Improving fidelity in atomic state teleportation via cavity decay

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We propose a modified protocol of atomic state teleportation for the scheme proposed by Bose *et al.* [Phys. Rev. Lett. **83**, 5158 (1999)]. The modified protocol involves an additional stage in which quantum information distorted during the first stage is fully recovered by a compensation of the damping factor. The modification makes it possible to obtain a high fidelity of teleported state for cavities that are much worse than that required in the original protocol, i.e., their decay rates can be over 25 times larger. The improvement in the fidelity is possible at the expense of lowering the probability of success. We show that the modified protocol is robust against dark counts.

I. INTRODUCTION

Quantum teleportation [1] is considered to be a perfect way of transferring qubits over long distances. It is particularly important to teleport qubits represented by the atomic states, which can store quantum information for sufficiently long time as to make it available for further quantum processing. However, in contrast to the teleportation of photonic states, the teleportation of atomic states over long distances is a difficult task. As yet, the longest distance achieved experimentally for atomic states is of the order of micrometers [2,3] while for photonic states is of the order of kilometers [4]. It is obvious that the distance of atomic states teleportation has to be orders of magnitude greater to make the teleportation useful in quantum communication. In order to make this distance greater, it is necessary to employ photons, which are the best long distance carriers of quantum information, to establish quantum communication between the remote atoms and complete the atomic state teleportation. Such a scheme of atomic state teleportation has been presented by Bose *et al.* [5]. They have proposed an additional stage of teleportation protocol—the preparation stage, in which the state of sender atom is mapped onto the sender cavity field state and therefore can be teleported in the next stage using well known linear optics techniques. The possibility of operating on atomic qubits with linear optics elements is the reason why a combination of atomic states and cavity field states has been recently suggested in many proposals, not only in proposals of teleportation protocols [6–8] but also in other schemes of quantum information processing [9–15]. Unfortunately, the state mapping and whole preparation stage is not perfect because of a destructive role played by cavity decay. The cavity decay reduces the fidelity of teleported state and the probability of success. Bose *et al.* [5] have suggested a way to minimize a destructive role of this imperfection by assuming very small cavity decay rate. However, the value of cavity decay rate required by their protocol is two orders of magnitude below of what is currently available [16–23].

Here, we present a protocol that reduces the effect of cavity decay on the fidelity. This protocol makes it possible to use cavities with larger decay rates without worsening the fidelity but at the expense of lowering success rates.

II. MODEL

The teleportation protocol that we propose in this paper is designed for the same device which Bose *et al.* [5] consider in their scheme. The device is depicted in Fig. 1. It is composed of two cavities C_A and C_B, a 50-50 beam splitter, two lasers L_A and L_B, and two single-photon detectors D_+ and D_-. The receiver, Bob, has the cavity C_B and the laser L_B. The other elements of the device are at the side of the sender— Alice. Inside each cavity there is one trapped atom, modeled by a three-level Λ system with two stable ground states $|0\rangle$ and $|1\rangle$, and one excited state $|2\rangle$ as shown in Fig. 2. Only the excited state decays spontaneously, therefore the ground states are ideal candidates for an atomic qubit. The spontaneous decay rate of the excited state is denoted by γ. Operations on the qubit coded in the superposition of both ground states are possible using two transitions: $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$. First of the transitions is coupled to the cavity mode with the coupling strength g while the second transition is coupled to a classical laser field with the coupling strength Ω. Since we want a population of the excited state to be negligible, the laser field and the cavity mode are detuned from the corresponding transition frequencies by Δ. Beside the spontaneous decay of the excited atomic state there is another decay mechanism. One mirror in each cavity is partially transparent and therefore photons leak out of the
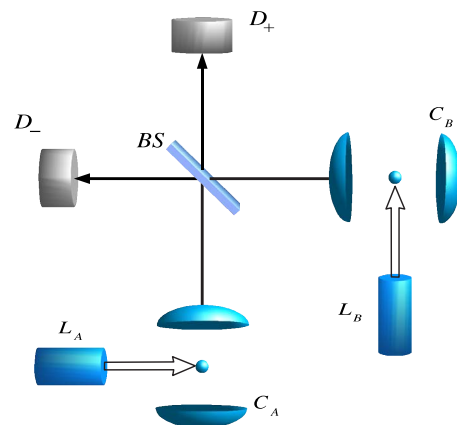

FIG. 1. (Color online) Schematic representation of the setup to realize long distance teleportation of atomic states via photons.
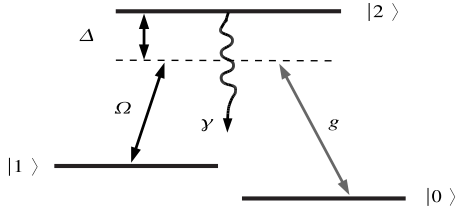
FIG. 2. Level scheme of the Λ atom interacting with the classical laser field and the quantized cavity mode.

cavities through these mirrors at a rate κ. The evolution of each atom-cavity system is governed by the effective non-Hermitian Hamiltonian ($\hbar=1$ here and in the following)

$$H = (\Delta - i\gamma)\sigma_{22} + (\Omega\sigma_{21} + g a\sigma_{20} + \text{H.c.}) - i\kappa a^\dagger a, \quad (1)$$

where a denotes the annihilation operator for Alice's cavity mode (a_A) or Bob's cavity mode (a_B). In Eq. (1) we introduce the flip operators $\sigma_{ij} \equiv |i\rangle\langle j|$. Both our protocol as well as the Bose *et al.* protocol work in the low saturation limit (g^2/Δ^2, $\Omega^2/\Delta^2 \ll 1$) and therefore the excited atomic state can be adiabatically eliminated [24–31]. Either of them require small values of the spontaneous decay rate ($\Delta \gg \gamma$ and $\gamma g^2/\Delta^2$, $\gamma\Omega^2/\Delta^2 \ll \kappa$) [32] which makes it possible to neglect γ as a first approximation. Finally, we assume the condition $\Omega = g$ which leads to a very simple form of the Hamiltonian

$$H = -\delta\sigma_{11} - \delta a^\dagger a\sigma_{00} - (\delta a\sigma_{10} + \text{H.c.}) - i\kappa a^\dagger a, \quad (2)$$

where $\delta = g^2/\Delta$. The evolution described by Eq. (2) is interrupted by collapses. Photon decays registered by detectors correspond to the action of the collapse operator

$$C = \sqrt{\kappa}(a_A + i\epsilon a_B), \quad (3)$$

where ϵ is equal to 1 for photon detection in D_+ and equal to -1 for photon detection in D_-.

The simple form of the Hamiltonian (2) allows for analytical solutions of the nonunitary Schrödinger equation and get expressions for the time evolution of quantum states which are used in both protocols. To give the expressions a more compact form we use the notation $|jn\rangle \equiv |j\rangle_{\text{atom}} \otimes |n\rangle_{\text{mode}}$ to describe the state of the atom-cavity system. During the whole teleportation process the time evolution of the system is restricted to the subspace spanned by the states: $|00\rangle$, $|10\rangle$, and $|01\rangle$. The state $|00\rangle$ experiences no dynamics because there is no operator in the Hamiltonian (2) which can change this state. Time evolution of the other two states is described by

$$e^{-iHt}|10\rangle = e^{i\delta t}e^{-\kappa t/2}\left[i\frac{2\delta}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t}{2}\right)|01\rangle + \left(\cos\left(\frac{\Omega_\kappa t}{2}\right)\right.\right.$$
$$\left.\left. + \frac{\kappa}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t}{2}\right)\right)|10\rangle\right],$$

$$e^{-iHt}|01\rangle = e^{i\delta t}e^{-\kappa t/2}\left[i\frac{2\delta}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t}{2}\right)|10\rangle + \left(\cos\left(\frac{\Omega_\kappa t}{2}\right)\right.\right.$$
$$\left.\left. - \frac{\kappa}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t}{2}\right)\right)|01\rangle\right], \quad (4)$$

where $\Omega_\kappa = \sqrt{4\delta^2 - \kappa^2}$. There are two important local operations we can perform on the system state via e^{-iHt}. First of them is to map the atomic state onto the cavity mode and second is the generation of the maximally entangled state of the atom and the cavity mode. The atomic state mapping one can obtain by turning the laser on for time t_A is given by

$$|10\rangle \rightarrow ie^{i\delta t_A}e^{-\kappa t_A/2}|01\rangle, \quad (5)$$

where $t_A = (2/\Omega_\kappa)[\pi - \arctan(\Omega_\kappa/\kappa)]$. In order to create the maximally entangled state the laser should be turned on for time $t_B = (2/\Omega_\kappa)\arctan[\Omega_\kappa/(2\delta - \kappa)]$

$$|10\rangle \rightarrow e^{i\delta t_B}e^{-\kappa t_B/2}\frac{2\delta}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t_B}{2}\right)(|10\rangle + i|01\rangle). \quad (6)$$

When the laser is turned off then $\Omega = 0$, and the Hamiltonian goes over into $H = -\delta a^\dagger a\sigma_{00} - i\kappa a^\dagger a$. Then all the terms of the Hamiltonian correspond to the diagonal elements in matrix representation, and the nonunitary Schrödinger equation can be easily solved. The evolution of the states $|10\rangle$ and $|01\rangle$, when the laser is turned off, are thus given by

$$e^{-iHt}|10\rangle = |10\rangle,$$

$$e^{-iHt}|01\rangle = e^{i\delta t}e^{-\kappa t}|01\rangle. \quad (7)$$

III. TELEPORTATION PROTOCOL

Both protocols start with the same initial state—the unknown state that Alice wants to teleport, which is stored in her atom. Bob's atom is prepared in the state $|1\rangle$ and the field modes of both cavities are empty, so we have

$$|\psi\rangle_A = \alpha|00\rangle_A + \beta|10\rangle_A, \quad (8)$$

$$|\psi\rangle_B = |10\rangle_B. \quad (9)$$

The teleportation protocol with improved fidelity consists of five stages: (A) the preparation stage, (B) the detection stage I, (C) the compensation stage, (D) the detection stage II, and (E) the recovery stage.

A. Preparation stage

The preparation stage is necessary because the quantum information encoded initially in Alice's atom is teleported by performing joint measurement on the field state of both cavities. Before of the detection stage Alice has to map the quantum information onto her cavity field state while Bob has to create the maximally entangled state of his atom and his cavity field. Alice and Bob achieve their goals by switching their lasers on for times t_A and t_B, respectively. After the

preparation stage the state of Alice's atom-cavity system is given by

$$|\widetilde{\psi}\rangle_A = \alpha|00\rangle_A + i e^{i\delta t_A} e^{-\kappa t_A/2} \beta|01\rangle_A, \qquad (10)$$

and Bob's system state becomes

$$|\widetilde{\psi}\rangle_B = e^{-\kappa t_B/2} \frac{2\delta}{\Omega_\kappa} \sin\left(\frac{\Omega_\kappa t_B}{2}\right)(|10\rangle_B + i|01\rangle_B). \qquad (11)$$

This first stage is successful only under the absence of photon detection event. The probability that no collapse occurs during Alice's operation is given by the squared norm of the state vector

$$P_A = |\alpha|^2 + e^{-\kappa t_A}|\beta|^2. \qquad (12)$$

Similarly, we can obtain appropriate expression for the probability of no collapse during Bob's operation

$$P_B = e^{-\kappa t_B} \frac{8\delta^2}{\Omega_\kappa^2} \sin^2\left(\frac{\Omega_\kappa t_B}{2}\right). \qquad (13)$$

It is evident that the state mapping is not perfect because of the damping factors that appear in expression (12) for P_A and in expression (10) for the state $|\widetilde{\psi}\rangle_A$. These damping factors reduce both the probability that the state mapping is successful and the fidelity of this operation. The quantum information after the mapping operation is also modified by the phase factor $ie^{i\delta t_A}$ but, in contrast to damping factors, the phase factors can later be easily compensated for and therefore they do not reduce the fidelity. In order to make the probability P_A and the fidelity close to unity Bose *et al.* assume that $\Omega_\kappa \gg \kappa$, which means that both κ and t_A values are small and the damping factor $e^{-\kappa t_A/2}$ is close to unity. Generally, however, the damping factor is not unity even for very small κ and t_A and, in consequence, the fidelity of the teleported state is diminished. Since high fidelities are required by quantum computation algorithms, we will show how to compensate for this factor in the next stages of the protocol.

B. Detection stage I

When the quantum information is mapped onto the state of Alice's cavity field and the maximally entangled state of Bob's cavity field and the target atom is created, then the joint measurement of both cavity fields can be performed. During this stage Alice and Bob perform the joint measurement just by waiting with their lasers turned off. The teleportation is successful if the detectors register one and only one photon. In successful cases the joint state of Alice's and Bob's systems becomes

$$|\widetilde{\phi}(t_d)\rangle = (i\epsilon\alpha|00\rangle_B + e^{i\delta t_A} e^{-\kappa t_A/2}\beta|10\rangle_B)|00\rangle_A$$

$$+ ie^{i\delta t_1} e^{-\kappa t_1/2}\beta e^{-\kappa t_d} e^{i\delta t_d}$$

$$\times (|01\rangle_B|00\rangle_A + i\epsilon|00\rangle_B|01\rangle_A), \qquad (14)$$

where t_d is the time of this detection stage. Until now the operations in both protocols are exactly the same. In the

protocol of Bose *et al.* it is assumed that time t_d is much longer than κ^{-1} and thus all unwanted states in expression (14) can be neglected. Finally, after removing a phase factor, the state of Bob's atom is given by $\alpha|0\rangle_B + e^{-\kappa t_A/2}\beta|1\rangle_B$. It is obvious that the fidelity of teleported state will never reach unity because of the factor $e^{-\kappa t_A/2}$. Moreover, in the protocol of Bose *et al.* the fidelity of teleported state decreases with increasing κ. In our protocol, we use one of the unwanted states to compensate for the factor $e^{-\kappa t_A/2}$. This compensation can be done if we choose the time of this detection stage such that $e^{i\delta t_d} = -1$. Then expression (14) can be rewritten as

$$|\widetilde{\phi}(t_d)\rangle = i\epsilon\alpha|00\rangle_B|00\rangle_A + e^{i\delta t_A} e^{-\kappa t_A/2}\beta e^{-\kappa t_d}\epsilon|00\rangle_B|01\rangle_A$$

$$+ e^{i\delta t_A} e^{-\kappa t_A/2}\beta(|10\rangle_B - ie^{-\kappa t_d}|01\rangle_B)|00\rangle_A. \qquad (15)$$

C. Compensation stage

In the compensation stage Bob compensates for the factor $e^{-\kappa t_A/2}$ by turning his laser on for time t_c. During the operation Alice's laser remains turned off. On condition that no photon detection occurs during time t_c, the unnormalized joint state at the end of this stage is given by

$$|\widetilde{\phi}(t_c)\rangle = e^{i\delta(t_A+t_c)} e^{-\kappa t_A/2}\beta e^{-\kappa(t_d+t_c)}\epsilon|00\rangle_B|01\rangle_A$$

$$- ie^{i\delta(t_A+t_c)}\beta e^{-\kappa(t_A+t_c)/2}\varphi(t_c)|01\rangle_B|00\rangle_A$$

$$+ e^{i\delta(t_A+t_c)}\beta e^{-\kappa(t_A+t_c)/2}\vartheta(t_c)|10\rangle_B|00\rangle_A$$

$$+ i\epsilon\alpha|00\rangle_B|00\rangle_A, \qquad (16)$$

where

$$\varphi(t_c) = e^{-\kappa t_d}\cos\left(\frac{\Omega_\kappa t_c}{2}\right) - \frac{2\delta + \kappa e^{-\kappa t_d}}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t_c}{2}\right),$$

$$\vartheta(t_c) = \cos\left(\frac{\Omega_\kappa t_c}{2}\right) + \frac{\kappa + 2\delta e^{-\kappa t_d}}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t_c}{2}\right). \qquad (17)$$

It is seen that this operation transfers population from the state $|01\rangle_B|00\rangle_A$, which is unwanted, to the state $|10\rangle_B|00\rangle_A$. Of course, we want the transfer to compensate for the factor $e^{-\kappa t_A/2}$ and therefore t_c has to fulfill the condition

$$e^{-\kappa(t_A+t_c)/2}\vartheta(t_c) = 1. \qquad (18)$$

D. Detection stage II

The population of one of the two unwanted states is already reduced after the previous stage, but it cannot be neglected yet. Moreover, the population of the second unwanted state is still considerable. Presence of the two unwanted states decreases the teleportation fidelity, so in the fourth stage of the protocol Alice and Bob have to eliminate them. All they have to do to achieve this goal is simply to wait for a finite time $t_D \gg \kappa^{-1}$. After time t_D the populations of both unwanted states are negligible and unnormalized joint state can be very well approximated by

$$|\widetilde{\phi}(t_D)\rangle = (i\epsilon\alpha|00\rangle_B + e^{i\delta(t_A+t_c)}\beta|10\rangle_B)|00\rangle_A. \qquad (19)$$
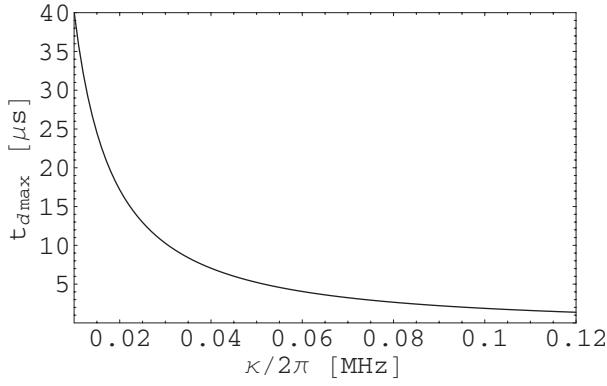
FIG. 3. The value of $t_{d\,\max}$ as a function of κ for $(\Delta;\Omega;g)/2\pi=(100;10;10)$ MHz calculated numerically using condition (21).

E. Recovery stage

Finally, Bob has to remove the phase shift factor $i\epsilon e^{-i\delta(t_A+t_c)}$ to recover the original Alice's state. To this end Bob adds to the state $|1\rangle_{\mathrm{atom}B}$ an extra phase shift with respect to the state $|0\rangle_{\mathrm{atom}B}$ using the Zeeman evolution [5]. After this operation the state of Bob's atom is exactly the same as the initial state of Alice's atom, i.e., $\alpha|0\rangle_{\mathrm{atom}B}+\beta|1\rangle_{\mathrm{atom}B}$, and thus the teleportation fidelity of this protocol can be very close to unity. This completes the teleportation protocol.

Now it is time to explain in detail how to choose the time t_d. The condition $e^{i\delta t_d}=-1$ leads to many solutions given by $t_d=\pi(2m+1)/\delta$, where m is a nonnegative integer. However, we cannot set m arbitrary because $\vartheta(t_c)$ and the probability of success in second stage are functions of t_d. It is obvious that the probability of observing one photon during detection time t_d increases with increasing t_d. On the other hand, we cannot choose this detection time too long because the population of unwanted state $|01\rangle_B|00\rangle_A$ can then be too small to compensate for the factor $e^{-\kappa t_A/2}$. Thus, t_d is limited by some time $t_{d\,\max}$. Let us now estimate $t_{d\,\max}$. Expression $e^{-\kappa(t_A+t_c)/2}\vartheta(t_c)$ takes its maximal value for the time of the compensation stage given by

$$t_{c\,\max}=\frac{2}{\Omega_\kappa}\arctan\left(\frac{2\delta\Omega_\kappa e^{-\kappa t_d}}{\Omega_\kappa^2+\kappa(\kappa+2\delta e^{-\kappa t_d})}\right). \qquad (20)$$

The factor $e^{-\kappa t_A/2}$ can be compensated for only under the condition that $e^{-\kappa(t_A+t_{c\,\max})/2}\vartheta(t_{c\,\max})\geq 1$. Since both $\vartheta(t_c)$ and $t_{c\,\max}$ depend on t_d, we can estimate the value of $t_{d\,\max}$ by finding numerically t_d satisfying the condition

$$e^{-\kappa(t_A+t_{c\,\max})/2}\vartheta(t_{c\,\max})=1. \qquad (21)$$

The problem of choosing t_d is much simpler when we want to compensate for the factor $e^{-\kappa t_A/2}$ for as large κ as possible. From Fig. 3 one can see that the limit $t_{d\,\max}$ decreases with increasing κ. Therefore, we should choose the smallest value of t_d by setting m to zero.

IV. NUMERICAL RESULTS

Let us now compare both protocols. For this purpose we compute the average probability of success and the average fidelity of teleported state for the same values of the detuning and both coupling strengths as in Ref. [5], i.e., $(\Delta;\Omega;g)/2\pi=(100;10;10)$ MHz. It is necessary to take average values over all input states because the probability of success in both protocols as well as the fidelity in the Bose *et al.* protocol all depend on the unknown moduli of the amplitudes α and β of the initial state. The fidelity in our protocol seems to be independent of the amplitudes of initial state and should be equal to unity. However, this is only true for the simplified model for which the excited state is eliminated. In more general model described by the Hamiltonian (1) the population of the excited state has a nonzero value during the evolution given by Eq. (4) even if the atom is initially prepared in its ground state. However, the population of the excited state remains zero for the initial state $|00\rangle$ of atom-cavity system because the state experiences no dynamics. If the initial state is a superposition given by Eq. (8) then the population of the excited state depends on the moduli of the amplitudes α and β. Since the population of the excited state reduces the fidelity, it is also necessary to average the fidelity in our protocol over all input states.

We compute all the averages numerically using the method of quantum trajectories [33,34] together with the Monte Carlo technique. Each trajectory starts with a random initial state and evolves according to a chosen teleportation protocol. If measurement indicates success then we calculate the fidelity of teleported state at the end of the protocol. Otherwise, we reject a trajectory as unsuccessful. After generating 20 000 trajectories we average the fidelity over all trajectories and calculate the average probability of success as a ratio of the number of successful trajectories to the number of all trajectories.

There are some problems that appear when we use the Hamiltonian (1) to simulate performance of our protocol. First, the fidelity is sensitive to the inaccuracy in calculations of phase shift factors. The compensation of the factor $e^{-\kappa t_A/2}$ requires the phase shift of the state $|01\rangle_B|00\rangle_A$ relative to the state $|10\rangle_B|00\rangle_A$ to be equal to $-i$ as shown in Eq. (15). Therefore the time t_d of the detection stage I has to satisfy the condition $e^{i\delta t_d}=-1$. However, the analytical expression for δ is derived from the Hamiltonian (2) and thus $\exp(i\delta t_d)$ is only an approximation to the real phase shift factor. Unfortunately, the population transfer that takes place in the compensation stage leads to an unknown extra phase shift in the final state (19) when the phase shift between states $|01\rangle_B|00\rangle_A$ and $|10\rangle_B|00\rangle_A$ differs from the expected value $-i$. Of course, this unknown phase shift cannot be compensated for in the recovery stage, which means that the fidelity in our protocol can even be smaller than the fidelity in the Bose *et al.* protocol. To overcome this problem we use a numerical optimization procedure which finds, for the more general model, such t_d that the joint state of Alice's and Bob's systems becomes as close to the expected state given by Eq. (15) as possible.

Second, a question arises: how to estimate the biggest value of κ for which the compensation is still possible? This
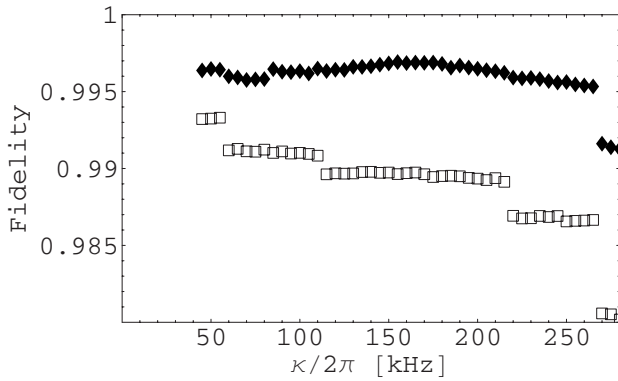
FIG. 4. The average fidelity of teleportation in the new protocol (diamonds) and in Bose *et al.* protocol (open squares) as functions of the cavity decay rate for $(\Delta;\Omega;g;\gamma)/2\pi=(100;10;10;0)$ MHz. The averages are taken over 20 000 trajectories.

value is very important because we want to know how good (or rather bad) cavities can be used for effective high fidelity teleportation. In the simplified model of our protocol governed by the Hamiltonian (2) this value can be computed from Eq. (21) and is about $\kappa/2\pi \approx 0.17$ MHz. However, the population of the excited atomic state changes this value because of the transfer of population from the state $|20\rangle_B|00\rangle_A$ to the state $|10\rangle_B|00\rangle_A$ in the compensation stage. To estimate the acceptable value of κ, we plot the average fidelity and the average probability of success as functions of κ. The population of the excited atomic state changes also the time t_c for which the improvement of the fidelity in our protocol is the best one and thus the value of t_c calculated from Eq. (18) can be used only as a starting point in the numerical computation of this time. From numerical results presented in Fig. 4 we find that there is a plateau in the fidelity of the modified protocol up to $\kappa/2\pi \approx 0.25$ MHz after which the fidelity jumps down. We consider the value of κ at the jump as the biggest value of κ.

Third, the population of the excited atomic state oscillates. Since the population of the excited state diminishes the fidelity of operations periodically, it is necessary to compute numerically, for all operations, such times that minimize the population simultaneously maximizing the fidelity. Until now we have assumed that times t_A and t_B can be calculated analytically as in the Bose *et al.* protocol. However, the analytical expressions are functions of κ, so, for different values of κ the population of the excited state and the fidelities of operations take different values. If we want to stabilize the average fidelity at a high level for different values of κ then we have to compute t_A and t_B numerically.

To begin with our calculations, we set the spontaneous decay rate of excited state to zero because we want to know how close to unity is the fidelity in the ideal case in which there is no possibility of photon emission to modes other than the cavity modes. Figure 4 shows that the modified protocol really stabilizes the fidelity of teleported state at a high level. The fidelity is reduced only by the nonzero population of the excited state and does not decrease with increasing κ until $\kappa/2\pi$ is about 0.25 MHz. The fidelity of teleported state in the protocol of Bose *et al.* is reduced by the population of excited state as well as by the factor
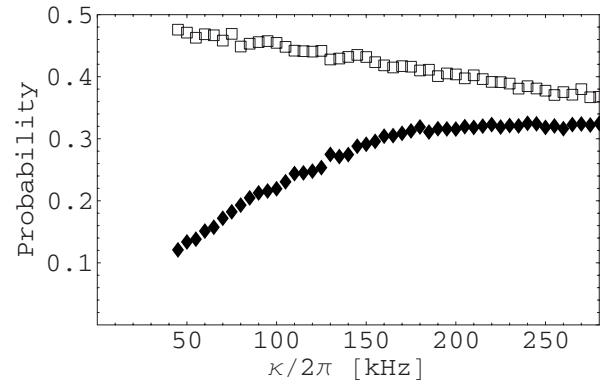

FIG. 5. The average probability of successful teleportation as a function of the cavity decay rate. The diamonds show the average probability of success in the new protocol. The open squares correspond to the average probability in Bose *et al.* protocol. The averages are taken over 20 000 trajectories. The parameters regime is $(\Delta;\Omega;g;\gamma)/2\pi=(100;10;10;0)$ MHz.

$\exp(-\kappa t_A/2)$ and, as expected, it decreases with increasing κ. It is seen from Fig. 4 that there are discontinuous jumps of the fidelity values. The discontinuities come from the numerical procedure finding such t_A for which the mapping fidelity is maximal. The time t_A of the mapping operation is a function of κ, and the mapping fidelity reaches its maximal value when the population of the excited state reaches its minimal value. Since the population of the excited state oscillates the numerically calculated t_A jumps, as κ increases, from one value for which the population of the excited state is minimal after the mapping operation to the next such value. Thus, the factor $\exp(-\kappa t_A/2)$ and the fidelity of the teleported state also exhibit discontinuous behavior. Figure 5 shows that the probability of success in the protocol with improved fidelity is always less than the probability of success in the protocol of Bose *et al.* Fortunately, there is only a small difference between the probabilities of both protocols for the biggest cavity decay rate for which compensation is still possible, i.e., for $\kappa/2\pi \approx 0.25$ MHz.

So far we have assumed that there is no possibility of photon emission to modes other than the cavity mode. Let us now relax this assumption and investigate the influence of the spontaneous emission decay rate of the excited state on both teleportation protocols. The spontaneous atomic emission destroys the quantum information which Alice wants to teleport to Bob. Such runs of the teleportation protocols are unsuccessful and should be rejected. However, an event of spontaneous atomic emission cannot be detected in both schemes and therefore the spontaneous decay rate of excited state reduces the average fidelities. We can only suppress this imperfection by taking $\gamma g^2/\Delta^2$, $\gamma\Omega^2/\Delta^2 \ll \kappa$ [32]. The biggest κ for which the compensation is still possible allows for the choice of $\gamma/2\pi=1$ MHz. We have generated 20 000 trajectories to compute the average fidelities and the average probabilities for the parameters $(\Delta;\Omega;g;\gamma;\kappa)/2\pi = (100;10;10;1;0.265)$ MHz. As a result we have obtained the average fidelity of 0.972 and the average probability of 0.36 for the Bose *et al.* protocol and the average fidelity of 0.978 and the success rate of 0.31 for the modified protocol.

The results indicate that the inability to distinguish the runs of protocols, in which spontaneous emission occurs, reduces only slightly the average fidelities when $\gamma/2\pi=1$ MHz. The average probabilities of success remain unchanged.

Other two important imperfections, which we have to take into account, are a finite detection quantum efficiency and the presence of dark counts. It is necessary to include such sources of noise in our numerical calculations because they are introduced by all real detectors. So far we have assumed in our analysis perfect detectors that are able to register all collected photons and do not produce any signal in the absence of photons. In practice, this assumption is not valid. The probability that a single photon reaching the detector is converted into the measurable signal, which is called the quantum efficiency and denoted by η, is less than unity for all real detectors [33,35]. Moreover, there are clicks, for all real detectors, even in the absence of light. They are called dark counts. These imperfections lead to lowering the average fidelity in both teleportation protocols because of randomness which they introduce to the measurement outcome. There is no way to distinguish the unsuccessful case of two photon emissions from the desired case of one photon emission when only one of the two emitted photons is detected. It is also not possible to recognize the unsuccessful case of no emission if one dark count occurs during the detection stage. The quantum information that Alice wants to teleport is destroyed in the unsuccessful cases. If one cannot reject such cases then the average fidelity is reduced. Therefore it is necessary to use detectors with very high efficiency η and a low enough dark count rate. As far as we know, the highest detector efficiency has been reported by Takeuchi *et al.* [36] and is equal to $\eta=0.88$. To study the effect of the detector inefficiency on the protocols under discussion, we have performed numerical calculations under the assumption that there are not dark counts first. We have used the same parameters as previously, i.e., $(\Delta;\Omega;g;\gamma;\kappa)/2\pi=(100;10;10;1;0.265)$ MHz and we have found that both protocols are sensitive to the detector inefficiency. The average fidelity is reduced to 0.894 in the Bose *et al.* protocol and to 0.905 in the modified protocol. Success rates remain almost unchanged—0.353 in the Bose *et al.* protocol and 0.306 in the modified protocol. It is obvious that the reliable teleportation requires detectors efficiency $\eta=0.88$ or higher. Unfortunately, the dark count rate of the detector increases roughly exponentially with the efficiency [36] and is as high as 20 kHz at the highest efficiency reported by Takeuchi *et al.* [36], i.e., $\eta=0.88$. The high efficiency of the detector means also the high rate of dark counts, which are not good for teleportation. To clarify the situation, we have also investigated the influence of the dark count rate on both teleportation protocols. Surprisingly, the protocol with improved fidelity has appeared to be less sensitive to this imperfection than the Bose *et al.* protocol. The average fidelity in the Bose *et al.* protocol appeared to be equal to 0.801 while the average fidelity in the modified protocol to be equal to 0.897, for the parameters $\eta=0.88$ and the dark count rate 20 kHz. The difference between the two protocols is quite impressive, but it has a simple explanation. In either protocol there is only one stage when the detection of one photon is expected—the detection stage in the Bose *et al.* protocol and the detection

stage I in the modified protocol. Only in these two stages occurrence of the dark count can be erroneously accepted as a successful measurement event because all other stages require no photon detection to be successful. Thus one can easily understand why the influence of the dark counts on both protocols is different by comparing the times of the two crucial stages—the time of the detection stage of the Bose *et al.* protocol (in our calculations we set $t_D=10\kappa^{-1}$) that is much longer than the time of the detection stage I ($t_d=\pi\delta^{-1}$) of the modified protocol. This means that there are many more rejected dark counts in the modified protocol than in the Bose *et al.* protocol. A bigger number of rejected runs with the dark count events leads to an increased average fidelity and at the same time to a decreased success rate. Therefore, the success rate is reduced more significantly in the modified protocol (0.237) than in the Bose *et al.* protocol (0.331).

Finally, we generalize our calculations to include losses in the mirrors and during the propagation. The absorption in the mirrors can be taken into account by making the replacement $\kappa=\kappa'+\kappa''$ in the Hamiltonian (1), where κ' is the decay rate corresponding to the photon transmission through the mirror and κ'' is the photon loss rate due to absorption in the mirrors. The evolution of the system is conditional, so we need also the collapse operators corresponding to the absorption of photons in the mirrors. The additional collapse operators are given by $C_A=\sqrt{2\kappa''}a_A$ and $C_B=\sqrt{2\kappa''}a_B$. As before, the collapse operators describing photon detections are given by Eq. (3) but with κ replaced by κ'. So, we now have two extra collapse operators describing evolution of the system. However, it can be checked that such evolution can be described without using the extra collapse operators when we make the replacement $\kappa=\kappa'+\kappa''$ in the collapse operators given by Eq. (3) and multiply the probability of photon detection by $\eta_a=(\kappa'/\kappa)$, which is the probability that a photon is detected despite the fact that there is absorption in the mirrors. The probability of detection in the presence of absorption is then $P_D'=\eta_a P_D$. The presence of absorption means effectively lower efficiency of the detector.

In the same way, we easily can take into account all photon losses during the propagation between the cavities and the detectors [5,37]. All we need to include such losses into consideration is to introduce additional efficiency factor η_p. Multiplying all the factors, we find the overall detection efficiency $\eta'=\eta_a\eta_p\eta$. To visualize the effect of such losses, we have plotted the average fidelity and the average probability for both protocols as functions of the overall detection inefficiency, i.e., as functions of $1-\eta'$. In order to make the average values reliable, we have generated 100 000 trajectories for each η'. From Fig. 6 it is clear that with increasing photon losses the average fidelity is reduced for both protocols. However, the advantage of the modified protocol to be less sensitive to the dark counts and the compensation for the factor $e^{-\kappa t_A/2}$ result in the fidelity improvement that is clearly visible for almost all values of η'. The difference between both protocols disappears only for such a small η' that most of the trajectories for which measurement indicates success are unsuccessful cases due to dark counts. Of course, in such a case the final state of Bob's atom is random and the average fidelity is 0.5.
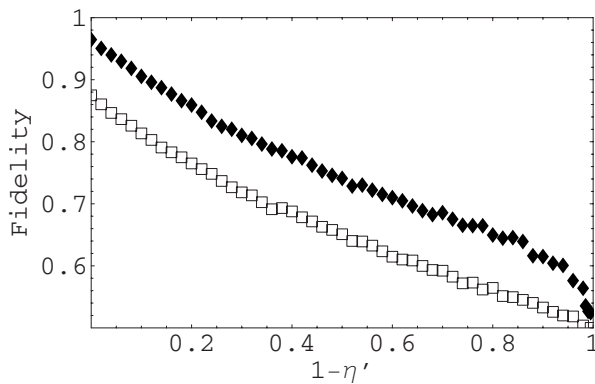
FIG. 6. The average fidelity including the effects of photon losses as a function of the overall detection inefficiency for $(\Delta;\Omega;g;\gamma;\kappa)/2\pi=(100;10;10;1;0.265)$ MHz and the dark count rate 20 kHz. The diamonds correspond to the new protocol and the open squares correspond to the Bose *et al.* protocol.
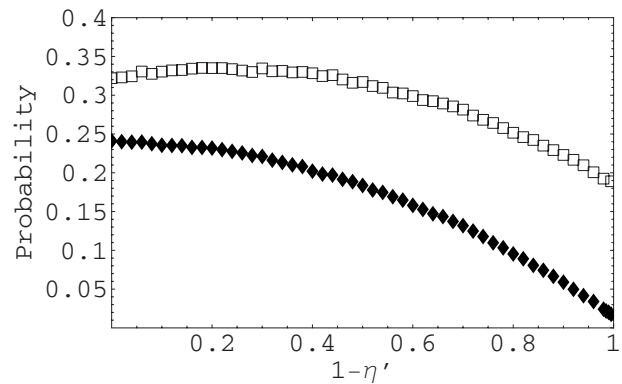
FIG. 7. Average probabilities that measurement indicates success for the modified protocol (diamonds) and for the Bose *et al.* protocol (open squares) as functions of the overall detection inefficiency. The parameter regime is $(\Delta;\Omega;g;\gamma;\kappa)/2\pi=(100;10;10;1;0.265)$ MHz; the dark count rate is 20 kHz.

From Fig. 7 it is visible that higher fidelity can be achieved by accepting lower success rates. The average probability of success in the modified protocol is always less than the average success rate in the Bose *et al.* protocol. This is the price we have to pay for higher fidelity.

V. CONCLUSIONS

We have presented the teleportation protocol for the device proposed by Bose *et al.* that improves the fidelity of teleported state. The improvement is obtained by compensating for the factor $e^{-\kappa t_A/2}$ which appears in the teleportation protocols. We have shown that this compensation makes it possible to stabilize the fidelity at a high level despite the increase in the cavity decay rate. The fidelity is stabilized until $\kappa/2\pi\approx0.25$ MHz. This means that the high fidelity teleportation can be performed for the values of the cavity decay rates over 25 times larger than the values assumed by Bose *et al.* The price we have to pay for more realistic values of the cavity decay rates is that we have to accept lower success rates. We have also shown that the modified protocol is less sensitive to the dark counts of detectors than the original protocol of Bose *et al.*

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