Macroscopic superposition and entanglement for displaced thermal fields induced by a single atom

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(Received 17 November 2006; published 30 March 2007)

We show that a cavity field can evolve from an initial displaced mixed thermal state to a macroscopic superpositions of displaced thermal states via resonant interaction with a two-level atom. As a macroscopic system (meter) is really in a mixed state before coupling with the microscopic system at some temperature, our result is important for studying the quantum measurement problem and decoherence under real conditions. For the two-mode case, entanglement of displaced thermal states between the modes can be obtained.

DOI: 10.1103/PhysRevA.75.032114 PACS number(s): 03.65.Ta, 42.50.Dv, 42.50.Pq

I. INTRODUCTION

The superposition principle is the most striking principle in quantum mechanics. The macroscopic superposition states, known as Schrödinger cat states [1], are of significance for exploring the boundary between quantum and classical worlds and understanding the decoherence effect in quantum information. The macroscopic superposition states also provide a test for quantum measurement theory. In the measurement model, the coupling between a macroscopic apparatus and a microscopic system results in their entanglement and produces a quantum superposition state of the whole system.

The Jaynes-Cummings model (JCM), the simplest system in quantum optics describing the interaction between light and matter, is an ideal system for studying quantum interference effects [2,3]. It has been shown that if the cavity field is initially in a coherent state with a large average photon number superpositions of coherent states can be obtained in the resonant JCM [4]. Recent advances in experiments involving the passage of single Rydberg atoms through a superconducting cavity have turned the JCM from a theoretical curiosity to a useful and testable enterprise [5]. Recently, mesoscopic superpositions of two coherent states for a cavity field [6–8] and the vibrational motion of a trapped ion [9] have been experimentally demonstrated.

Previous researches on the macroscopic quantum-interference states concentrate on superpositions of coherent states with different phases or amplitudes since coherent states are considered to be the quantum states close to classical ones. However, coherent states are not strictly classical states since they are pure states. Before coupling with the microscopic system, due to interaction with the environment a macroscopic apparatus is in fact in a mixed state, instead of a pure state [10]. On the other hand, thermal states, which do not exhibit any nonclassical property, are real classical states. In fact, thermal states are true representation of the state of a field at a finite temperature. The problem naturally arises: can a thermal state evolve to a macroscopic superposition? Does such a mixed state superposition exhibit quantum interference? How do we detect the coherence and decoherence of this state? These questions are closely related with the quantum measurement process and decoherence under real conditions.

In this paper, we answer the above mentioned important questions, showing that a cavity field can evolve to a macroscopic superposition through interaction with a resonant two-level atom even if it is initially in a thermal state. This is remarkable since it is believed that in order to obtain a macroscopic superposition state one should initially prepare the cavity mode in a pure coherent state. The coherence of the thermal state superposition can be revealed by the collapses and induced revivals of the Rabi oscillation. Our study opens a prospect for studying the quantum measurement problem and exploring the quantum-classical transition at a finite temperature. For the two-mode case, entanglement of thermal states between two modes can be obtained. Our work is not only important for studying how macroscopic quantum interference effects can persist at a finite temperature, but also useful for quantum information processing with mixed states.

The paper is organized as follows. In Sec. II, we show that an initial displaced thermal state can evolve to superpositions of displaced thermal states correlated with the atomic states via resonant atom-field interaction. In Sec. III, we discuss the problem of detecting the coherence between the displaced thermal states. In Sec. IV, we generalize the idea to the two-mode case and show that entanglement of displaced thermal states between the modes can be obtained. A summary appears in Sec. V.

II. GENERATION OF SUPERPOSITIONS OF DISPLACED THERMAL STATES

We consider a thermal field, whose state is described by the density operator ρ_{th}

$$\rho_{\text{th}} = \frac{1}{\pi \bar{n}_{\text{th}}} \int e^{-|\alpha|^2/\bar{n}_{\text{th}}} |\alpha\rangle\langle\alpha| d^2\alpha, \tag{1}$$

where $\bar{n}_{\text{th}} = 1/(e^{\hbar\omega/k_B T} - 1)$ is the mean photon-number of the thermal field. We first displace the cavity field by an amount α, leading to the density operator $D(\alpha)\rho_{\text{th}}D^{+}(\alpha)$, with $D(\alpha)$ being the displacement operator. We here assume that α is a real number. We let a two-level atom interact with the single-

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mode cavity field. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar=1$)

$$H=g(a^{\dagger}S^{-}+aS^{\dagger}), \tag{2}$$

where $S^{\dagger}=|e\rangle\langle g|$, $S^{-}=|g\rangle\langle e|$, with $|e\rangle$ and $|g\rangle$ being the excited and ground states of the atom, a^{+} and a are the creation and annihilation operators for the cavity mode, and g is the atom-cavity coupling strength.

The evolution of the system is given by

$$\rho=U(t)D(\alpha)|\phi_a\rangle\langle\phi_a|\otimes\rho_{\text{th}}D^{\dagger}(\alpha)U^{\dagger}(t), \tag{3}$$

where

$$U(t)=e^{-iHt}. \tag{4}$$

We can rewrite Eq. (3) as

$$\rho=D(\alpha)U_d(t)|\phi_a\rangle\langle\phi_a|\otimes\rho_{\text{th}}U_d^{\dagger}(t)D^{\dagger}(\alpha), \tag{5}$$

where

$$U_d(t)=D^{\dagger}(\alpha)U(t)D(\alpha). \tag{6}$$

Replacing Eq. (4) into (6) we obtain

$$U_d(t)=e^{-iH_dt}, \tag{7}$$

where

$$H_d=\frac{1}{2}g[(a^{\dagger}+\alpha^{*})S^{-}+(a+\alpha)S^{\dagger}]. \tag{8}$$

Define the new atomic basis [11]

$$|+\rangle=\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle),\quad |-\rangle=\frac{1}{\sqrt{2}}(|e\rangle-|g\rangle). \tag{9}$$

Then we can rewrite H_d as

$$H_d=\frac{1}{2}g[a^{\dagger}(2\sigma_z+\sigma^{\dagger}-\sigma^{-})+a(2\sigma_z+\sigma^{-}-\sigma^{\dagger})]+2\Omega\sigma_z, \tag{10}$$

where $\sigma_z=\frac{1}{2}(|+\rangle\langle+|-|-\rangle\langle-|)$, $\sigma^{\dagger}=|+\rangle\langle-|$, $\sigma^{-}=|-\rangle\langle+|$, and $\Omega=\alpha g$. We can rewrite the displaced evolution operator $U_d(t)$ as

$$U_d(t)=e^{-i2\Omega\sigma_z t}e^{-iH_it}, \tag{11}$$

where

$$H_i=\frac{1}{2}g[a^{\dagger}(2\sigma_z+e^{i2\Omega t}\sigma^{\dagger}-e^{-i2\Omega t}\sigma^{-})+a(2\sigma_z+e^{-i2\Omega t}\sigma^{-}-e^{i2\Omega t}\sigma^{\dagger})]. \tag{12}$$

Assuming that $\Omega\gg g$, i.e., $\alpha\gg 1$, we can neglect the terms oscillating fast. Then H_i reduces to

$$H_i=g(a^{\dagger}+a)\sigma_z=\frac{g}{2}(a^{\dagger}+a)(S^{-}+S^{\dagger}). \tag{13}$$

Equation (13) reveals the striking feature that, under the large displacement condition, the dynamics of the normal JCM in the displaced picture is described by the combination of the JCM and anti-JCM.

The displacement transformation alters the JCM evolution, resulting in the competition of the excitation and deexcitation of the atomic state accompanying the creation or annihilation of a photon in the initial thermal field. After the displacement transformation, the excitation number $N_e=|e\rangle\langle e|+a^{\dagger}a$ does not conserve since the displacement operator, involving the competition between the annihilation and creation operators, does not commute with the excitation number operator.

We now assume that the atom is initially in the state $|g\rangle$. $|g\rangle$ can be rewritten as

$$|g\rangle=\frac{1}{\sqrt{2}}(|+\rangle-|-\rangle). \tag{14}$$

Using Eqs. (5) and (11), we obtain evolution of the system after an interaction time τ

$$\rho=D(\alpha)|\phi(\tau)\rangle\langle\phi(\tau)|\otimes\rho_{\text{th}}D^{\dagger}(\alpha), \tag{15}$$

$$\begin{aligned}|\phi(\tau)\rangle&=\frac{1}{\sqrt{2}}[e^{-i\theta}D(-\beta)|+\rangle-e^{i\theta}D(\beta)|-\rangle]\\&=\frac{1}{2}\{[e^{-i\theta}D(-\beta)+e^{i\theta}D(\beta)]|g\rangle\\&\quad+[e^{-i\theta}D(-\beta)-e^{i\theta}D(\beta)]|e\rangle\},\end{aligned} \tag{16}$$

where

$$\theta=\Omega\tau,$$

$$\beta=ig\tau/2. \tag{17}$$

We can rewrite the density operator ρ as

$$\begin{aligned}\rho=&\frac{1}{4}D(\alpha)[e^{-i\theta}D(-\beta)+e^{i\theta}D(\beta)]\rho_{\text{th}}[e^{-i\theta}D(-\beta)\\&+e^{i\theta}D(\beta)]D^{\dagger}(\alpha)\otimes|g\rangle\langle g|+\frac{1}{4}D(\alpha)[e^{-i\theta}D(-\beta)\\&-e^{i\theta}D(\beta)]\rho_{\text{th}}[e^{i\theta}D(\beta)-e^{-i\theta}D(-\beta)]D^{\dagger}(\alpha)\otimes|e\rangle\langle e|\\&+\frac{1}{4}D(\alpha)[e^{-i\theta}D(-\beta)+e^{i\theta}D(\beta)]\rho_{\text{th}}[e^{i\theta}D(\beta)\\&-e^{-i\theta}D(-\beta)]D^{\dagger}(\alpha)\otimes|g\rangle\langle e|+\frac{1}{4}D(\alpha)[e^{-i\theta}D(-\beta)\\&-e^{i\theta}D(\beta)]\rho_{\text{th}}[e^{-i\theta}D(-\beta)+e^{i\theta}D(\beta)]D^{\dagger}(\alpha)\otimes|e\rangle\langle g|.\end{aligned} \tag{18}$$

The unnormalized states $D(\alpha)[e^{-i\theta}D(-\beta)\pm e^{i\theta}D(\beta)]\times\rho_{\text{th}}[e^{i\theta}D(\beta)\pm e^{-i\theta}D(-\beta)]D^{+}(\alpha)$ are superpositions of two displaced thermal states. The coherence arises from the superposition of two different displacement operators. The average photon-number of the displaced thermal state depends upon the amount of the initial displacement: $\bar{n}=\bar{n}_{\text{th}}+|\alpha|^2$. The quantum superposition persists in the classical limit $|\alpha|^2\to\infty$.

III. DETECTION OF THE COHERENCE BETWEEN THE DISPLACED THERMAL STATES

In order to detect the coherence between two displaced thermal states we could use the echo method proposed by Morigi *et al.* [12]. We divide the whole duration into two parts. After an interaction time t, the evolution of the system is given by the unitary operator $U_1=e^{-iHt}$. Then the atom undergoes an instantaneous phase kick, corresponding to the application of the inversion operator $S_z=(|e\rangle\langle e|-|g\rangle\langle g|)$. For the remaining time t', the JCM evolution operator is $U_1=e^{-iHt'}$. The whole evolution operator is

$$U=U_2S_zU_1=S_ze^{-iH(t-t')}. \quad (19)$$

We here have used the relation $S_zHS_z=-H$. Therefore, the phase kick leads to the reversal of the unitary evolution of the system. After the duration $2t$, the system evolves back to the initial state.

At the time τ ($\tau<t$), the probability for the atom in the state $|e\rangle$ is given by

$$\begin{aligned}P_e&=\mathrm{Tr}\{D(\alpha)[e^{-i\theta}D(-\beta)-e^{i\theta}D(\beta)]\\&\quad\times\rho_{\mathrm{th}}[e^{i\theta}D(\beta)-e^{-i\theta}D(-\beta)]D^{\dagger}(\alpha)\}\\&=\frac{1}{2}(1-e^{-(g\tau)^2(\bar{n}_{\mathrm{th}}+2)/4}\cos(2\Omega\tau)].\end{aligned} \quad (20)$$

The oscillation arises from the interference between the two displaced thermal states $D(\alpha)D(-\beta)\rho_fD(\beta)D^{+}(\alpha)$ and $D(\alpha)D(\beta)\rho_fD(-\beta)D^{+}(\alpha)$. The distance between the two displaced states increases with the interaction time. When $g\tau\sqrt{\bar{n}_{\mathrm{th}}+2}/2>1$, the two displaced states are approximately orthogonal and thus the oscillation collapses. After the phase kick, the evolution is reversed. At the time $2t$, the two components merge again into a single state and the atom returns to the initial state $|g\rangle$. Then the Rabi oscillation resumes. The process can be interpreted in term of complementarity. Due to the interaction with the cavity mode, there exist two paths for the atom to reach a definite state, with one path associated with the displaced thermal state $D(\alpha)D(-\beta)\rho_fD(\beta)D^{\dagger}(\alpha)$ and the other associated with $D(\alpha)D(\beta)\rho_fD(-\beta)D^{+}(\alpha)$. In the case $g\tau/2\ll 1$, the difference between the two displaced thermal states is negligible and thus no path information can be obtained. With the increase of the interaction time, the two displaced thermal states become distinguishable and the path information is recorded on the cavity mode, destructing the interference. After the phase kick, the distance between the two displaced thermal states decreases. At time $2t$, the two components are recombined and the path information is erased, resulting the reappearance of the interference. The contrast of the oscillation is independent of the value of α. Therefore, collapses and revivals occur for the displaced thermal states even when $\bar{n}\rightarrow\infty$.

With the decoherence being considered, the contrast of the oscillation is reduced. The initial displaced thermal state can be expressed in terms of the Fock states

$$D(\alpha)\rho_{\mathrm{th}}D^{\dagger}(\alpha)=\sum_{m,n=0}^{\infty}\rho_{m,n}|m\rangle\langle n|, \quad (21)$$

where

$$\rho_{m,n}=\sum_{\nu=0}^{\infty}\frac{\bar{n}^{\nu}}{(1+\bar{n})^{\nu+1}}C_{\nu,m}C_{\nu,n}^{*}, \quad (22)$$

$$C_{\nu,m}=\sqrt{\nu!m!}e^{-\alpha^2/2}\sum_{\nu=0}^{\infty}\frac{1}{l!(\nu-l)!(m-l)!}(-\alpha)^{\nu-l}\alpha^{m-l},l\leqslant m. \quad (23)$$

Set $\kappa\ll g$ and $\kappa t\ll 1$, with κ being the cavity decay rate. At time T, due to the cavity decay the contrast of the oscillation is reduced by

$$\begin{aligned}&\sum_{n=0}^{\infty}\kappa\sqrt{1+2\bar{n}_{\mathrm{th}}}\rho_{nn}\left\{\frac{t}{4}(2n-1)+\frac{\sin(gt\sqrt{n})}{4g\sqrt{n}}-\frac{\sin(gt\sqrt{n-1})}{4g\sqrt{n-1}}\right.\\&\quad-\frac{1}{4g}[\sqrt{n}(4n-3)\sin(gt\sqrt{n})\cos(gt\sqrt{n-1})\\&\quad\left.-\sqrt{n-1}(4n-1)\sin(gt\sqrt{n-1})\cos(gt\sqrt{n})]\right\}.\end{aligned} \quad (24)$$

IV. GENERATION OF ENTANGLEMENT BETWEEN TWO THERMAL CAVITY MODES

We note the idea can also be generalized to generate entanglement between two thermal cavity modes. We consider two degenerate cavity modes initially in the thermal states with the density operators $\rho_{\mathrm{th},1}$ and $\rho_{\mathrm{th},2}$. We first displace each cavity mode by a large amount $\alpha(\alpha\gg 1)$, leading to the density operator $D_1(\alpha)\rho_{\mathrm{th},1}D_1^{\dagger}(\alpha)\otimes D_2(\alpha)\rho_{\mathrm{th},2}D_2^{\dagger}(\alpha)$. The two-level atom resonantly interact with the two modes. In the interaction picture, the Hamiltonian is

$$H=\ (g_1a^{\dagger}+g_2b^{\dagger})S^{-}+(g_1a+g_2b)S^{\dagger}], \quad (25)$$

where a and b are the annihilation operators for the two modes, and g_1 and g_2 are the corresponding coupling strengths. After the atom exits the cavity the whole system is in the state

$$\rho=D(\alpha,\alpha)|\phi(\tau)\rangle\langle\phi(\tau)|\otimes\rho_{\mathrm{th},1}\rho_{\mathrm{th},2}D^{\dagger}(\alpha,\alpha), \quad (26)$$

where

$$|\phi(\tau)\rangle=\frac{1}{\sqrt{2}}\left[e^{-2i\theta}D(-\beta_1,-\beta_2)|+\rangle-e^{2i\theta}D(\beta_1,\beta_2)|-\rangle\right], \quad (27)$$

$$D(\beta_1,\beta_2)=D_1(\beta_1)D_2(\beta_2), \quad (28)$$

$$\beta_1=ig_1\tau/2,$$

$$\beta_2=ig_2\tau/2. \quad (29)$$

We can rewrite the state as

$$
\begin{aligned}
\rho = {} & \frac{1}{4}D(\alpha,\alpha)D_+\rho_{\mathrm{th},1} \otimes \rho_{\mathrm{th},2}D_+D^\dagger(\alpha,\alpha) \otimes |g\rangle\langle g| \\
& - \frac{1}{4}D(\alpha,\alpha)D_-\rho_{\mathrm{th},1} \otimes \rho_{\mathrm{th},2}D_-D^\dagger(\alpha,\alpha) \otimes |e\rangle\langle e| \\
& - \frac{1}{4}D(\alpha,\alpha)D_+\rho_{\mathrm{th},1} \otimes \rho_{\mathrm{th},2}D_-D^\dagger(\alpha,\alpha) \otimes |g\rangle\langle e| \\
& + \frac{1}{4}D(\alpha,\alpha)D_-\rho_{\mathrm{th},1} \otimes \rho_{\mathrm{th},2}D_+D^\dagger(\alpha,\alpha) \otimes |e\rangle\langle g|, \quad (30)
\end{aligned}
$$

where

$$
\begin{aligned}
D_+ &= e^{-2i\theta}D(-\beta_1,-\beta_2) + e^{2i\theta}D(\beta_1,\beta_2), \\
D_- &= e^{-2i\theta}D(-\beta_1,-\beta_2) - e^{2i\theta}D(\beta_1,\beta_2). \quad (31)
\end{aligned}
$$

The states $D(\alpha,\alpha)D_\pm\rho_{\mathrm{th},1}\otimes\rho_{\mathrm{th},2}D_\pm D^+(\alpha,\alpha)$ are entangled displaced thermal states. The entanglement results from the superposition of the two-mode displacement operators $D(-\beta_1,-\beta_2)$ and $D(\beta_1,\beta_2)$. At the time τ, the probability for the atom in the state $|e\rangle$ is given by

$$
P_e = \frac{1}{2}[1 - e^{-[(g_1\tau)^2+(g_2\tau)^2](\bar{n}+2)/4}\cos(4\Omega\tau)]. \quad (32)
$$

The oscillation arises from the interference between the two two-mode thermal states $D(\alpha,\alpha)D(-\beta_1,-\beta_2)\rho_{\mathrm{th},1}\otimes\rho_{\mathrm{th},2}D(\beta_1,\beta_2)D^\dagger(\alpha,\alpha)$ and $D(\alpha,\alpha)D(\beta_1,\beta_2)\rho_{\mathrm{th},1}\otimes\rho_{\mathrm{th},2}D(-\beta_1,-\beta_2)D^\dagger(\alpha,\alpha)$. The distance between the two two-mode displacement operators does not decrease as the average photon numbers of the displaced thermal states increases. Therefore, the entanglement survives when the average photon numbers of the two fields approach infinity. The idea opens promising prospects for entangling two macroscopic mixed systems and investigate the decoherence. The idea can be easily generalized to produce entanglement for two or more thermal fields located in separated cavities.

V. SUMMARY

In conclusion, we show that the macroscopic thermal state superposition can be induced by a resonant two-level atom. The quantum coherence can be revealed by the collapses and induced revivals of the Rabi oscillation. In contrast with previous studies concentrating on pure coherent states, our research provides a way for investigating the quantum measurement model and decoherence phenomena under real conditions. For the two mode case, the quantum entanglement between two thermal cavity modes can be obtained. The entanglement survives even if the average photon-numbers of the modes go to infinity. The required experimental techniques for demonstrating the idea include the passage of single Rydberg atoms through a high-quality cavity, displacement, phase kick, and atomic state measurement. All these techniques are presently available [6–8] and thus the implementation of the idea appears experimentally feasible.

ACKNOWLEDGMENTS

This work was supported by the National Fundamental Research Program Under Grant No. 2001CB309300, and the National Natural Science Foundation of China under Grant No. 10674025.

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