Quantum interference in absorption and dispersion of a four-level atom in a double-band photonic crystal

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The probe absorption-dispersion spectrum of a double V-type four-level atom in a double-band photonic crystal is investigated. In the model used, the double V-type transitions are, respectively, coupled by the free vacuum modes and the photonic band gap modes, leading to the two possible types of quantum interference. Three types of zero absorption (transparency) appear in this model. In the first type, there exist two zeroes at the band edge frequencies in the case of isotropic photonic band gap. In the second type, a zero emerges at the middle frequency of two upper levels in the case that the quantum interference takes place in both V-type transitions. Finally, in the third type, a zero occurs at or around the middle frequency of two upper levels in the case that the quantum interference takes place only in the V-type transitions coupled to the free vacuum modes. Ultimately, the probe absorption-dispersion spectrum in the case of the single-band photonic band gap reservoirs compared to those of the double-band photonic band gap reservoirs. The results show that the dispersion property of the system depends on the two types of the quantum interference and the density of states of the photonic band gap reservoir.

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I. INTRODUCTION

It is well known that quantum and nonlinear optical phenomena in atoms embedded in photonic band gap (PBG) materials lead to the prediction of many interesting effects. As examples, one can mention the localization of light and the formation of atom-photon bound states [1], transient lasing without inversion [2], suppression and even complete cancellation of spontaneous emission [3], population trapping in two-atom systems [4], the phase dependent behavior of the population dynamics [5], etc. Recently, it has been found that a modified reservoir with a rapidly varying density of modes (DOM) in a single-band structure [6] can induce transparency and transient gain without population inversion in an open Λ system. This transparency scheme does not require any external driving field to induce atomic coherence, which is an advantage over common electromagnetically induced transparency (EIT). Based on this, photon-photon correlation and entanglement in a photonic crystal (PC) doped with four-level atoms are also investigated [7].

Meanwhile, great attention has recently been devoted to the study of spontaneous emission spectrum of atoms embedded in PBG materials. Of particular interest, the spontaneous emission of a three-level Λ-type atom with both transitions coupled to a single PBG reservoir has been investigated by Jiang *et al.* [8]. The dynamics of a two-level atom immersed in the modified radiation field of a PBG material using non-Markovian stochastic Schrödinger equation has been studied by Vega *et al.* [9].

Recently, Zhang *et al.* [9] studied the effect of the fine structure of the lower level on the spontaneous emission and absorption-dispersion spectrum of a three-level and a four-level atom embedded in a double-band PBG material [10]. They found that the new features of four (two) transparency with two (one) spontaneous emission peaks resulted from the fine structure of the lower levels of the atom, in the case of isotropic PBG modes.

Radeonychev *et al.* have investigated the response of a three-level atomic system driven by a resonant coherent field acting on a transition near the photonic band edge of a PBG material [11]. Their analysis is based on the master equations for elements of the density matrix in the semiclassical dressed-state representation. They show that the strong frequency dependence of the radiation mode density at the scale of the driving-field Rabi frequency near the photonic band edge leads to some essential and controllable changes in the refractive index, as well as to EIT and LWI effects.

Mahi has studied the phenomenon of EIT when a four-level atom is located in a PBG material [12]. Quantum interference is introduced by driving the two upper levels of the atom with a strong pump laser field. The top level and one of the ground levels are coupled by a weak probe laser field, and absorption takes place between these two states. The susceptibility due to the absorption for this transition has been calculated by using the master equation method in linear response theory. Mahi showed that when resonance frequencies lie within the band, the medium becomes transparent under the action of the strong pump laser field. When the resonance frequencies lie at the band edge, the medium becomes nontransparent even under a strong pump laser field. On the other hand, when the resonance frequencies lie within the band gap, the medium becomes transparent even under a weak pump laser field.

Focusing on the quantum interference of an atoms without external driving fields, Zhu *et al.* [13] have reported the quantum interference of spontaneous emission and probe absorption of an atom coupled to vacuum space reservoirs. If a similar atomic system were coupled to a PBG reservoir rather than with a vacuum-space reservoir, what would be the resulting absorption spectrum near the edges of the pho-

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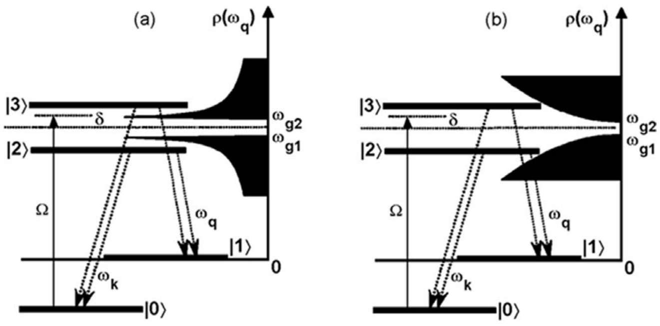

FIG. 1. Schematic diagram of a double V-type four-level atom in (a) a double-band isotropic PBG reservoir; (b) a double-band anisotropic PBG reservoir.

tonic band? Zhang *et al.* have reported the quantum interference in the spontaneous emission spectrum of an atom embedded in a double-band PC [14].

While the absorption-dispersion spectra of atom embedded in PBG have been mentioned in a few papers [15], most discussions are limited to the isotropic PBG case or the existence of defect modes and external coupling fields. In contrast, in this paper, we focus on the effects of the quantum interference on the absorption-dispersion spectrum of a double V-type four-level atom embedded in a double-band PBG material. We assume that the double V-type transitions are coupled, respectively, by the free vacuum modes and the PBG modes, leading to the two possible types of quantum interference. Most interestingly, it is shown that three types of transparency can occur in our system. Furthermore, we realize that the dispersion property of the system depends on the two types of the quantum interference and the DOS of the PBG reservoir. The effects of system parameters on the interference are discussed in detail. The results are also compared with those of the atomic system coupled to a free vacuum reservoir. Moreover, the width of the band gap adopted in this paper is of the same order as the natural transition width. A photonic crystal with a narrow band gap is easier to fabricate and thus the effects predicted here are experimentally feasible. In Sec. II, the equations to describe the probe absorption-dispersion spectrum of the four-level atom in a photonic crystal are deduced. The general results and analysis are presented in Sec. III. The major conclusions are summarized in Sec. IV.

II. EQUATIONS FOR THE PROBE ABSORPTION-DISPERSION SPECTRUM

We consider a double V-type four-level atom with the upper levels labeled by $|2\rangle$ and $|3\rangle$, and the lower levels labeled by $|0\rangle$ and $|1\rangle$, as shown in Fig. 1. The V-type transitions from the upper levels to the lower level $|0\rangle$ are coupled by free vacuum modes (ω_k) and a weak probe field (ω_p), while the atom in the V-type transitions from the upper levels to the lower level $|1\rangle$ are coupled by the modes of a modified reservoir (ω_q). The modified reservoir can be a double-band isotropic PBG reservoir, the double-band anisotropic PBG reservoir or the free vacuum reservoir, respectively. In Fig. 1, $\rho(\omega_q)$ denotes the density of states (DOS) of the PBG modes where ω_{g1} and ω_{g2} are the lower and upper frequencies of the forbidden gap, respectively, Ω is the Rabi frequency of the probe field, and δ is the detuning of frequency of the probe field from the middle of the upper levels.

The dispersion relations near the photonic band edges are approximated by [16]

$$\omega_q = \omega_{g1} - A_1(q-q_0)^2, \quad q < q_0,$$

$$\omega_q = \omega_{g2} + A_2(q-q_0)^2, \quad q > q_0, \tag{1}$$

$$\omega_q = \omega_{g1} - A_1|(\vec{q}-\vec{q}_0)|^2, \quad |\vec{q}| < |\vec{q}_0|,$$

$$\omega_q = \omega_{g1} + A_2|(\vec{q}-\vec{q}_0)|^2, \quad |\vec{q}| > |\vec{q}_0| \tag{2}$$

for isotropic and anisotropic dispersion relations, respectively, where $A_i \approx \omega_{gi}/q_0$ $(i=1,2)$, and ω_{g1} and ω_{g2} are the lower and the upper frequencies of the forbidden gap, respectively.

The four-level atom is initially prepared in the lower level $|0\rangle$. At time $t=0$, this atom starts to interact with a probe field of frequency ω_p. The dynamics of the system can be described by using the Schrödinger equation. Then the wave function of the system at time t can be expressed in terms of the state vectors as

$$|\psi(t)\rangle = a_0(t)|0,\{0\}\rangle + a_2(t)e^{-i\delta_2 t}|2,\{0\}\rangle + a_3(t)e^{-i\delta_3 t}|3,\{0\}\rangle + \sum_{ke} a_{ke}(t)|0,\{ke\}\rangle + \sum_{qe} a_{qe}(t)|1,\{qe\}\rangle, \tag{3}$$

where k and q denote the momentum vectors of the emitted photons, e denotes the polarization of the emitted photons, $\delta_2=\omega_p-\omega_{20}$ and $\delta_3=\omega_p-\omega_{30}$. The function $a_j(t)$ $(j=0,2,3)$ gives the probability amplitude to find the atom in the excited state $|j\rangle$ and the photon reservoir in the vacuum state. On the other hand, $a_{ke}(t)$ and $a_{qe}(t)$ give the probability amplitudes to find the atom on the lower states $|0\rangle$ and $|1\rangle$, respectively, and a single photon of polarization e and wave vectors k and q in the photon reservoir, respectively.

The Hamiltonian describing the dynamics of this system in the interaction picture and the rotating wave approximation can be presented as

$$H_I = \Bigg(\sum_{ke} g_{ke}^{20} e^{-i\delta_k t}|2\rangle\langle 0|\hat{b}_{ke} + \sum_{ke} g_{ke}^{30} e^{-i\delta'_k t}|3\rangle\langle 0|\hat{b}_{ke} + \sum_{qe} g_{qe}^{21} e^{-i\delta_q t}|2\rangle\langle 1|\hat{b}_{qe} + \sum_{qe} g_{qe}^{31} e^{-i\delta'_q t}|3\rangle\langle 1|\hat{b}_{qe} + \Omega_2 e^{i\delta_2 t}|0\rangle\langle 2| + \Omega_3 e^{i\delta_3 t}|0\rangle\langle 3| + \text{H.c.} \Bigg). \tag{4}$$

Here Ω_2 and Ω_3 are the probe field Rabi frequencies, which are considered real for convenience in our problem. $\delta_k=\omega_k-\omega_{20}$ ($\delta'_k=\omega_k-\omega_{30}$) represents the detuning of the radiation mode frequency ω_k from the atomic transition frequency ω_{20} (ω_{30}), and $\delta_q=\omega_q-\omega_{21}$ ($\delta'_q=\omega_q-\omega_{31}$) represents the detuning of the frequency of the radiation mode ω_q from the atomic transition frequency ω_{21} (ω_{31}). $g_{\lambda e}^{ij}$ is the frequency dependent coupling constant between the atomic transition $|i\rangle \to |j\rangle$ and the mode $\{\lambda e\}$ of the radiation field. More precisely

$$g_{\lambda e}^{ij} = \frac{\omega_{ij} d_{ij}}{\hbar}\left(\frac{\hbar}{2\varepsilon_0 \omega_\lambda V}\right)^{1/2} \vec{e}_{\lambda e}\cdot \vec{d}_{ij}, \tag{5}$$

where $\vec{d}_{ij}$ is the atomic dipole moment unit vector for the transition $|i\rangle \to |j\rangle$, $\vec{e}_{\lambda e}$ is the polarization unit vector of the radiation field, V is the sample volume, and ε_0 is the Coulomb constant.

The probability amplitude equations can be obtained from Eqs. (3) and (4) as follows:

$$i\dot{a}_0(t) = \Omega_2 a_2(t) + \Omega_3 a_3(t), \tag{6}$$

$$i\dot{a}_2(t) = \sum_{ke} g_{ke}^{20} e^{-i(\delta_k-\delta_2)t} a_{ke}(t) + \sum_{qe} g_{qe}^{21} e^{-i(\delta_q-\delta_2)t} a_{qe}(t) - \delta_2 a_2(t) + \Omega_2 a_0(t), \tag{7}$$

$$i\dot{a}_3(t) = \sum_{ke} g_{ke}^{30} e^{-i(\delta_k'-\delta_3)t} a_{ke}(t) + \sum_{qe} g_{qe}^{31} e^{-i(\delta_q'-\delta_3)t} a_{qe}(t) - \delta_3 a_3(t) + \Omega_3 e^{-i\delta_3 t} a_0(t), \tag{8}$$

$$i\dot{a}_{ke}(t) = g_k^{02} e^{i(\delta_k-\delta_2)t} a_2(t) + g_k^{03} e^{i(\delta_k'-\delta_3)t} a_3(t), \tag{9}$$

$$i\dot{a}_{qe}(t) = g_q^{12} e^{i(\delta_q-\delta_2)t} a_2(t) + g_q^{13} e^{i(\delta_q'-\delta_3)t} a_3(t). \tag{10}$$

We proceed by performing a formal time integration of Eqs. (9) and (10), and substitute the result into Eqs. (7) and (8) to obtain the integral-differential equations

$$\dot{a}_2(t) = i\delta_2 a_2(t) - i\Omega_2 a_0(t) - \frac{\gamma_{20}}{2} a_2(t) - \frac{\eta}{2}\sqrt{\gamma_{20}\gamma_{30}} a_3(t) - \int_0^t a_2(t') e^{i\delta_2(t-t')} K_{22}(t-t') dt' - \int_0^t a_3(t') e^{i\delta_2(t-t')} K_{23}(t-t') dt', \tag{11}$$

$$\dot{a}_3(t) = i\delta_3 a_3(t) - i\Omega_3 a_0(t) - \frac{\eta}{2}\sqrt{\gamma_{20}\gamma_{30}} a_2(t) - \frac{\gamma_{30}}{2} a_3(t) - \int_0^t a_2(t') e^{i\delta_3(t-t')} K_{32}(t-t') dt' - \int_0^t a_3(t') e^{i\delta_3(t-t')} K_{33}(t-t') dt'. \tag{12}$$

Because the reservoir with modes k is assumed to be Markovian (free vacuum reservoir), we use the usual Weisskopf-Wigner result [17], i.e.,

$$\sum_{ke} g_{ke}^{i0} g_{ke}^{0j} e^{-i(\omega_k-\omega_{j0})(t-t')} = \frac{\eta_{ij}}{2}\sqrt{\gamma_{i0}\gamma_{j0}}\,\delta(t-t'), \tag{13}$$

where $\eta_{ij} = \delta_{ij} + \eta(1-\delta_{ij})$ $(i,j=2,3)$. Here η indicates the quantum interference in transitions coupled to the free vacuum modes, which can be defined as $\eta = (3/8\pi)\int(\cos\alpha_{20}\cos\alpha_{30} + \cos\beta_{20}\cos\beta_{30}) d\Omega$ where $\{\alpha_{i0}, \beta_{i0}, \theta_{i0}\}$ $(i=2,3)$ are the directional angles of the dipole moment unit vector $\vec{d}_{i0}$ in a coordinate system defined by the unit victors $\{\vec{e}_{k1}, \vec{e}_{k2}, \vec{k}\}$, and $d\Omega$ is the solid angle. So $\eta=1.0$ when $\vec{d}_{20}\vec{d}_{30} = \pm 1$, and $\eta=0$ when $\vec{d}_{20}\vec{d}_{30}=0$; otherwise, $0<\eta<1.0$. γ_{i0} is effective decay rate for the transition from the upper level $|i\rangle$ $(i=2,3)$ to the lower level $|0\rangle$.

The summations associated with the PBG reservoir modes in Eqs. (11) and (12), i.e., $K_{ij}(t-t') = \sum_{qe} g_{qe}^{i1} g_{qe}^{1j} e^{-i(\omega_q-\omega_{j1})(t-t')}$ $(i,j=2,3)$ can be expressed in the following forms [16]:

$$K_{ij}(t-t') = \frac{p_{ij}}{2}\sqrt{\beta_{i1}^{3/2}\beta_{j1}^{3/2}}\left(\frac{e^{i[\delta_{g2}^{j}(t-t')-\pi/4]}}{\sqrt{\pi(t-t')}} + \frac{e^{i[\delta_{g1}^{j}(t-t')+\pi/4]}}{\sqrt{\pi(t-t')}}\right), \tag{14}$$

$$K_{ij}(t-t') = \frac{p_{ij}}{2}\sqrt{\alpha_{i1}\alpha_{j1}}\left(\frac{e^{i[\delta_{g2}^{j}(t-t')+\pi/4]}}{\sqrt{4\pi(t-t')^3}} + \frac{e^{i[\delta_{g1}^{j}(t-t')-\pi/4]}}{\sqrt{4\pi(t-t')^3}}\right), \tag{15}$$

for the isotropic and anisotropic PBG reservoirs, respectively, with $p_{ij} = \delta_{ij} + p(1-\delta_{ij})$. Here p represents the quantum interference in transitions coupled to the modified reservoir and, as above, $0 \leqslant p \leqslant 1.0$. $\delta_{gi}^{j} = \omega_{j1} - \omega_{gi} = \delta_{gi} - (-1)^j 0.5\omega_{32}$ $(i=1,2$ and $j=2,3)$ where $\delta_{gi} = 0.5(\omega_{31}+\omega_{21}) - \omega_{gi}$. The definitions of the parameters are

$$\beta_{i1}^{3/2} = \frac{1}{2\pi\varepsilon_0}\frac{\omega_{i1}^2 d_{i1}^2 \omega_{gi}^{3/2}}{3\hbar c^3}, \tag{16}$$

$$\alpha_{i1} = \frac{1}{2\pi\varepsilon_0}\frac{\omega_{i1}^2 d_{i1}^2 \omega_{gi}^{1/2}}{3\hbar c^3}. \tag{17}$$

The aim here is to investigate the absorption-dispersion properties of our system for a weak probe laser field. The linear susceptibility can be given by [18]

$$\chi(\delta) = -4\pi N\left[\frac{|d_{02}|^2}{\Omega_2} a_0(t\to\infty) a_2^*(t\to\infty) + \frac{|d_{03}|^2}{\Omega_3} a_0(t\to\infty) a_3^*(t\to\infty)\right], \tag{18}$$

where $\delta = \omega_p - 0.5(\omega_{20}+\omega_{30}) = \delta_3 + 0.5\omega_{32} = \delta_2 - 0.5\omega_{32}$ and N is the atomic density. We assume that the interaction between the probe laser and the atom is very weak $(\Omega_2, \Omega_3 \ll \beta, \alpha)$ so that $a_0(t) \approx 1$ for all times. With the use of the Laplace transform we can obtain from Eqs. (11) and (12),

$$s\tilde{a}_2(s) = -i\frac{\Omega_2 D(s) - \Omega_3 B(s)}{A(s)D(s) - B(s)C(s)}, \tag{19}$$

$$s\tilde{a}_3(s) = -i\frac{-\Omega_2 C(s) + \Omega_3 A(s)}{A(s)D(s) - B(s)C(s)}, \tag{20}$$

where

$$A(s) = s - i\delta_2 + \tilde{K}_{22}(s - i\delta_2) + \gamma_{20}/2, \tag{21}$$

$$B(s) = \tilde{K}_{23}(s - i\delta_3) + \eta\sqrt{\gamma_{20}\gamma_{30}}/2, \tag{22}$$

$$C(s) = \tilde{K}_{32}(s - i\delta_2) + \eta\sqrt{\gamma_{20}\gamma_{30}}/2, \tag{23}$$

$$D(s) = s - i\delta_3 + \tilde{K}_{33}(s - i\delta_3) + \gamma_{30}/2. \tag{24}$$

Here $\tilde{a}_i(s)=L[a_i(t)]$, $\tilde{K}_{ij}(s)=L[K_{ij}(t)]$ $(i,j=2,3)$, s is the Laplace variable, and

$$\tilde{K}_{ij}(s) = \frac{p_{ij}}{2}\sqrt{\beta_{i1}^{3/2}\beta_{j1}^{3/2}}\left(\frac{i}{\sqrt{is+\delta_{g1}^{j}}} + \frac{1}{\sqrt{is+\delta_{g2}^{j}}}\right), \tag{25}$$

$$\tilde{K}_{ij}(s) = \frac{p_{ij}}{2}\sqrt{\alpha_{i1}\alpha_{j1}}(-i\sqrt{is+\delta_{g1}^{j}} + \sqrt{is+\delta_{g2}^{j}}), \tag{26}$$

for the double-band isotropic and anisotropic PBG reservoirs, respectively, with $p_{ij}=\delta_{ij}+p(1-\delta_{ij})$. The long time behavior of the probability amplitudes $a_2(t\to\infty)$ and $a_3(t\to\infty)$ can be obtained from Eqs. (19) and (20) by using the final value theorem as follows:

$$a_2(t\to\infty) = -i\frac{\Omega_2 D(0) - \Omega_3 B(0)}{A(0)D(0) - B(0)C(0)}, \tag{27}$$

$$a_3(t\to\infty) = -i\frac{-\Omega_2 C(0) + \Omega_3 A(0)}{A(0)D(0) - B(0)C(0)}. \tag{28}$$

We use Eq. (18) obtained above, and calculate the absorption-dispersion spectrum for several parameters of the system.

III. RESULTS AND DISCUSSION

Figures 1(a) and 1(b) show a double V-type four-level atom in a double-band isotropic and anisotropic PBG, respectively. In this model, the double V-type transitions are, respectively, coupled by the free vacuum reservoir and a modified reservoir, leading to the two possible types of quantum interference. The first is ascribed to the atomic transitions coupled to the free vacuum modes, which is shown by η in Eqs. (22) and (23), and the second arises from the atomic transitions coupled to the modified reservoir modes which is shown by p in Eqs. (25) and (26). In order to investigate the effects of the two types of the quantum interference on the probe absorption spectrum, we plot the imaginary part of the linear susceptibility [Eq. (18)] as a function of detuning δ. As far as we know, there are no reports on the effects of the two types of quantum interference on the absorption spectrum of an atom embedded in a PBG material.

In the following part, we study the case where an atom is initially prepared in the lower level $|0\rangle$ and embedded in a double-band PC. In Figs. 2(a)–2(c), the absorption spectra are shown for the cases of the transitions $|3\rangle\to|1\rangle$ and $|2\rangle\to|1\rangle$ coupled to the isotropic PBG reservoir, anisotropic PBG reservoir, and free-space vacuum reservoir, respectively. Here, symmetric values of parameters (i.e., the same detuning $\delta_{g1}=-\delta_{g2}=1.0$, the same coupling constants $\beta_{21}=\beta_{31}=1.0$, $\alpha_{21}=\alpha_{31}=1.0$, $\gamma_{21}=\gamma_{31}=1.0$, and $\gamma_{20}=\gamma_{30}=1.0$) for the double V-type structure are employed. The plots at the positions $N=1,2,3,4$ correspond to ($\eta=1$, $p=1$), ($\eta=1$, $p=0$), ($\eta=0$, $p=1$), and ($\eta=0$, $p=0$), respectively. For convenience, we use the symbol ηp to represent the two types of quantum interference. So $\eta p=\{ij\}$ with $(i,j=1,0)$ represents $\eta=i$ and $p=j$. From Fig. 2, we realize that there are three types of zero absorption. The first type that appears only in the case of the double-band isotropic PBG [Fig. 2(a)] includes two zeros at the band edge. Because of very high DOS at the isotropic photonic band edges, the population of the upper levels transfers very effectively to the lower level $|1\rangle$. Consequently, the medium becomes transparent to the probe field at these frequencies and the zeros of type (1) appear in the absorption spectrum. This fact is reflected by the singularities in the Laplace transform of the delayed Green's function [Eq. (25)].

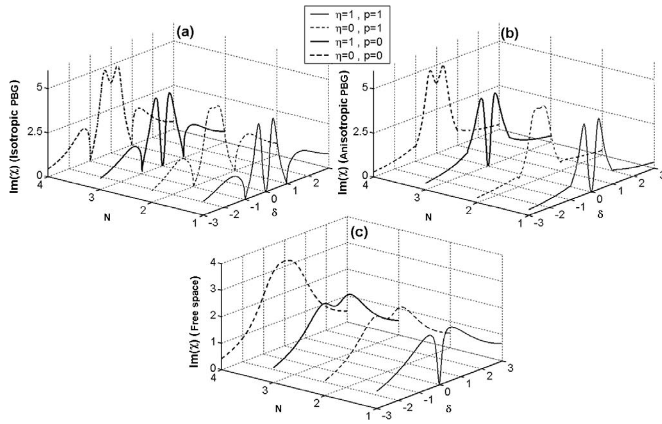

FIG. 2. The steady-state absorption spectrum $\{-\text{Im}[\chi(\delta)]\}$ (in arbitrary units) in the cases of (a) the double-band isotropic PBG reservoir and $\beta_{21}=\beta_{31}=1.0$; (b) the double-band anisotropic PBG reservoir and $\alpha_{21}=\alpha_{31}=1.0$; (c) the free vacuum reservoir and $\gamma_{21}=\gamma_{31}=1.0$. The parameters used are $\delta_{g1}=-\delta_{g2}=1.0$, $\gamma_{20}=\gamma_{30}=1.0$, $a_0(0)=1.0$, and $\omega_{32}=1.0$. All parameters in this paper are in units of γ_{20}.

The second type, occurring at $\delta=0.0$, appears only in simultaneous presence of the two kinds of quantum interference ($\eta p=\{11\}$). This zero exists in all the three cases of the double-band isotropic PBG reservoir, the double-band anisotropic PBG reservoir, and the free vacuum reservoir [the plots at the position $N=1$ in Figs. 2(a)–2(c)]. The investigation of the spontaneous emission spectrum of this system shows there is a black dark line at central frequency of the two upper levels in the case of $\eta p=\{11\}$ [14]. This dark line indicates that the quantum interference in both V-type transitions is destructive. Therefore, the population is trapped in the upper levels and hence the medium becomes transparent at $\delta=0.0$ [the zero of type (2)].

Finally, in the third type there is a zero absorption at $\delta=0.0$ for $\eta p=\{10\}$ in the cases of the double-band isotropic and anisotropic PBG reservoirs [the plots at the position $N=2$ in Figs. 2(a) and 2(b)]; whereas, at the same condition this zero absorption vanishes in case of the free vacuum reservoir [Fig. 2(c)]. This means that the quantum interference in transitions $|3\rangle\to|0\rangle$ and $|2\rangle\to|0\rangle$ is not completely destructive. Consequently, some population transfers to the

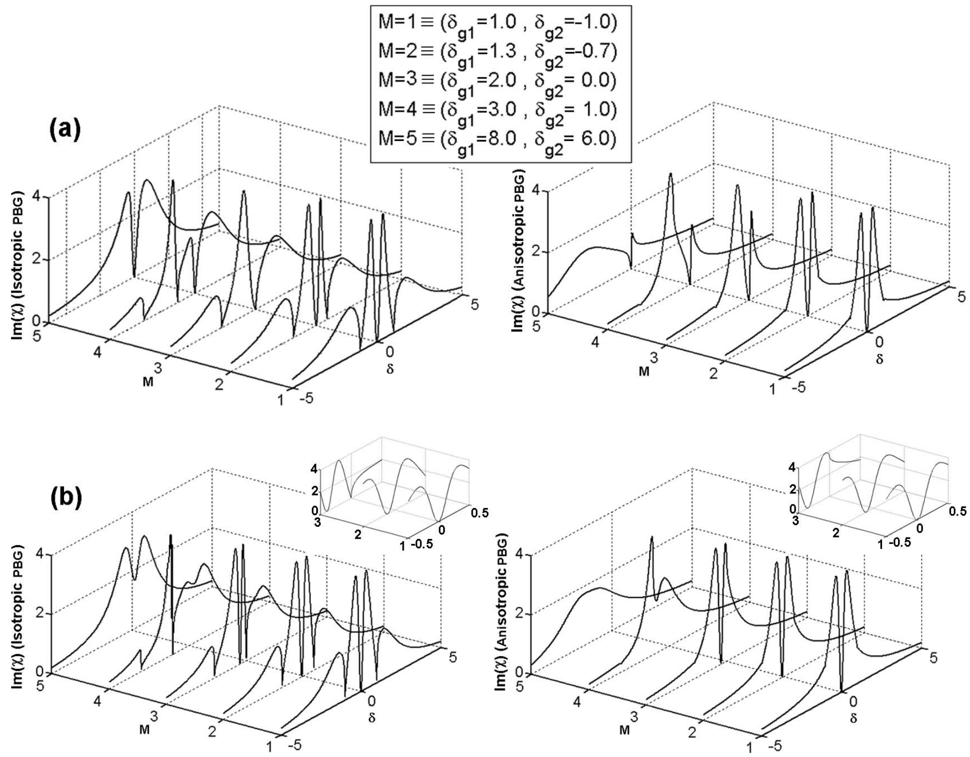

FIG. 3. The steady-state absorption spectra $\{-\text{Im}[\chi(\delta)]\}$ (in arbitrary units) for different detuning of the atomic transitions $|3\rangle \to |1\rangle$ and $|2\rangle \to |1\rangle$ from the band gap edges. (a) The case of $\eta p = \{11\}$; (b) the case of $\eta p = \{10\}$, for the double-band isotropic (left) and anisotropic (right) PBG. The other parameters used are the same as Fig. 2.

lower level $|0\rangle$ and absorbs the probe field at $\delta = 0.0$. However, this quantum interference is completely destructive in the cases of the double-band isotropic and anisotropic PBG. It seems that this destructiveness is due to the presence of the PBG reservoir.

Since both kinds of the quantum interference are responsible for the zero absorption of type (2), its position must be independent of the width and the location of the PBG. On the other hand, we expect the place of the zero absorption of type (3) depends on the width and the location of the PBG. In order to investigate these assumptions, in Fig. 3, the absorption spectra are shown for different values of the detuning of the atomic transitions $|3\rangle \to |1\rangle$ and $|2\rangle \to |1\rangle$ from the band gap edges (Fig. 3), and the absorption spectra are compared in the cases of the double-band and single-band PBG reservoirs (Fig. 4).

Figure 3 shows that the zero absorption of type (2) occurs at $\delta = 0.0$ irrespective of the values of δ_{g1} and δ_{g2}, while the zero of type (3) appears at or around $\delta = 0.0$ only in the case that at least one of the atomic transition frequencies from the upper levels to the lower level $|1\rangle$ (namely ω_{31}, ω_{21}) lies within the PBG; otherwise, this zero absorption suppresses.

Furthermore, the zero absorption occurs at $\delta = 0.0$ for symmetric values of δ_{g1}, δ_{g2} ($\delta_{g1} = -\delta_{g2}$) [the plots at the position $M = 1$ in Fig. 3(b)], whereas it appears around $\delta = 0.0$ for asymmetric values of δ_{g1}, δ_{g2} ($\delta_{g1} \neq -\delta_{g2}$) [the plots at the positions $M = 2, 3$ in Fig. 3(b)]. Similar results are found in Ref. [19]. In their paper, Zhou *et al.* have investigated the quantum interference in probe absorption of a V-type three-level atom in the free space reservoir. They showed that in the case of quantum interference, zero absorption appears at the middle frequency of the upper levels if the spontaneous decay rates of the two upper levels were equal, otherwise the zero absorption occurs around the middle frequency of the upper levels. In our case, the total spontaneous decay rates of the upper levels depend on the detuning of the atomic transitions $|3\rangle \to |1\rangle$ and $|2\rangle \to |1\rangle$ from the band gap edges. In asymmetric cases of $\delta_{g1} \neq -\delta_{g2}$, the total spontaneous decay rates of the upper levels are not equal. Thus, the zero absorption of type (3) appears around $\delta = 0.0$.

For more discussion, we plot the absorption spectra in the cases of the double-band and the single-band PBG. For comparison, we assume that in both cases ω_{21} and ω_{31} lie within the same band. In Fig. 4 the absorption spectra are shown for (a) $\eta p = \{11\}$ and (b) $\eta p = \{10\}$, respectively, in the cases of the single-band (solid lines) and double-band (dashed lines) isotropic (left) and anisotropic (right) PBG reservoirs. Figure 4(a) shows that the zero absorption of type (2) appears again for the parameter used at $\delta = 0.0$ when $\eta p = \{11\}$ in all cases of the single-band and double-band isotropic and anisotropic PBG. The zero absorption of type (3) vanishes in the same condition when $\eta p = \{10\}$ [Fig. 4(b)]. Nevertheless, zero absorptions due to the singularities in the Laplace transform of the delayed Green's function are present in both cases of the single-band and the double-band isotropic PBG.

The discussion above indicates that the zero absorption of type (2) is the consequence of the quantum interference in both V-type transitions (namely $[|3\rangle \to |0\rangle, |2\rangle \to |0\rangle]$ and $[|3\rangle \to |1\rangle, |2\rangle \to |1\rangle]$). On the other hand, we speculate the combined effects of the quantum interference in the V-type

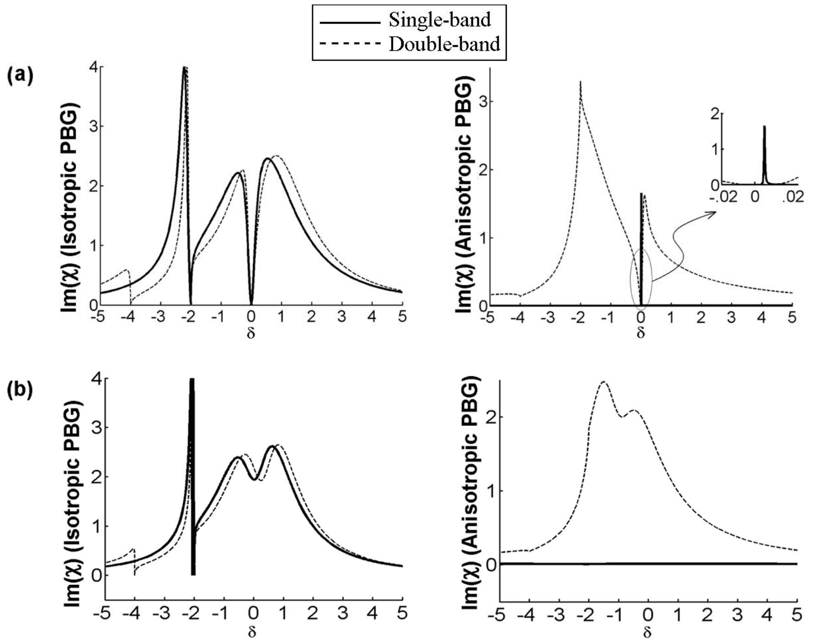

FIG. 4. The steady-state absorption spectrum $\{-\mathrm{Im}[\chi(\delta)]\}$ (in arbitrary units) in the cases of (a) $\eta p=\{11\}$ and (b) $\eta p=10$, respectively, for the single-band and double-band isotropic (left) and anisotropic (right) PBG. The parameters used are $\delta_{g1}=4.0$, $\delta_{g2}=2.0$ (for the double-band PBG) and $\delta_g=2.0$ (for the single-band PBG). The other parameters used are the same as Fig. 2.

transitions coupled to the free space reservoir (namely $|3\rangle\rightarrow|0\rangle$, $|2\rangle\rightarrow|0\rangle$) and the DOS of the PBG reservoir are responsible for the zero absorption of type (3), but further studies are needed to clarify the physical nature of the zero absorption of this type.

Further dissection of Fig. 4 shows that, unlike the isotropic case, the absorption spectra in the case of the single-band anisotropic PBG are remarkably different from those of the double-band anisotropic PBG. These similarities and differences are reflected in the dispersion spectra, too (see Fig. 5).

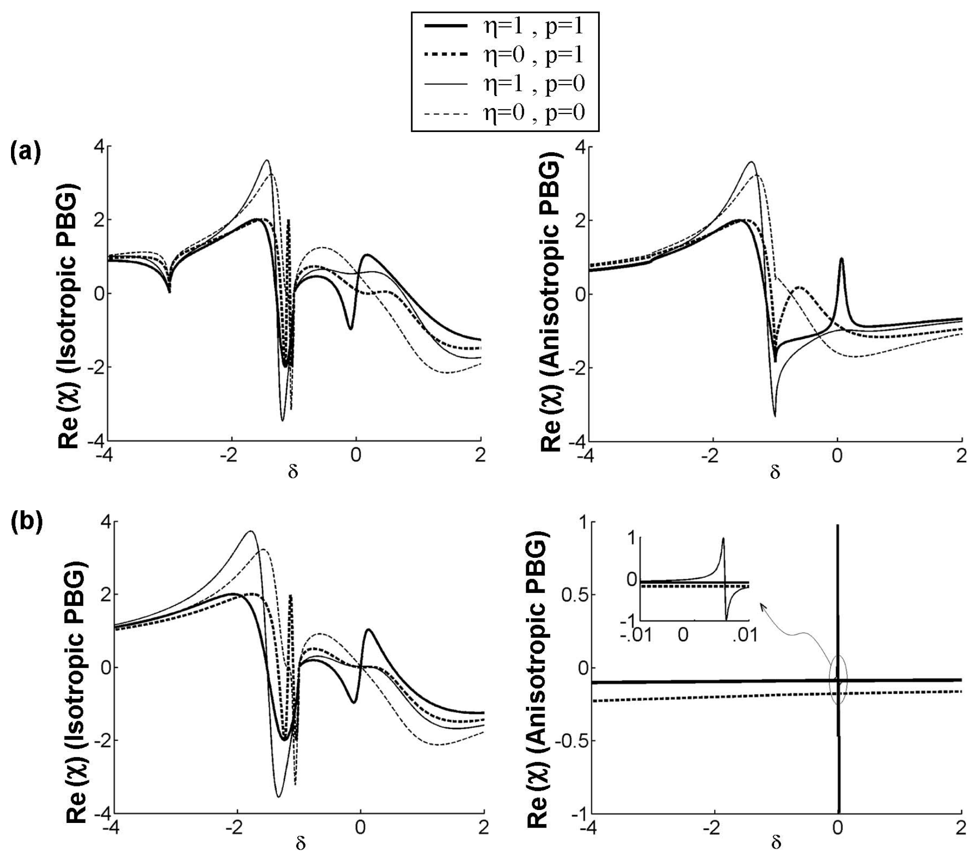

FIG. 5. The steady-state dispersion $\{\mathrm{Re}[\chi(\delta)]\}$ (in arbitrary units) for (a) the double-band PBG, and (b) the single-band PBG reservoir, in the cases of isotropic (left) and anisotropic (right) PBG. The parameters used are $\delta_{g1}=3.0$, $\delta_{g2}=1.0$, (for the double-band PBG) and $\delta_g=2.0$ (for the single-band PBG). The other parameters used are the same as Fig. 2.

To show this, we consider the dispersion property of the model used, and assume ω_{21} and ω_{31} are located in the upper band. In Figs. 5(a) and 5(b), the dispersion spectra are shown in the cases of the double-band and single-band isotropic (left) and anisotropic (right) PBG for (ηp={11}) (thick solid lines), (ηp={10}) (thin solid lines), (ηp={01}) (thick dashed lines), and (ηp={00}) (thin dashed lines), respectively. As in the absorption property, we see that the dispersion property in the case of the single-band isotropic PBG is very similar to those of the double-band isotropic PBG. But the situation is different for anisotropic PBG due to the DOS of the single-band PBG reservoir.

It is seen that the dispersion curves are smooth anywhere in the case of the single-band anisotropic PBG for ηp={10}, ηp={01}, and ηp={00}, respectively [see Fig. 5(b)]. However, the dispersion curve varies very sharply near δ=0.0 for ηp={11}. The quantum interference in both V-type transitions and the DOS of the single-band photonic crystal reservoir are responsible for this behavior. Since the slope of dispersion curve around δ=0.0 is negative, it can lead to the superluminal propagation of light if both kinds of the quantum interference take place.

IV. CONCLUSION

In this paper, we treated absorption-dispersion properties of a double V-type four-level atom embedded in a double-band photonic crystal. We assumed that the double V-type transitions are coupled, respectively, by the free vacuum modes and the PBG modes, leading to the two possible types of quantum interference. The results show that three types of zero absorption (transparency) appear in this model. In the first type, there exist two zeros at the band edge frequencies in the case of the isotropic PBG. In the second type, a zero emerges at the middle frequency of the two upper levels in the case that the quantum interference takes place in both V-type transitions. This zero absorption is independent of the PBG position. Finally, when the quantum interference takes place only in the V-type transitions coupled to the free vacuum modes, a zero occurs at or around the middle frequency of the two upper levels as the third type. It is shown that the first type originates from the two singularities in the Laplace transform of the delayed Green's function, which is equivalent to the DOS of the isotropic PBG modes. The quantum interference in both V-type transitions is responsible for the second type. Ultimately, the third type is due to the quantum interference in the V-type transitions coupled to the free space reservoir and the DOS of the PBG. The results show that the dispersion property of the model used depends on the quantum interference and the DOS of the PBG reservoir. It is shown that the dispersion property of the system (the slope of the dispersion curve) depends on the two types of the quantum interference and the DOS of the PBG reservoir. Specially, the slope of the dispersion curve can be very negative in the case of the single-band anisotropic PBG reservoir, if the two types of quantum interference take place.

Such coherent phenomena are similar to those in general EIT systems but here the atomic coherence is established by the quantum interference and the modified reservoir. This kind of system may have many advantages over ordinary EIT systems and promises to be applicable for a wide variety of quantum optical or nonlinear optical phenomena. In this respect, the application in connection with lasing without inversion and potential use in logic elements for optical computers and optical commutators can be mentioned.

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