP can then be written as

$$i\text{SWAP} = i \text{ SWAP CZ}[R_z(\pi/2) \otimes R_z(\pi/2)], \tag{39}$$

with

$$R_z(\phi) = e^{-i\sigma_z\phi/2} = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}. \tag{40}$$

Since it is difficult to perform local operations on the cavity qubit, say in step k, we make use of the SWAP gate and apply it instead in step $k+1$ after the SWAP gate on qubit $k+1$. Fortunately, $R_z(\pi/2)$ commutes with the CZ gate and we obtain the recipe shown in Fig. 12(b). The last unitary on the ancilla can be ignored because it has no influence on the desired multiqubit state. In order to obtain the cluster state as defined in Eq. (25), we have to compensate for the local unitaries on the atoms and prepare all atoms initially in superpositions of $(|0\rangle+|1\rangle)/\sqrt{2}$.

B. No ancilla: Atoms interact via cavity mode

Using the cavity mode as an ancilla has the disadvantage that it may decay during the time-interval between two successive atoms. This problem can be avoided by a direct interaction between the atomic qubits via the common cavity mode. As sketched in Fig. 13, there are always two atoms at the same time inside the cavity. Zheng and Guo [43] proposed the implementation of a $\sqrt{i\text{SWAP}}$ between the atomic qubits via a dispersive scheme. Note that in order to implement a controlled-NOT (CNOT) gate, an additional atomic level would be required [43]. Recent experiments [7,9] raise hope that it will soon be possible to move two atoms into a cavity in a well controlled manner. Then the $\sqrt{i\text{SWAP}}$ accompanied by local unitaries suffices to perform an arbitray two-qubit operation.

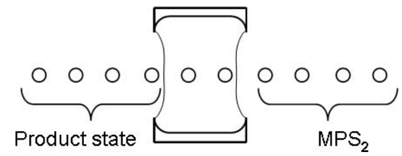

FIG. 13. A stream of uncorrelated atoms passes a cavity. Two atomic qubits couple to each other via the common cavity mode. If an arbitrary unitary operation can be realized between them, the class of entangled multiqubit atomic states at the output are quivalent to the class of MPS_2.

If we consider that both atoms cross the cavity only once [44], where an $\sqrt{i\text{SWAP}}$ has been demonstrated, the question arises again: which mutliqubit atomic states can be sequentially generated in this manner? Since the atomic qubits can be manipulated locally before and after the gate with Ramsey zones, we gain more possibilities than in the corresponding ancilla case.

The generation of the W and cluster states follows the lines of the previous case of Sec. IV B, since we only replace the ancilla by the neighbor qubit and the superfluous SWAP gate does not affect the desired output state. In order to engineer a GHZ state, we use another decomposition of iSWAP gate, given by

$$i\text{SWAP} = i\text{SWAP}[R_z(\pi/2) \otimes R_z(\pi/2)][\mathbb{1} \otimes H]\text{CNOT}[\mathbb{1} \otimes H], \tag{41}$$

where the Hadamard gate is given by

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{42}$$

Since we can compensate the local unitaries acting on the atomic qubits, we end up with a sequential application of a CNOT gate followed by a SWAP gate on all neighboring qubits as depicted in Fig. 14.

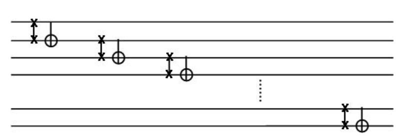

FIG. 14. The iSWAP gate between neighboring qubits is equivalent to a SWAP gate followed by a CNOT gate up to local unitaries. The latter can be compensated since local operations can be applied to the atomic qubits with Ramsey zones.

Now, let us assume that our initial state $|\psi_I\rangle$ has all atoms prepared in level $|a\rangle$, except the first which enters the cavity in the superposition $(|a\rangle+|b\rangle)/\sqrt{2}$,

$$|\psi_I\rangle \to \frac{1}{\sqrt{2}}|a\cdots a\rangle \otimes (|a,a\rangle + |b,b\rangle) \to \cdots \to \frac{1}{\sqrt{2}}(|a,\ldots,a\rangle + |b,\ldots,b\rangle). \tag{43}$$

Here, the arrows in step i indicate the application of a CNOT gate and a SWAP gate between qubits i and $i+1$, producing at the end the desired GHZ state.

Further experimental efforts will make use of two consecutive cavities in the same setup [45]. This opens up new possibilities in terms of state engineering in the light of the present work. Three cavities in a row, which are crossed by atoms such that unitary operations may be performed between the cavities, would even allow for arbitrary two-qubit operations between the atoms and therefore lead to a class of states equivalent to the class of MPS_2.

V. CONCLUSIONS

We developed a formalism to describe the sequential generation of entangled multiqubit states. It classifies the states achievable by a D-dimensional ancilla as MPS with D-dimensional bonds and provides a recipe for the sequential generation of any state. It turns out that all states that can be generated nondeterministically with a D-dimensional ancilla also belong to the class of MPS with D-dimensional bonds. Therefore, we will be able to provide a recipe for their deterministic generation as well. Remark that the formalism applies also to a situation where qubits interact directly in a sequential manner.

For the generation of entangled multi-qubit photonic states by a cavity QED single-photon source, we propose am implementation of an arbitrary interaction between the atom and a cavity qubit defined either by the absence and the presence of a photon or by the polarization of single excitation in the cavity. Moreover, we discussed the case of an adiabatic passage for the photon generation as being employed for single-photon generation in current cavity QED experiments [4,5,9].

For coherent microwave cavity QED experiments [28], where atoms sequentially cross a cavity and interact with the same cavity mode, we give a recipe for the generation of W-type states as well as cluster states. We considered also the case of direct coupling of successive atoms via the cavity mode. For this case we show how to generate GHZ states, in addition to the W-type and cluster states. Other physical scenarios as a light pulse crossing several atomic ensembles [46] or trapped ion experiments, where each ion interacts sequentially with a collective mode of the motion [34,35,47], may also be described by the present formalism.

ACKNOWLEDGMENTS

We would like to thank G. Giedke and F. Verstraete for useful and stimulating discussions. This work was supported by the European Union projects COVAQIAL, QAP, CONQUEST, and SCALA. C.S. acknowledges support from the U.K. Engineering and Physical Sciences Research Council and partial support from the Quantum Information Processing Interdisciplinary Research Collaboration (QIP IRC). K.H. acknowledges support from the Austrian Science Foundation. E.S. acknowledges financial support from DFG SFB 631 and EU EuroSQIP project.

[1] B. Lounis and M. Orrit, Rep. Prog. Phys. **68**, 1129 (2005).
[2] M. Oxborrow and A. G. Sinclair, Contemp. Phys. **46**, 173 (2005).
[3] H. Walther, Prog. Phys. **54**, 617 (2006).
[4] A. Kuhn, M. Hennrich, and G. Rempe, Phys. Rev. Lett. **89**, 067901 (2002).
[5] J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, and H. J. Kimble, Science **303**, 1992 (2004).
[6] B. Darquie, M. P. A. Jones, J. Dingjan, J. Beugnon, S. Bergamini, Y. Sortais, G. Messin, A. Browaeys, and P. Grangier, Science **309**, 454 (2005); J. Beugnon, M. P. A. Jones, J. Dingjan, B. Darquie, G. Messin, A. Browaeys, and P. Grangier, Nature (London) **440**, 779 (2006).
[7] S. Nußmann, K. Murr, M. Hijlkema, B. Weber, A. Kuhn, and G. Rempe, Nat. Phys. **1**, 122 (2005).
[8] T. Aoki, B. Dayan, E. Wilcut, W. P. Bowen, A. S. Parkins, T. J. Kippenberg, K. J. Vahala, and H. J. Kimble, Nature (London) **443**, 671 (2006).
[9] M. Keller, B. Lange, K. Hayasaka, W. Lange, and H. Walther, Nature (London) **431**, 1075 (2004).
[10] B. B. Blinov, D. L. Moehring, L. M. Duan, and C. Monroe, Nature (London) **428**, 153 (2004).
[11] B. Lounis and W. E. Moerner, Nature (London) **407**, 491 (2000).
[12] M. Pelton, C. Santori, J. Vuckovic, B. Y. Zhang, G. S. Solomon, J. Plant, and Y. Yamamoto, Phys. Rev. Lett. **89**, 233602 (2002); C. Santori, D. Fattal, J. Vuckovic, G. S. Solomon, and Y. Yamamoto, Nature (London) **419**, 594 (2002).
[13] J. P. Reithmaier, G. Sek, A. Loffler, C. Hofmann, S. Kuhn, S. Reitzenstein, L. V. Keldysh, V. D. Kulakovskii, T. L. Reinecke, and A. Forchel, Nature (London) **432**, 197 (2004).
[14] P. Lodahl, A. F. van Driel, I. S. Nikolaev, A. Irman, K. Overgaag, D. L. Vanmaekelbergh, and W. L. Vos, Nature (London) **430**, 654 (2004).
[15] R. M. Stevenson, R. J. Young, P. Atkinson, K. Cooper, D. A. Ritchie, and A. J. Shields, Nature (London) **439**, 179 (2006).
[16] R. Brouri, A. Beveratos, J. P. Poizat, and P. Grangier, Opt. Lett. **25**, 1294 (2000).
[17] C. Kurtsiefer, S. Mayer, P. Zarda, and H. Weinfurter, Phys. Rev. Lett. **85**, 290 (2000).
[18] C. Schön, E. Solano, F. Verstraete, J. I. Cirac, and M. M. Wolf, Phys. Rev. Lett. **95**, 110503 (2005).

[19] D. Perez-García, F. Verstraete, M. M. Wolf, and J. I. Cirac, e-print quant-ph/0608197 (2006).
[20] A. Klümper, A. Schadschneider, and J. Zittartz, J. Phys. A **24**, L955 (1991); Z. Phys. B: Condens. Matter **87**, 281 (1992); Europhys. Lett. **24**, 293 (1993).
[21] M. Fannes, B. Nachtergaele, and R. F. Werner, Commun. Math. Phys. **144**, 443 (1992).
[22] I. Affleck *et al.*, Chem. Phys. **115**, 477 (1988).
[23] S. Östlund and S. Rommer, Phys. Rev. Lett. **75**, 3537 (1995).
[24] G. Vidal, Phys. Rev. Lett. **93**, 040502 (2004).
[25] F. Verstraete, D. Porras, and J. I. Cirac, Phys. Rev. Lett. **93**, 227205 (2004).
[26] Y. Delgado, L. Lamata, J. León, D. Salgado, and E. Solano, e-print quant-ph/0607105.
[27] J. Zhang, J. Vala, S. Sastry, and K. B. Whaley, Phys. Rev. A **69**, 042309 (2004).
[28] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J.-M. Raimond, and S. Haroche, Science **288**, 2024 (2000).
[29] S. Brattke, B. T. H. Varcoe, and H. Walther, Phys. Rev. Lett. **86**, 3534 (2001).
[30] G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003).
[31] K. M. Gheri, C. Saavedra, P. Törma, J. I. Cirac, and P. Zoller, Phys. Rev. A **58**, R2627 (1998); C. Saavedra, K. M. Gheri, P. Törmä, J. I. Cirac, and P. Zoller, *ibid.* **61**, 062311 (2000).
[32] T. Wilk, H. P. Specht, S. C. Webster, G. Rempe, and A. Kuhn, e-print quant-ph/0603083.
[33] A. Beige, Y. L. Lim, and C. Schön, J. Mod. Opt. **54**, 397 (2007).
[34] D. Kielpinski, C. Monroe, and D. J. Wineland, Nature (London) **417**, 709 (2002).
[35] S. Gulde, M. Riebe, G. P. T. Lancaster, C. Becher, J. Eschner, H. Häffner, F. Schmidt-Kaler, I. L. Chuang, and R. Blatt, Nature (London) **421**, 48 (2003).
[36] M. F. Santos, E. Solano, and R. L. de Matos Filho, Phys. Rev. Lett. **87**, 093601 (2001).
[37] M. F. Santos, Phys. Rev. Lett. **95**, 010504 (2005).
[38] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).
[39] D. M. Greenberger, M. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), pp. 69–72.
[40] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).
[41] F. Verstraete and J. I. Cirac, Phys. Rev. A **70**, 060302(R) (2004).
[42] N. Sangouard, X. Lacour, S. Guerin, and H. R. Jauslin, Phys. Rev. A **72**, 062309 (2005).
[43] S. B. Zheng and G. C. Guo, Phys. Rev. Lett. **85**, 2392 (2000).
[44] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J.-M. Raimond, and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001).
[45] S. Haroche (private communication).
[46] B. Julsgaard, C. Schori, J. L. Sørensen, and E. S. Polzik, Quantum Inf. Comput. **3**, 518 (2003).
[47] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).