Generation and discrimination of Greenberger-Horne-Zeilinger states using dipole-induced transparency in a cavity-waveguide system

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We propose an efficient scheme to build an arbitrary multipartite Greenberger-Horne-Zeilinger state and discriminate all the universal Greenberger-Horne-Zeilinger states using parity measurement based on dipole-induced transparency in a cavity-waveguide system. A prominent advantage is that initial entangled states remain after nondetective identification and they can be used for successive tasks. We analyze the performance and possible errors of the required single-qubit rotations and emphasize that the scheme is reliable and can satisfy the current experimental technology.

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In recent years, quantum information science [1] has made rapid progress in entanglement preparation, scalable quantum networks, and quantum communication in several candidates including photons, atoms, quantum dots, etc. [2]. Entangled states are an indispensable resource which is widely applied in many aspects such as the quantum key distribution [3], quantum logic gates [4], and so on. The Greenberger-Horne-Zeilinger (GHZ) state is one of the most important multiparticle entangled states for quantum information processing and for addressing the Einstein-Podolsky-Rosen (EPR) conflict of local realism with quantum mechanics [5]. Several proposals have been put forward to generate GHZ states in the cavity-QED realm [6–8]. On many occasions, one needs to discriminate entangled states to post-select relevant quantum operations to complete a given task. A simple scheme has been presented to identify the two typical GHZ states from other variations under local flip operations using linear-optics elements [9]. Unfortunately, the photons are absorbed by the detectors and the initial entangled states are destroyed finally because of destructive measurements. Therefore, nondestructive analysis of entangled states is also an important and open problem and more effective strategies are expected to overcome the difficulties.

Very recently Waks *et al.* proposed an interesting method, which was called dipole-induced transparency (DIT), to generate and detect two-qubit entangled states in a cavity-waveguide system [10]. They showed that when a dipole (atoms, quantum dots, etc.) is placed in a drop-filter cavity, the input field remains in the original waveguide; if, however, there is no dipole in the cavity, the input field is transmitted to the another waveguide in the critical coupling [11]. Surprisingly, such an effect is valid even in the low-Q regime where the coupling strength between the cavity mode and the dipole transition is much less than the cavity decay rate. Thus the constraint condition on the strong coupling in the cavity-dipole system is unnecessary and the conditions for realizing this effect can be met in a more practical parameter range for solid-state materials. Furthermore, they showed that DIT can be used to generate entanglement between spatially separated dipoles conditionally and to perform full Bell-state measurement on two dipoles, which are extremely useful for quantum repeaters [12]. A very interesting application is that DIT can be used to perform a parity check for two dipoles without destroying the states of the dipoles; that is, one can check whether the states of two spatially separated dipoles are the same (even parity) or different (odd parity) through measuring the light fields in the waveguide. This parity measurement is very useful for entanglement discrimination because one can probe the information of the qubits without destroying their states.

In this paper, we present a simple and efficient scheme to generate an arbitrary multiqubit GHZ state and propose a quite different scheme to thoroughly realize the identification of universal GHZ states based on DIT in the cavity-waveguide system. We also show how DIT can be used to joint different chains of GHZ states. The feasibility of our scheme is demonstrated with the reach of current experimental technology.

Now let us discuss how to generate an arbitrary multiqubit GHZ state using parity measurement based on DIT. Figure 1 shows the schematic setups we are considering. A cavity contains a single dipole emitter and is evanescently coupled to up and down waveguides. Each dipole is assumed to have three relevant energy levels: a ground state $|g\rangle$, an excited state $|e\rangle$, and a long-lived metastable state $|m\rangle$. The photon detectors a_{even} and a_{odd} are used to check the parity of the dipoles. Waks *et al.* showed that DIT can be used to generate an entangled state between two dipoles $|gm\rangle-|mg\rangle$; here and in what follows, the normalized factors are ignored for simplicity. We rotate the second dipole under a local transformation: $|g\rangle\to-|m\rangle, |m\rangle\to|g\rangle$. Thus, the above two-qubit state becomes $|gg\rangle+|mm\rangle$. With

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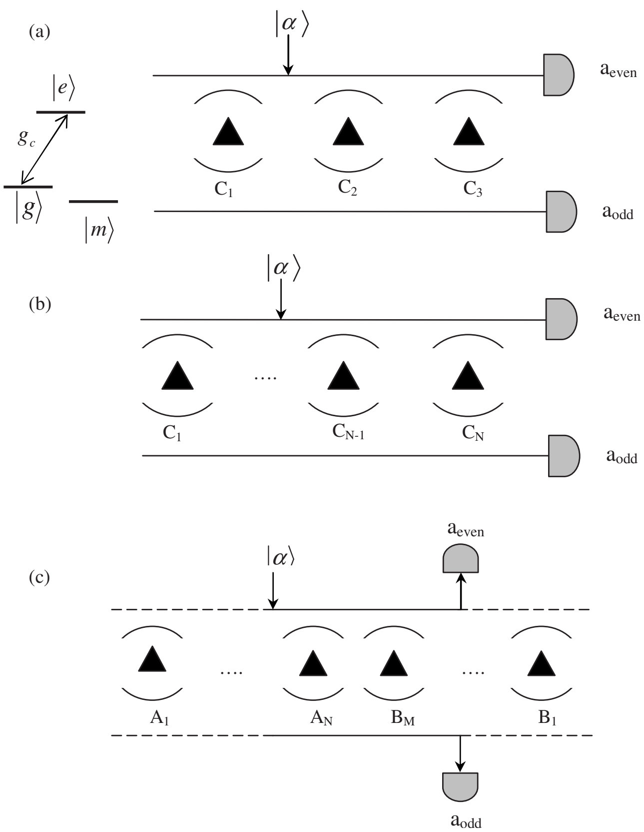

FIG. 1. (a) Generating a three-dipole GHZ state from a Bell state by measuring the parity of the dipoles in C_2 and C_3. Each dipole has three relevant energy level $|g\rangle$, $|m\rangle$, and $|e\rangle$. The cavity mode interacts with the transition between $|g\rangle$ and $|e\rangle$ resonantly. (b) Generating an N-dipole GHZ state from an $(N-1)$-dipole GHZ state by checking the parity of the last two dipoles. (c) Joining two GHZ-state chains by measuring the parity of the end qubits of the two chains.

this state and another qubit we can generate a three-dipole GHZ state $|ggg\rangle+|mmm\rangle$ through a parity measurement. As shown in Fig. 1(a), we add a cavity in which the third dipole is prepared in the state $|g\rangle+|m\rangle$ and the total state of the three dipoles is

$$(|g_1g_2\rangle+|m_1m_2\rangle)\otimes(|g_3\rangle+|m_3\rangle) = |g_1g_2g_3\rangle+|m_1m_2m_3\rangle+|g_1g_2m_3\rangle+|m_1m_2g_3\rangle. \quad (1)$$

Now we check the parity of the dipoles in the cavities C_2 and C_3 by injecting a probe field $|\alpha\rangle$ into the waveguide before the cavity C_2. We find immediately that the total state of the three dipoles will reduce to the former two terms of the right-hand side of Eq. (1) if the states of the dipoles in C_2 and C_3 are the even parity and we obtain a GHZ state $|ggg\rangle+|mmm\rangle$. On the other hand, if the dipoles in C_2 and C_3 are found in the odd-parity state, the total state collapses to $|ggm\rangle+|mmg\rangle$. We then carry out a local flip operation on the third dipole, $|g_3\rangle\leftrightarrow|m_3\rangle$, to obtain a three-dipole GHZ state $|ggg\rangle+|mmm\rangle$. Furthermore, as shown in Fig. 1(b), we can generalize this method to build a multidipole GHZ state of an arbitrary length. Similar to the case of three dipoles, we add a cavity-dipole subsystem in which the state of the dipole is $(|g\rangle+|m\rangle)$ and input a probe field to measure the parity of the last two dipoles. For example, in order to achieve an N-dipole GHZ state by using an $(N-1)$-dipole GHZ state $|g_1g_2\cdots g_{N-1}\rangle+|m_1m_2\cdots m_{N-1}\rangle$, we only need to attach a cavity-dipole subsystem [the state of the dipole is $(|g_N\rangle+|m_N\rangle)$] to the terminal of the preceding $(N-1)$ cavity-dipole system and input a weak coherent field into the fiber just before the cavity C_{N-1} which is used to check the parity of the dipoles in the cavity C_{N-1} and C_N. If the outcome of the odd parity is obtained, we apply a local flip operation on the dipole in C_N, or else we do nothing. Therefore, we will get an N-dipole GHZ state in a deterministic way no matter whether the parity of the last two dipoles is even or odd.

In addition, as shown in Fig. 1(c), we can join two GHZ-state chains which include N and M qubits, respectively, to form a longer chain of GHZ states with $(N+M)$ dipoles by applying a parity measurement on the end qubits of the two chains. When the parity of the two qubits in the neighbor cavities A_N and B_M is even, we obtain a GHZ state with $(N+M)$ dipoles. If the parity of the end qubits is odd, we will get the GHZ state by performing local flip operations on each dipole of one of the two chains or checking the parity of the end qubits again. Thus we can use parity measurements to join two GHZ-state chains. These operations are extremely attractive for long-distance quantum communication and quantum repeater.

Next let us show in detail how a universal GHZ state [9] for matter qubits can be discriminated with parity measurements and single-qubit rotations. It has been demonstrated that the universal GHZ state with arbitrary number of photons can be effectively distinguished in the weak nonlinearity regime [13]. The universal GHZ states with three dipoles are given by

$$|\Phi^{\pm}\rangle = (|g_1g_2g_3\rangle \pm |m_1m_2m_3\rangle)/\sqrt{2}, \quad (2a)$$

$$|\Psi_1^{\pm}\rangle = (|m_1g_2g_3\rangle \pm |g_1m_2m_3\rangle)/\sqrt{2}, \quad (2b)$$

$$|\Psi_2^{\pm}\rangle = (|g_1m_2g_3\rangle \pm |m_1g_2m_3\rangle)/\sqrt{2}, \quad (2c)$$

$$|\Psi_3^{\pm}\rangle = (|g_1g_2m_3\rangle \pm |m_1m_2g_3\rangle)/\sqrt{2}. \quad (2d)$$

We adopt a two-step strategy to fulfill the analysis of a universal three-dipole GHZ state: (i) we can distinguish the four classes in Eq. (2) from each other only through two parity measurements; (ii) single-qubit rotation and a parity check of three dipoles are used to separate two states in each class with the relevant phase π. The setup is depicted in Fig. 2. In the first step, we utilize DIT to measure the parities of the dipoles in C_1 and C_2 and in C_2 and C_3, respectively. For the first parity measurement, two mirrors are inserted into the optical path to guide the probe beam $|\alpha_1\rangle$ to the detectors D_1 and D_4 and avoid the coupling between $|\alpha_1\rangle$ and the cavity C_3. Then for the second parity measurement, the mirrors are removed and the probe beam $|\alpha_2\rangle$ is input just before the cavity C_2. The results of parity measurements are listed in Table I. Four classes of universal GHZ states result in different combinations of the photon detectors' clicks:

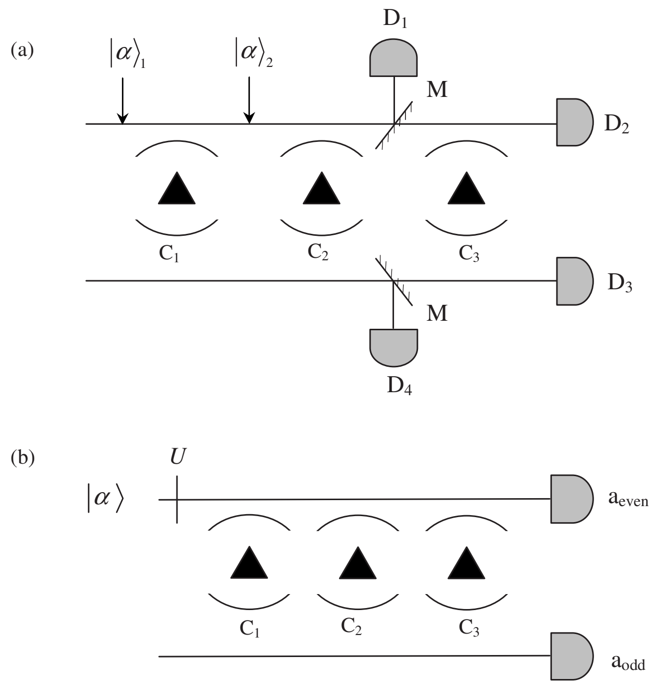

FIG. 2. (a) Separating the three-dipole GHZ state into four classes by two successive parity measurements. For the first measurement, the mirrors guide the probe field $|\alpha_1\rangle$ into the detector D_1 or D_4. For the second measurement, after the mirrors are removed, $|\alpha_2\rangle$ passes through C_2 and C_3 and clicks D_2 or D_3 to check the parity of the dipoles in them. (b) After single-qubit rotations U on three dipoles, a probe field is used to distinguish two states which have a relative phase of π in the same class.

$D_1D_2(|\Phi^{\pm}\rangle)$, $D_4D_2(|\Psi_1^{\pm}\rangle)$, $D_4D_3(|\Psi_2^{\pm}\rangle)$ and $D_1D_3(|\Psi_3^{\pm}\rangle)$. Next, in the second step, we perform single-qubit unitary operations U on each of the dipoles: $|g\rangle\rightarrow(|g\rangle+i|m\rangle)$ and $|m\rangle\rightarrow(|g\rangle-i|m\rangle)$. For example, the states $|\Phi^{\pm}\rangle$ evolve into

$$|\Phi^{+}\rangle \rightarrow |g_1g_2g_3\rangle - |g_1m_2m_3\rangle - |m_1g_2m_3\rangle - |m_1m_2g_3\rangle, \tag{3a}$$

$$|\Phi^{-}\rangle \rightarrow i(|g_1g_2m_3\rangle + |g_1m_2g_3\rangle + |m_1g_2g_3\rangle - |m_1m_2m_3\rangle). \tag{3b}$$

The evolutions of $|\Psi_{1,2,3}^{\pm}\rangle$ are similar to (3). The initial $|\Phi^{+}\rangle$ and $|\Psi_{1,2,3}^{+}\rangle$ (we call them $|+\rangle$ states) result in the total state

TABLE I. The results of photon detectors in two successive parity measurements using DIT. According to this, eight GHZ states are separated into four classes.

GHZ states	First parity measurement ($|\alpha_1\rangle$)	Second parity measurement ($|\alpha_2\rangle$)
$|\Phi^{\pm}\rangle$	Even/D_1 [a]	Even/D_2
$|\Psi_1^{\pm}\rangle$	Odd/D_4	Even/D_2
$|\Psi_2^{\pm}\rangle$	Odd/D_4	Odd/D_3
$|\Psi_3^{\pm}\rangle$	Even/D_1	Odd/D_3

[a]Which means the parity of the dipoles are even and the detector D_1 clicks.

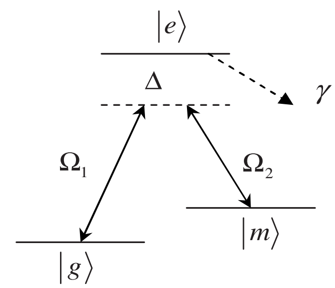

FIG. 3. Single-qubit rotation in two-photon Raman configuration. The detunings between two optical fields and the dipole transitions are large enough to cancel the exited state adiabatically.

$|ggg\rangle\oplus|gmm\rangle\oplus|mgm\rangle\oplus|mmg\rangle$; otherwise, $|\Phi^{-}\rangle$ and $|\Psi_{1,2,3}^{-}\rangle$ (we call them $|-\rangle$ states) result in $|mmm\rangle\oplus|mgg\rangle\oplus|gmg\rangle\oplus|ggm\rangle$, where $\oplus$ means the coherent superposition of different product states of the dipoles. Apparently, in the case of $|+\rangle$ states, one or three dipoles are in $|g\rangle$ and two or zero dipoles are in $|m\rangle$, while, on the contrary, the states $|-\rangle$ lead to one or three dipoles in $|m\rangle$ and two or zero dipoles in $|g\rangle$. Thus we design a very simple setup to separate the states $|\pm\rangle$ [see Fig. 2(b)]. A weak coherent light goes through the three cavities and $|+\rangle$ ($|-\rangle$) makes the probe beam into the even (odd) detector. So far, the eight GHZ states are distinguished in a deterministic way. Finally, we can perform the inverse transformation U^{-1} on each of the dipoles to recover the initial GHZ states. The extension to the N-dipole case is straightforward, and our two-step strategy is also valid.

There are several inevitable factors that will reduce the successful probability and the fidelity of entangled states. First, in the presence of cavity loss, since a spontaneously emitted photon reveals the "which-way" information of the qubit, there is a trade-off between the fidelity of the final state and the probability of a successful parity measurement [14]. Second, we must consider a practical error in a single-quibt operation in our scheme. In Fig. 3, we use a two-photon Raman transition to control the coherent transfer between the two qubit states $|g\rangle$ and $|m\rangle$ [15,16]. To alleviate the influence of spontaneous emission, the detunings between two optical fields and the corresponding dipole transitions are large enough. The Rabi frequencies of the applied fields are $\Omega_1\sim\Omega_2=\Omega$, the detunings are Δ (we assume that two-photon detuning is zero), and the spontaneous emission rate of the excited state is γ. When $\Delta\gg\Omega,\gamma$, the three-level system is equivalent to the two-level qubit with the effective Rabi frequency $\Omega_{eff}=\Omega^2/\Delta$ between the metastable states $|g\rangle$ and $|m\rangle$. For the time of single-qubit operations $T=\pi/4\Omega_{eff}$, the fidelity of the unitary transformation U is $(1+e^{-\pi\gamma/2\Delta})^2/4$. For example, for $\Delta>200\gamma$, the fidelity is larger than 99% and the error probability due to the decay from the exited state $P_e^{sp}\leqslant 1\%$. An appropriate candidate may be a defect dipole cavity in a planar photonic cavity coupled to a quantum dot [17]. For typical experimental parameters using quantum dots in photonic crystal cavities, we set $\gamma=1$ GHz [18] and $\Delta>200$ GHz is easy to realize in the experiments. The Rabi frequency Ω has no influence on the fidelity of single-qubit gates, but determines the time of gate operations. In addition, in the case that the qubit in the cavity is quantum-dot spin, conduction-band-hole Raman transitions induced by laser field and cavity modes can be used to realize arbitrary single-qubit rotations [19,20]. Third, when the separation of the qubits is very large, which is very common in quantum networks, the photon loss in the waveguide has a nontrival disadvantage that decreases the intensity of the probe field and reduces the successful probability of the parity measurement. But it can be overcome if we choose probe fields with appropriate intensity and low-loss waveguides.

In summary, we propose a novel scheme utilizing DIT to generate the multiparticle entangled states and to discriminate all of the universal N-qubit GHZ states. These are useful to quantum communication, quantum secret key sharing, and quantum state tomography.

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