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Coherent control of self-trapping of cold bosonic atoms

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We study the behavior of ultracold bosonic atoms held in an optical lattice. We first show how a self-trapping transition can be induced in the system either by increasing the number of atoms occupying a lattice site or by raising the interaction strength above a critical value. We then investigate how applying a periodic driving potential to the self-trapped state can be used to coherently control the emission of a precise number of correlated bosons from the trapping site. This allows the preparation and transport of entangled bosonic states, which are of great relevance to novel technologies such as quantum-information processing.

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INTRODUCTION

Ultracold atoms held in optical lattices are currently at the center of intense theoretical and experimental investigation. The experimental parameters of these systems can be controlled extremely precisely, and in addition, the high degree of isolation from the environment permits their quantum dynamics to remain coherent over long time scales. Consequently such systems provide an attractive way of investigating quantum many-body physics and provide an excellent starting point for engineering and manipulating entangled states, which are vital for implementations of quantum-information processing.

Bosons confined in an optical lattice provide an almost ideal realization [1] of the Bose-Hubbard (BH) model, and a recent experiment directly observed the quantum phase transition between a superfluid and a Mott insulator [2]. Theoretical work [3,4] has shown how this transition may be induced in an alternative way: by applying an additional oscillatory driving field to suppress the intersite tunneling by means of the quantum interference effect termed coherent destruction of tunneling (CDT) [5]. In this work we show how such an oscillating driving field can be used to manipulate the dynamics of a different ground state of the BH model—the *self-trapped* state. In particular we demonstrate an effect analogous to photon-assisted tunneling (PAT), in which certain driving frequencies induce a coherent oscillation of an integer number of bosons between the trapping site and its nearest neighbors. Combining this effect with CDT allows us to control the emission of a definite number of bosons from the trapping site and manipulate their speed of propagation through the optical lattice, and thus enables the self-trapped state to be used as a quantum beam splitter or as a coherent source of entangled bosons.

STATIC PROPERTIES

The BH model is described by the Hamiltonian

$$H_{BH} = -J\sum_{\langle j,k\rangle}[a_j^\dagger a_k + \mathrm{H.c.}] + \frac{U}{2}\sum_j n_j(n_j-1), \qquad (1)$$

where $a_j(a_j^\dagger)$ are the standard annihilation (creation) operators for a boson on site j, $n_j = a_j^\dagger a_j$ is the number operator, J is the tunneling amplitude between neighboring sites, and U is the repulsion between a pair of bosons occupying the same site. This description is valid [1] if the largest potential energies present in the system are smaller than the excitation energy to the first excited Bloch band, and that the interaction can be described by a short-range pseudopotential. We initialize the system in a state in which all the bosons occupy a *single* lattice site. As the Hubbard interaction is repulsive, it might be thought that such a state would be extremely unstable. Surprisingly, however, this is not necessarily the case and, depending on the strength of the interaction and the filling of the lattice site, this highly localized configuration can persist for long times. In such cases the bosons are said to be "self-trapped" [6].

Self-trapping has been recently observed experimentally in Bose-Einstein condensates of roughly 1000 atoms [7,8] and can be understood qualitatively by an energetics argument. The presence of the optical lattice causes the energy spectrum of noninteracting bosons to be confined to a Bloch band of width $4J$. Consequently, if the potential energy per particle of the trapped condensate is much higher than this, it cannot be converted into the kinetic energy of free bosons and the trapped state cannot decay. Its stability thus depends critically on the absence of dissipative processes in optical lattice systems.

To describe this effect quantitatively, we consider a two-site BH model holding N bosons. If the system is initialized in the state $|N,0\rangle$ (a Fock state with N bosons occupying one site with the other site empty), then the primary tunneling process will be to the state $|N-1,1\rangle$. Truncating the Hilbert space to just these two states produces an effective two-level model

$$H_{\text{two-lev}} = \begin{pmatrix} V(N) & J\sqrt{N} \\ J\sqrt{N} & V(N-1) \end{pmatrix}, \qquad (2)$$

where $V(n) = Un(n-1)/2$ is the potential energy of n bosons occupying one site. It is useful to visualize the time evolution of the system geometrically by making use of the Bloch sphere representation. Parametrizing $H_{\text{two-lev}}$ in terms of the Pauli matrices,

$$H_{\text{two-lev}} = \frac{U}{2}(N-1)^2 I + J\sqrt{N}\sigma_x + \frac{U}{2}(N-1)\sigma_z, \qquad (3)$$

reveals that we can interpret it as an interaction between the Bloch vector $\underline{\sigma}$ and a fictitious magnetic field

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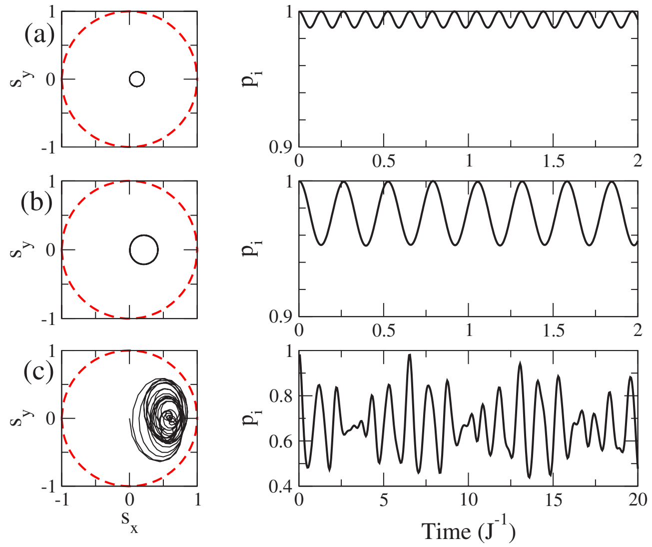

FIG. 1. (Color online) Time evolution of a two-site system containing seven bosons. The system is initialized in the state $|7,0\rangle$ (the north pole of the Bloch sphere) and evolves under the full BH Hamiltonian (1). To the left we show the evolution on the Bloch sphere, projected onto the (s_x, s_y) plane, and to the right the overlap of the system with the initial state $p_i(t)$. (a) Strong interaction, $U/J=8$. The system is described well by the two-level model (3) and periodically traces out a circle on the Bloch sphere. Correspondingly, p_i makes high-frequency oscillations of small amplitude, indicating that self-trapping is occurring. (b) Intermediate interaction, $U/J=4$. The radius of the circular orbit is larger, but the two-level approximation is still good. The oscillations in p_i are now of larger amplitude, showing that the trapping effect is reduced. (c) Weak interaction, $U/J=1$. The two-level approximation now breaks down, and the system's time evolution is correspondingly more complicated. The value of p_i rapidly drops as the trapping effect has now been completely lost.

$\underline{B}=(J\sqrt{N},0,U(N-1)/2)$. Thus under the influence of the Hamiltonian the Bloch vector will simply make a Larmor orbit about $\underline{B}$. This form of time evolution is shown in Fig. 1(a) for a strongly interacting seven-boson system. It can be clearly seen that, as expected, the Bloch vector traces out a circular orbit centered on $\underline{B}$. Since for these parameters $\underline{B}$ is almost parallel to the s_z axis, the radius of this orbit is rather small, and thus the system exhibits a high degree of self-trapping.

To assess the degree of self-trapping more precisely we measure the overlap of the system's state with the initial state, $p_i(t)=|\langle\psi_i|\psi(t)\rangle|^2$, since when self-trapping occurs this quantity is close to unity. The circular motion shown in Fig. 1(a) equates to a very-small-amplitude sinusoidal oscillation of p_i. As U is reduced, the angle between $\underline{B}$ and the s_z axis increases, and as a result the radius of the Larmor orbit made by the Bloch vector increases [Fig. 1(b)]. Consequently the degree of trapping is reduced and the amplitude of oscillations of p_i is larger. If U is decreased even further [Fig. 1(c)], the self-trapping effect is lost and the two-level approximation breaks down. In this case the Bloch vector rapidly leaves the surface of the Bloch sphere and its erratic time evolution corresponds to an irregular quasiperiodic behavior of p_i, which can take very low values.

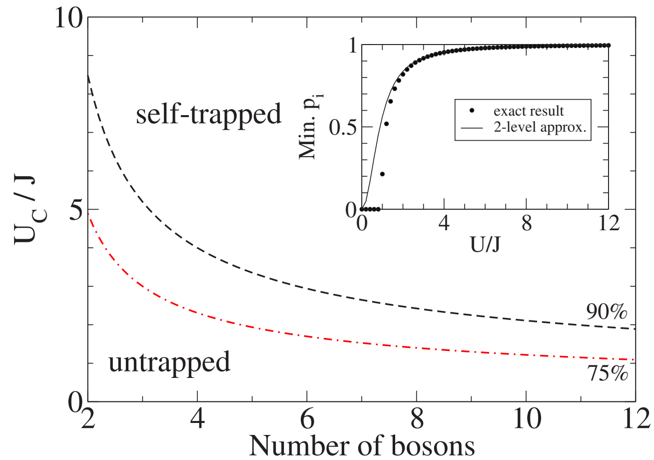

FIG. 2. (Color online) The dashed lines plot the value of the Hubbard interaction U_C [Eq. (4)] at which the occupation of the self-trapped site can fall to 75% (dash-dotted line) and 90% (dashed line) in an undriven two-site system. These provide weak and strong estimates for the boundary between the untrapped and self-trapped regimes. Inset: comparison of the two-level approximation with the full BH Hamiltonian for a seven-boson system. For $U/J<1$ the two-level approximation breaks down, but becomes increasingly accurate as U is increased.

In the inset of Fig. 2 we plot the minimum value of p_i attained in the seven-boson system during a time interval of 100 as a function of the interaction strength. For small values of U no self-trapping occurs, and thus $p_i^{\min}$ is zero. As U is increased, however, the oscillations in p_i are quenched, and for $U/J>3$ it can be seen that the effective two-level model describes the dynamics extremely well. From Eq. (3) we can obtain a criterion for the crossover to the self-trapped regime, by defining the transition to occur when $p_i^{\min}$ drops below a value α. This yields a value for the critical value of U,

$$\frac{U_C}{J} = \frac{2}{N-1}\sqrt{\frac{N\alpha}{1-\alpha}}. \tag{4}$$

This boundary is plotted in Fig. 2 for $\alpha=0.90$ and 0.75. Although the self-trapping regime is more easily achievable for large boson numbers since $U_C \sim 1/\sqrt{N}$, it is even possible in a system of just two bosons if the interaction strength can be raised sufficiently. This has recently been achieved experimentally [9,10] in gases of trapped rubidium atoms.

DYNAMICAL PROPERTIES

We now consider dynamically controlling the self-trapped state by applying a harmonic driving potential

$$H(t) = H_{BH} + K \sin \omega t \sum_j j n_j, \tag{5}$$

where K is the amplitude of the driving field and ω is its frequency. Such a potential can be applied to an optical lattice by applying a phase modulation to one of the laser fields which provide the standing-wave potential [11]. We first consider the case of extracting a *single* boson from the trapping site. As the occupation of the trapping site changes from N to

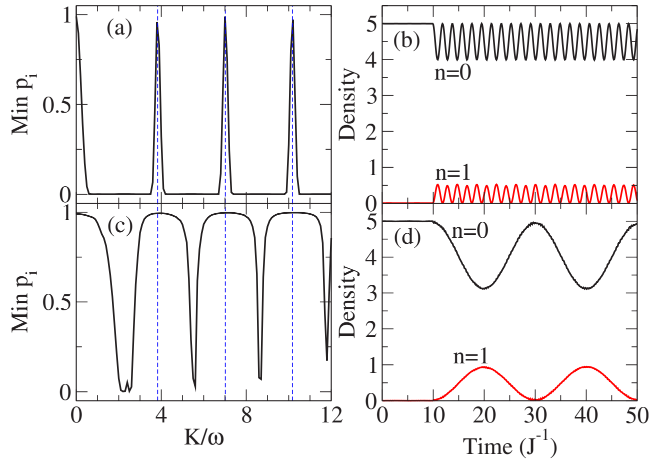

FIG. 3. (Color online) Response of a five-site system, holding five bosons ($U=16$), to a sinusoidal driving field. (a) For a driving frequency $\omega=4U$, the value of $p_i^{\min}$ rapidly drops as K is increased from zero, indicating that the initial state is rapidly destroyed by PAT. At a sequence of sharp peaks centered on the zeros of $\mathcal{J}_1(K/\omega)$ (vertical dashed lines) CDT instead causes the system to be frozen in its initial state. (b) Time dependence of the occupation numbers of the lattice sites. For $t<10$, K/ω is set to be the first zero of $\mathcal{J}_1$, and CDT suppresses any oscillations. For $t>10$ we set $K/\omega=2.40$ which produces a sinusoidal oscillation of a single boson from the central site ($n=0$) to its two neighbors ($n=\pm 1$). (c) As in (a) but for a frequency of $\omega=3U$. Again $p_i^{\min}$ peaks at the zeros of $\mathcal{J}_1(K/\omega)$, although the peaks are rather broader. (d) As in (b), but for $\omega=3U$. When K/ω switches to a value of 2.40 the number densities show a slow sinusoidal oscillation, in which the occupation of the central site varies between 5 and 3—i.e. a *two-boson* oscillation.

$N-1$ there is a corresponding loss of potential energy $\Delta E=V_N-V_{N-1}=U(N-1)$, where V_n is defined as in Eq. (2). When a system possesses such a large energy gap, Floquet analysis may be used to show [4,12] that extremely fine control over the tunneling dynamics is possible at *multiphoton resonances*—that is, when $m\omega=\Delta E$, $m=1,2,\ldots$. In general when this condition is satisfied, the system is able to exchange energy with the driving field to overcome the energy gap, and so tunneling is restored (PAT) [13]. However, at particular values of the amplitude of the driving field, CDT will occur when the Floquet quasienergies of the system become degenerate and the dynamics of the system will be frozen. For a sinusoidal driving potential these degeneracies occur at the zeros of $\mathcal{J}_m(K/\omega)$, the mth Bessel function of the first kind. Thus at a photon resonance it is possible to produce dramatic differences in the tunneling rate by making small changes in the amplitude of the driving field to move the system between CDT and PAT.

In Fig. 3(a) we plot $p_i^{\min}$ obtained in a five-boson system driven at a frequency of $\omega=4U$. This corresponds to the $m=1$ photon resonance. For $K=0$ the system is self-trapped, and consequently p_i remains near unity. Increasing K, however, causes the value of $p_i^{\min}$ to rapidly drop to zero, demonstrating how PAT overcomes the self-trapping effect. As K is increased further, $p_i^{\min}$ exhibits a number of extremely sharp peaks centered on $K/\omega=3.83$, 7.01, and 10.17—the zeros of $\mathcal{J}_1(K/\omega)$.

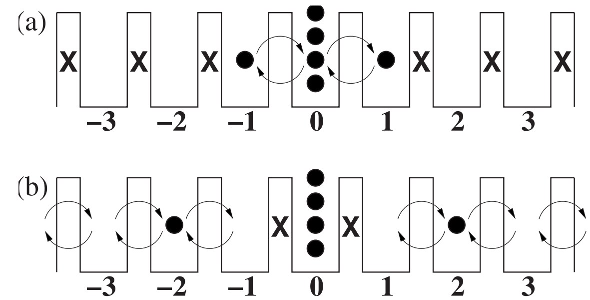

FIG. 4. (a) Schematic version of the process illustrated in Fig. 3(b). The driving field excites a boson from the trapping site ($n=0$). If $\mathcal{J}_0(K/\omega)=0$, CDT suppresses the single-boson tunneling processes (marked by X), and so a Rabi-like oscillation occurs between the trapping site and its neighbors. (b) Changing the driving parameters to stabilize the self-trapping isolates the remaining bosons in the trapping site, but the escaped particle is able to propagate through the optical lattice with a renormalized tunneling amplitude J_{eff}.

Away from these zeros, the driving field causes a single boson to tunnel symmetrically from the trapping site to its neighboring lattice sites and from there to continue propagating through the optical lattice with a renormalized tunneling [5] $J_{\text{eff}}=J\mathcal{J}_0(K/\omega)$. If we therefore choose a value of K such that $J_{\text{eff}}=0$, the particle will not be able to propagate further and will thus just make a Rabi-like oscillation between the trapping site and its neighbors. This situation is illustrated schematically in Fig. 4(a). We show in Fig. 3(b) the time dependence of the occupation of the trapping site and its neighbor. Initially we set $K/\omega=3.83$ (a zero of $\mathcal{J}_1$) and no tunneling occurs: for this value of K/ω, CDT reinforces the self-trapping effect. At $t=10$ we alter the amplitude of the driving to $K/\omega=2.40$ (a zero of $\mathcal{J}_0$) and a very clear particle oscillation occurs, in which the occupation of the trapping site cycles between 5 and 4 and that of the neighboring sites oscillates between 0 and 0.5.

We can apply a similar method to induce the emission of an larger number of bosons, N_{em}, by noting that the energy difference per particle takes the remarkably simple form

$$\Delta E/N_{em}=[V_N-(V_{N-N_{em}}+V_{N_{em}})]/N_{em}=U(N-N_{em}). \quad (6)$$

Thus by driving the system at the correct frequency we can induce the emission of a given number of particles. In Fig. 3(c) we show the response of the five-boson system to a driving field of frequency $\omega=3U$, which induces the emission of *two* bosons. As before we can observe peaks in $p_i^{\min}$ centered on the zeros of $\mathcal{J}_1(K/\omega)$ at which CDT occurs, while between them PAT causes $p_i^{\min}$ to take low values. Figure 3(d) shows the effect of switching the amplitude of the driving field to a value of $K/\omega=2.40$. We can again see that this has the effect of inducing a Rabi-like oscillation between the trapping site and its neighbors, but in this case the oscillation indeed consists of two bosons, and has a longer period.

In Fig. 5 we show how combining the PAT effect with CDT allows the population of the self-trapped state to be reduced step by step. The system is initialized with five

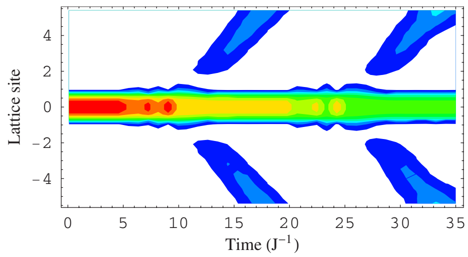

FIG. 5. (Color online) Time evolution of the particle density of a five-boson system. The driving parameters are chosen (see text) to produce the emission of single bosons at $t \simeq 5$ and 25, and thus the occupation of the trapping site falls in steps as $5 \rightarrow 4 \rightarrow 3$. The emitted particle bursts move away from the central site at a roughly constant speed and disperse slightly as they propagate through the lattice.

bosons in the central site and is initially driven at $\omega = 4U$. Driving the system at $K/\omega = 3.80$ induces CDT which stabilizes the self-trapped state. At $t = 5$ the value of K/ω is changed to 2.40 which induces the Rabi-type oscillation between the trapping site and its neighbors, as schematically shown in Fig. 4(a). After a *half-integer* number of these oscillations we then alter the driving parameters to $\omega = 3U$, $K/\omega = 3.80$, which traps the remaining four bosons in the trapping site. For these parameters, however, the single-particle tunneling is *not* quenched and so the ejected boson is able to propagate through the optical lattice away from the trapping site, as shown in Fig. 4(b). The emitted particle smears out to an extent as it moves through the lattice but nonetheless the atom pulse remains quite clearly defined after propagating through several lattice spacings. By repeating this procedure with appropriate driving frequencies we can successively reduce the occupation of the trapping site in integer steps and thereby produce a sequence of well-defined, phase-coherent atom pulses.

CONCLUSIONS

We have shown how self-trapping arises in cold bosons confined in an optical lattice. The lifetime of this state will be principally limited by losses from three-body collisions, which increase rapidly with particle number. For $N = 5$ a lifetime of 0.2 s has been measured [14], which indicates that coherent driving effects should nonetheless be observable. Applying a resonant driving field to the self-trapped state can either stabilize the trapping (when CDT occurs) or can induce a Rabi-like particle oscillation. The interplay between these effects makes it possible to control the emission and propagation of a precisely defined number of particles and thus enables the self-trapped state to be used as a controllable source of mesoscopic entangled states. These have many possible applications in quantum information, such as linking distant quantum registers and providing a channel for quantum communication. Further practical issues to consider include the role of the background trapping potential, and finite-temperature effects.

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