Antisymmetric entangled two-photon states generated in nonlinear GaN/AlN photonic-band-gap structures

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The properties of an entangled two-photon state antisymmetric in frequencies are studied. At a beam splitter, two entangled photons are perfectly anticorrelated. In addition, they cannot be detected at the same time instant despite the fact that their detection times are confined to a narrow time window, i.e., they are temporally antibunched. Using nonlinear photonic-band-gap structures made of GaN/AlN, two schemes for generating such states are described.

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I. INTRODUCTION

In the process of spontaneous parametric down-conversion, photons are generated in pairs into a signal and an idler fields. Because of uncertainty in the generated frequency and/or polarization and/or wave vector of the down-converted fields and because of energy and momentum conservation, a state describing a photon pair is entangled. This means that two photons constituting a photon pair cannot be described separately and leads to unusual behavior of photon pairs.

Entanglement in frequencies manifests itself in the fact that both photons appear in time very close to each other. The width of a time window characterizing the appearance of two entangled photons is called an entanglement time and can be measured in a Hong-Ou-Mandel interferometer [1,2]. A Hong-Ou-Mandel interferometer is based on the fact that if two photons with the same time profile (they need not be entangled; see, e.g. [3,4] for interference of two independent photons) reach two ports of a beam splitter at the same time instant, they both exit the beam splitter from the same output port. This occurs because the paths leaving one photon in one output port and the other photon in the other output port cancel due to destructive interference. We note that in fact it is not the overlap of two photons at a beam splitter, but their overlap in the area of detectors measuring a coincidence count that is required to observe this behavior [5]. However, in most of the experimental setups, an overlap at a beam splitter insures also overlap at the detectors. This correlation (coalescence) is unusual in classical physics in which two statistically independent photons choose their output ports randomly and so they may be observed in different output ports with a probability of 50%. In a Hong-Ou-Mandel interferometer, a mutual time delay is introduced between two photons that decreases the mutual overlap of the photons. Subsequently, an entanglement time can be determined from the width of a typical dip in the fourth-order coincidence-count interference rate. In order to observe 100 percent visibility of the interference pattern, two photons have to be identical (or perfectly indistinguishable). For this reason, a lot of attention has been paid to develop methods for the generation of photon pairs containing identical signal and idler photons [6,7]. Considering usual entangled two-photon states (i.e., states more or less symmetric in their independent variables), it may be deduced from the shape of the fourth-order coincidence-count interference pattern that the probability of detecting an idler photon at a given time instant is constant (in cw regime) in a certain time window provided the signal photon has been detected at a given time instant, or decreases (for femtosecond pumping) as the difference of two time instants increases.

The above-mentioned picture changes when two photons are generated in a state antisymmetric in a variable describing this state. Provided that two photons are generated in an antisymmetric polarization Bell state $|\psi^{-}\rangle$ and enter a beam splitter in different input ports they exit the beam splitter in different output ports (see, e.g., in [6]), and we speak about anticorrelation (anticoalescence) of two photons. This property is exploited in many teleportation schemes [8].

If two photons are generated in a state antisymmetric in frequencies, they are anticorrelated at a beam splitter. They also show temporal antibunching, i.e., if a signal photon has been detected at a given time instant, then the probability of detecting an idler photon at a given time instant increases with the increasing difference of two time instants for small values of the difference (see Fig. 3 later). These states are investigated in detail in this paper.

States antisymmetric in frequencies represent a temporal (spectral) analogy of the states that show spatial antibunching as a consequence of antisymmetry of their two-photon amplitude with respect to the exchange of the signal- and

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idler-field wave vectors. Several methods for the generation of spatially antibunched entangled states have been proposed and experimentally verified [9–12].

We note that entangled two-photon states generated in spontaneous parametric down-conversion have been essential for numerous important experiments; we mention testing of Bell and other nonclassical inequalities [13–15], quantum teleportation [16], quantum cloning [17], and generation of Greenberger-Horne-Zeilinger states [18] to name a few. Also, important applications of entangled two-photon states have been developed [19,20].

II. ENTANGLED TWO-PHOTON STATES ANTISYMMETRIC IN FREQUENCIES

In an entangled two-photon state antisymmetric in frequencies signal- and idler-field spectra may be considered as composed of two peaks of equal heights, and ideally no field is generated at the central frequencies of the signal and idler fields. In other words, their two-photon amplitude is antisymmetric with respect to the signal-field frequency and idler-field frequency exchange. We note that methods for controlling correlations between the signal- and idler-field frequencies have been suggested [21,22].

Nonlinear photonic-band-gap structures [23–25] represent a suitable source of entangled two-photon states antisymmetric in frequencies. The wave function $|\psi\rangle_{s,i}^{(2)}$ of a state generated in a photonic-band-gap structure can be naturally decomposed into four contributions that take into account the propagation directions of the generated photons ([26], and details may be found in [27]):

$$|\psi(t)\rangle_{s,i}^{(2)} = |\psi_{s,i}^{FF}(t)\rangle + |\psi_{s,i}^{FB}(t)\rangle + |\psi_{s,i}^{BF}(t)\rangle + |\psi_{s,i}^{BB}(t)\rangle, \quad (1)$$

where the superscripts F and B distinguish forward and backward propagation of down-converted photons with respect to an incident pump beam. Introducing the probability amplitudes $\phi^{mn}(\omega_s,\omega_i)$ of generating a signal photon at frequency ω_s and an idler photon at frequency ω_i, the contributions $|\psi_{s,i}^{mn}\rangle$ can be written as follows:

$$|\psi_{s,i}^{mn}(t)\rangle = \int_0^{\infty} d\omega_s \int_0^{\infty} d\omega_i \phi^{mn}(\omega_s,\omega_i)\hat{a}_{s_m}^{\dagger}(\omega_s)\hat{a}_{i_n}^{\dagger}(\omega_i) \times \exp(i\omega_s t)\exp(i\omega_i t)|\text{vac}\rangle,$$

$$m,n = F,B. \quad (2)$$

The symbols $\hat{a}_{s_m}^{\dagger}$ and $\hat{a}_{i_n}^{\dagger}$ stand for creation operators of a photon into a given mode of the quantized optical field with vacuum state $|\text{vac}\rangle$.

For simplicity, we consider cw pumping with central frequency ω_p^0. The probability amplitude ϕ^{mn} of our entangled photon pair can be written in the general form that follows (see also [6]):

$$\phi^{mn}(\omega_s,\omega_i) = \delta(\omega_p^0 - \omega_s - \omega_i)[f(\omega_s - \omega_s^0) - f(-\omega_s + \omega_s^0)], \quad (3)$$

with the complex function f determined by the geometry. This form of ϕ^{mn} ensures that the signal- and idler-field spectra are composed of two symmetrically positioned peaks having the same height.

An entangled photon pair in the time domain is conveniently described using a two-photon amplitude $\mathcal{A}^{mn}$ [28–30] defined as

$$\mathcal{A}^{mn}(\tau_s,\tau_i) = \langle \text{vac}|\hat{E}_{s_m}^{(+)}(t_0+\tau_s)\hat{E}_{i_n}^{(+)}(t_0+\tau_i)|\psi_{s,i}^{mn}(t_0)\rangle, \quad (4)$$

where the symbols $\hat{E}_{s_m}^{(+)}$ and $\hat{E}_{i_n}^{(+)}$ denote positive-frequency components of electric-field amplitude operators for signal and idler fields. Considering the state described by the probability amplitude ϕ^{mn} in Eq. (3), we arrive at

$$\mathcal{A}^{mn}(\tau_s,\tau_i) = \frac{\hbar\sqrt{\omega_s^0\omega_i^0}}{4\sqrt{2\pi}^3\epsilon_0 c\mathcal{B}} \exp(-i\omega_s^0\tau_s)\exp(-i\omega_i^0\tau_i) \times[\tilde{f}(\tau_s-\tau_i) - \tilde{f}(\tau_i-\tau_s)], \quad (5)$$

where $\hbar$ is the reduced Planck constant, ϵ_0 is the permittivity of vacuum, c is the speed of light, and $\mathcal{B}$ gives the area of the transverse profiles of the interacting beams. The symbol $\tilde{f}$ denotes the Fourier transform of the function f and ω_s^0 (ω_i^0) means a central frequency of the signal (idler) field. Provided that $\tau_s = \tau_i$ in Eq. (5), the two-photon amplitude $\mathcal{A}^{mn}$ is zero. Because $|\mathcal{A}^{mn}(\tau_s,\tau_i)|^2$ gives the probability of simultaneous detection of the signal photon at time τ_s and the idler photon at time τ_i, the signal photon and its idler twin cannot be simultaneously detected at the same time instant.

Anticorrelation of entangled photons at a beam splitter may be deduced from the behavior of the normalized coincidence-count rate R_n^{mn} [27] in a Hong-Ou-Mandel interferometer. We derive the following expression for the normalized coincidence-count rate R_{n}^{mn} assuming a state with the probability amplitude ϕ^{mn} given in Eq. (3):

$$R_{\text{n}}^{mn}(\tau_l) = 1 + \frac{\text{Re}[\exp[-i(\omega_s^0 - \omega_i^0)\tau_l]g(\tau_l)]}{g(0)}, \quad (6)$$

where

$$g(\tau) = \int_{-\infty}^{\infty} d\omega |f(\omega) - f(-\omega)|^2 \exp(-2i\omega\tau) \quad (7)$$

and Re represents the real part of an expression. If the mutual time delay τ_l between two photons in a pair is zero, i.e., both photons perfectly overlap at the beam splitter, $R_n = 2$ according to Eq. (6). This means that both entangled photons must leave the beam splitter from different output ports in order to have twice the coincidence counts per second compared to the case of two independent photons ($\tau_l \to \infty$).

III. TWO SCHEMES FOR THE GENERATION OF AN ENTANGLED TWO-PHOTON STATE ANTISYMMETRIC IN FREQUENCIES IN GaN/AlN STRUCTURES

Photonic-band-gap structures made of GaN/AlN (for characteristics, see, e.g. [31]) offer two different schemes for generating such states. The first scheme relies on the vectorial character of spontaneous parametric down-conversion and uses destructive interference between two paths that dif-

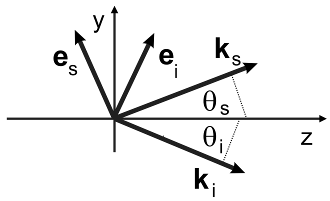

FIG. 1. Vectorial scheme of a generated photon pair. A signal (idler) photon with wave vector $\mathbf{k}_s$ ($\mathbf{k}_i$) propagates with p polarization (in yz plane) along the angle θ_s (θ_i) and is linearly polarized along vector $\mathbf{e}_s$ ($\mathbf{e}_i$).

fer in polarization properties. The second scheme is based on the observation [27] that an efficient photon-pair generation occurs at frequencies that lie at resonance peaks of the linear transmission spectrum of the structure. This suggests a design of a photonic-band-gap structure such that photon pairs are efficiently generated at two neighboring transmission peaks.

As an example, we consider a structure composed of 25 nonlinear layers of GaN (thickness 110 nm) among which there are 24 linear layers of AlN (thickness 60 nm). The boundaries are perpendicular to the z crystallographic axis of GaN that coincides with the z axis of the coordinate system (see Fig. 1). The structure is designed for pumping at the wavelength of 395 nm at normal incidence.

The first scheme based on destructive interference between two quantum paths occurs in this structure when the pump, signal, and idler fields are p polarized (polarization vectors lie in the yz plane; see Fig. 1) for photon pairs composed of copropagating photons (i.e., both photons are forward-propagating or backward-propagating). A two-peak character of frequency dependence of the photon-pair generation rate η_S^{FF} for forward-propagating photons is shown in Fig. 2.

Here, no photon pair with a signal photon at the central frequency $\omega_p^0/2$ can be generated due to completely destructive interference. GaN has two nonzero elements of nonlinear susceptibility $\chi^{(2)}$, $\chi^{(2)}_{y,y,z}$ and $\chi^{(2)}_{y,z,y}$, that are responsible for the generation of photon pairs in this configuration. Considering, e.g., the element $\chi^{(2)}_{y,y,z}$, one photon is generated with polarization along the y axis, while its twin has polarization along the z axis (see Fig. 1). The state of a photon pair with a signal photon having wave vector $\mathbf{k}_s$ and an idler photon with wave vector $\mathbf{k}_i$ is created by interference of two paths: Either a photon with polarization along the y axis becomes a signal photon and then a photon with polarization along the z axis has to be an idler photon or vice versa. The probability amplitudes of these two paths have a different sign due to the vectorial character of nonlinear interaction (see Fig. 1; the z components of the polarization vectors of the signal and idler photons differ in sign) and as a result no photon pair can be generated at the degenerate frequencies of the down-converted fields (for $\omega_s=\omega_i=\omega_p^0/2$) due to symmetry. Photon pairs are then generated symmetrically around the degenerate central frequencies (see Fig. 2).

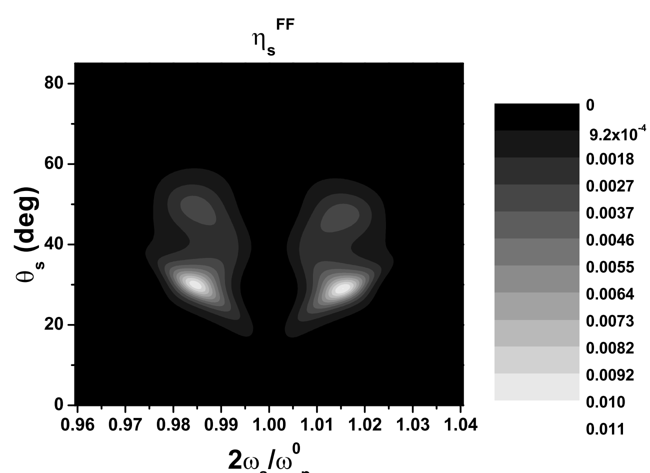

FIG. 2. Photon-pair generation rate η_s^{FF} as a function of normalized signal-field frequency $2\omega_s/\omega_p^0$ and angle θ_s of signal-photon emission for forward-propagating signal and idler photons; cw pumping and p polarization of all fields are assumed.

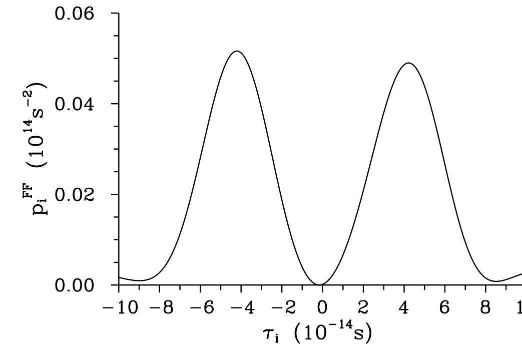

FIG. 3. Probability p_i^{FF} of detecting an idler photon at time τ_i provided that its signal photon is detected at time $\tau_s=0$ s is shown for forward-propagating down-converted photons with a signal photon emitted along the angle $\theta_s=30°$; cw pumping and p polarization of all fields are assumed.

The two photons in this entangled state cannot be detected at the same time instant despite the fact that their possible detection times are confined within a sharp time window (of typical duration in hundreds of fs), as documented in Fig. 3, where the probability $p_i^{FF}(\tau_i)$ of detecting an idler photon at time τ_i is plotted, provided that the signal photon is detected at a given time τ_s.

Perfect anticorrelation of two entangled photons in this state at a beam splitter is evident from the normalized coincidence-count rate R_n in a Hong-Ou-Mandel interferometer shown in Fig. 4, where for $\tau_l=0$ s the coincidence-count rate R_n for both photons propagating forward is two times greater in comparison with that for two uncorrelated photons ($\tau_l\to\infty$). Oscillations in the coincidence-count rate R_n around the main peak in Fig. 4 indicate the difference of the central frequencies of two peaks.

Generation of these states occurs due to the fact that the nonlinear GaN has a wurtzite structure. We note that materials with a cubic symmetry cannot provide these states.

The generation of an antisymmetric entangled photon pair along *the second scheme* occurs in the considered structure in the configuration with s-polarized idler and pump fields (their polarization vectors are orthogonal to the yz plane) and

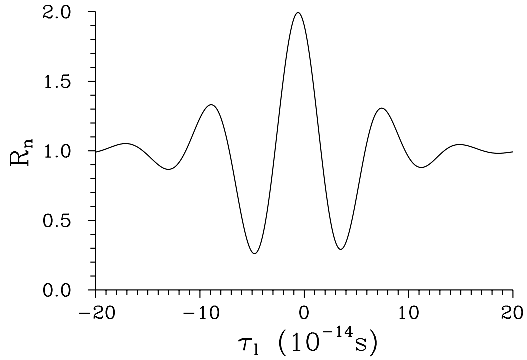

FIG. 4. Normalized coincidence-count rate R_n as a function of relative time delay τ_l in a Hong-Ou-Mandel interferometer for forward-propagating signal and idler photons for the angle of signal-field emission $\theta_s=30°$. All fields have p polarization and cw pumping is assumed.

a p-polarized signal field. Pumping is at normal incidence. Photon pairs are now generated owing to a nonzero value of the element $\chi^{(2)}_{x,z,x}$. A two-peak structure of the photon-pair generation rate η_s^{FF} with maxima at signal-field emission angles θ_s equal to 30° and 50° (see Fig. 5) has its origin in the properties of the photonic-band-gap structure that allows an efficient generation for frequencies lying in the areas around the transmission peaks of the linear transmission spectrum. However, different generation rates for frequencies around different peaks exist. This is caused by the fact that waves with frequencies around different transmission peaks have a different degree of localization (density of modes) inside the structure, and this affects the efficiency of the nonlinear process. As a consequence, a photonic-band-gap structure cannot provide the entangled state in its perfect form. For instance, the number of coincidences for $\tau_l=0$ s in a Hong-Ou-Mandel interferometer exceeds that for $\tau_l\to\infty$ by ~60% for the considered structure and $\theta_s=50°$.

Considering s-polarized pump field at normal incidence and detecting a signal photon as well as an idler photon in

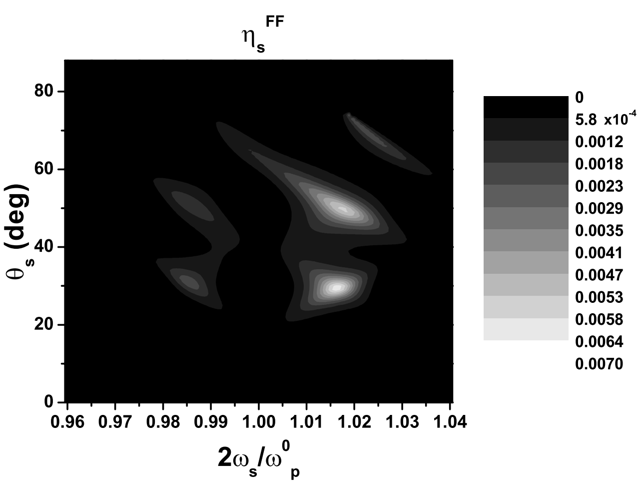

FIG. 5. Photon-pair generation rate η_s^{FF} as a function of normalized signal-field frequency $2\omega_s/\omega_p^0$ and angle θ_s of signal-photon emission for forward-propagating signal and idler photons; cw pumping, p-polarized signal field, and s-polarized idler and pump fields are assumed.

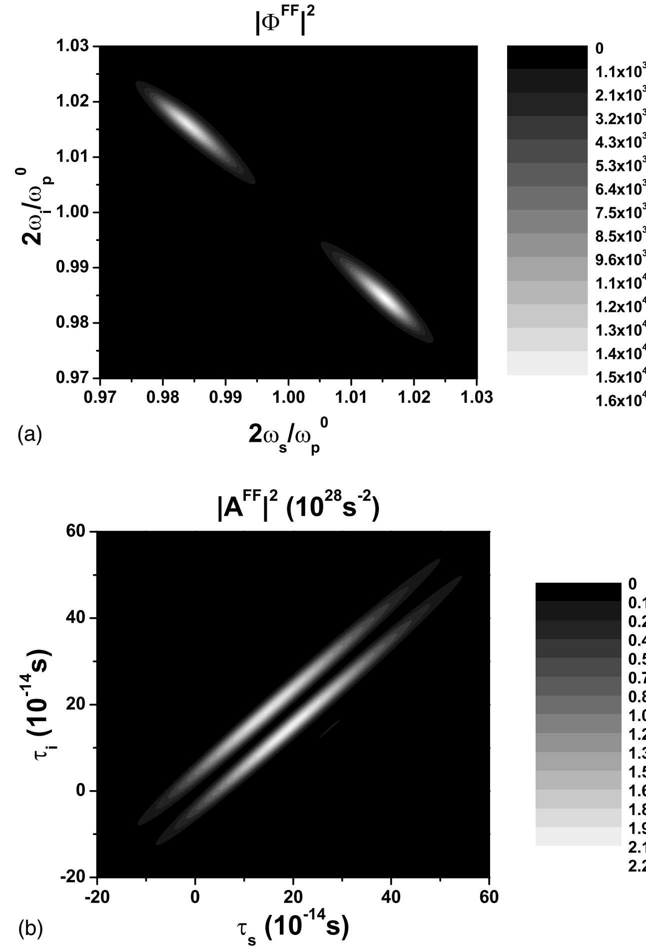

FIG. 6. (a) Probability $|\Phi^{FF}|^2$ of generating a two-photon state with a signal photon at normalized frequency $2\omega_s/\omega_p^0$ and an idler photon at normalized frequency $2\omega_i/\omega_p^0$ and (b) probability $|\mathcal{A}^{FF}|^2$ of detecting a signal photon at time τ_s and an idler photon at time τ_i for forward-propagating down-converted photons and the angle of signal-field emission $\theta_s=30°$. All fields are p polarized and pumping by a Gaussian pulse with 200-fs duration is assumed. Normalization of the probabilities is such that one photon pair is emitted within the plotted ranges.

polarization directions rotated by 45° (with respect to p-polarization direction) for $\theta_s=30°$ and 50°, we have again the antisymmetric entangled state generated along the first scheme, now due to destructive interference of two contributions originating in elements $\chi^{(2)}_{x,z,x}$ and $\chi^{(2)}_{x,x,z}$. This means that the effect of anticorrelation at a beam splitter and temporal antibunching is perfect. We note that the considered structure with s-polarized pumping at normal incidence represents a source of photon pairs entangled in polarization, i.e., either a signal photon is s polarized and its twin p polarized, or vice versa.

Pumping the structure *with an ultrashort pulse* causes the above described properties of the generated entangled photons not only to survive, but also to be enhanced. For example, in Fig. 6(a) we depict the probability $|\phi^{FF}|^2$ of generating a signal photon at the normalized frequency $2\omega_s/\omega_p^0$ together with its twin at the normalized frequency $2\omega_i/\omega_p^0$ for a Gaussian pump pulse with duration 200 fs. Splitting of the generated photon pair (using the first scheme) into two separated spectral regions is clearly visible. On the other hand, the squared modulus of the two-photon amplitude $\mathcal{A}^{FF}$ in Fig. 6(b) shows that temporal antibunching of entangled photons is even more pronounced in the pulsed regime. We note that the down-converted fields now occur in the form of pulses with a typical duration in the hundreds of fs.

IV. CONCLUSION

In summary, we have shown that entangled two-photon states antisymmetric with respect to the exchange of frequencies of the signal and idler fields can be generated in nonlinear photonic-band-gap structures. The photons comprising a pair exhibit anticorrelation at a beam splitter. Despite the fact that both photons exist within a narrow time window, they cannot be detected at the same time instant. Photonic-band-gap structures offer two different schemes for their generation: One exploits vectorial character of the nonlinearly interacting fields together with destructive interference between two quantum paths and the other is based upon the generation of two adjacent transmission peaks. These properties might be useful in future quantum-information protocols [32]. Splitting of an entangled two-photon state into two perfectly separated spectral regions even for femtosecond pumping is especially promising for further consideration. Experimental realization of these states using GaN/AlN structures is in progress.

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