Controllable coupling of superconducting transmission-line resonators

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Realization of controllable interaction between subsystems is one of the major problems in solid-state quantum computing. We study a current-biased Josephson junction (CBJJ) as a tunable coupler for superconducting transmission-line resonators (TLR). By modulating the bias current, the junction can be tuned in and out of resonance with the TLRs connected to it. Various inter-TLR quantum operations can be reliably implemented by controlling the mediating CBJJ. The main decoherence sources are analyzed in detail. This work may offer improvement to scalable quantum computing in coupled TLR-cavity array system.

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During the past few years there has been tremendous progress in realizing quantum electrodynamics (QED) in superconducting devices. It has been suggested that the interaction of a Cooper pair box (CPB) [1–3] and a transmission line resonator (TLR) could be described by the Jaynes-Cummings (JC) model [4], where the CPB and TLR play the roles of atom and cavity. Compared with conventional optical cavity, this implementation goes beyond Lamb-Dick limit due to the fixed location of the artificial "atom" inside the TLR. The strong coupling limit of this circuit QED architecture has already been observed [5–7]. Coherent dynamics of a flux qubit coupled to a LC circuit has also been demonstrated [8,9]. These systems are attracting increasing interest because of their potential use in testing quantum theory at macroscopic level and realizing quantum computation.

Motivated by these exciting advances, we study further steps towards realizing the potential of quantum information and quantum computing in these systems. One of the critical next steps is to implement quantum gates between distant subsystems. In optics, to engineer entanglement between atoms trapped in distant cavities, several schemes of connecting cavities via optical fiber have been proposed [10,11]. Very recently, there are also studies of coupled macroscopic quantum resonators, including coupling a nanomechanical resonator to a TLR by a dc SQUID and coherent single photon transfer in TLRs connected by capacitors [12,13].

Similar to coupling Josephson charge qubits [14,15], the most straightforward way of coupling TLRs is to connect two TLRs via a capacitor. The Coulomb interaction between charge on the two coplanar waveguides thus induce energy transfer between TLRs, i.e., photons can leak from one TLR to the other. However, since the intercavity coupling strength is much smaller than the frequency of a single TLR, the direct capacitive coupling can only be used to couple TLRs with the same frequency. Moreover, the coupling strength is determined by fabrication and cannot be switched on and off. These disadvantages of the capacitive coupling make it nonideal, especially from the view of scalable quantum computing.

In this paper, we propose an alternative controllable interconnection scheme between TLRs. Our idea is inspired by previous tunable coupling schemes for Josephson qubits [16,17]. In our scheme, two TLRs are capacitively coupled to a current biased Josephson junction (CBJJ) [18,19] which plays the role of a data bus. The CBJJ could be described as a tunable two-level atom and the TLR-CBJJ-TLR interaction can be switched on and off by tuning the CBJJ's level splitting resonant and off-resonant with the TLRs. This system is flexible enough to allow for various intercavity quantum operations such as quantum state transfer and entanglement generation. Since long lifetimes for both the TLRs and the CBJJ have already been achieved, these proposed quantum gates could be realized with very high fidelity in this system.

Let us illustrate our idea intuitively. The system we study is two TLRs with identical length L coupled to a CBJJ by coupling capacitors C_c and to external input and/or output leads by wiring capacitors C_0 from left and right, as shown in Fig. 1. For simplicity we assume that the leads and TLRs have identical inductance F and capacitance c per unit length. Since in reality the wiring and coupling capacitances are very small compared with Lc, in the following we treat the TLRs as independent systems with small couplings as perturbation [20].

The individual TLRs. A single TLR is well described by an infinite series of inductors with each node capacitively connected to ground, as shown in Fig. 2. To transmit input and output signals the TLR should be coupled to the external

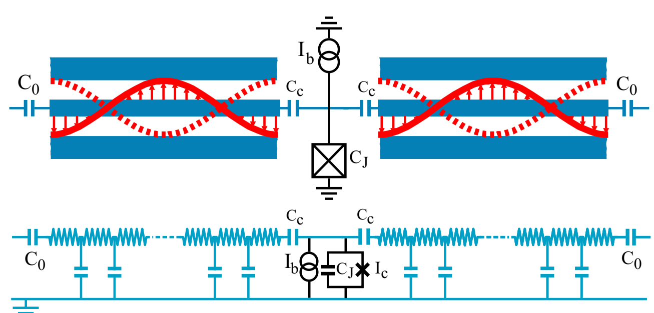

FIG. 1. (Color online) Coupled system of a CBJJ and two TLRs. Two TLRs are connected to a CBJJ from left and right by coupling capacitors C_c. The CBJJ acts as a tunable coupler between the two TLRs.

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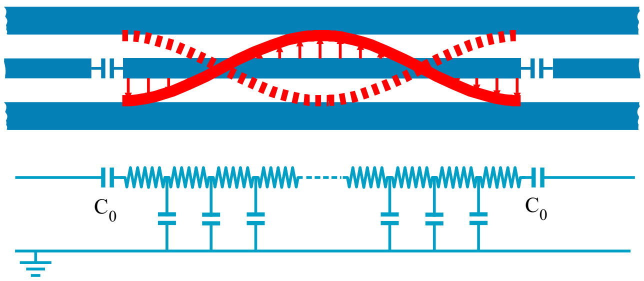


FIG. 2. (Color online) Schematic diagram and equivalent lumped circuit representation of a single TLR cavity. The TLR consists of a full-wave section of superconducting coplanar waveguide. The red line represents the full wave cavity mode. The coupling capacitors connected to the input and output wiring slightly modify the frequency and phase of the TLR's modes.

subsystem by capacitors. This capacitive coupling mechanism has been studied extensively in Ref. [20]. Denoting $\epsilon_1=C_0/Lc$, $\epsilon_2=C_c/Lc$ and focusing only on the full wave mode, we get the Hamiltonian of the two individual TLRs,

$$H_{\text{cavity}} = \hbar\,\omega(a^\dagger a + b^\dagger b), \tag{1}$$

where a and b are the annihilation operators of full wave modes in the left and right TLRs, respectively. The frequency of these modes is renormalized by the wiring and coupling capacitors as $\omega\approx\omega_0(1-\epsilon_1-\epsilon_2)$ where $\omega_0=2\pi/(L\sqrt{Fc})$. The voltage distributions in the left and the right TLRs are

$$V_L(x) = \sqrt{\hbar\,\omega/Lc}(a^\dagger + a)\cos(kx+\delta), \tag{2a}$$

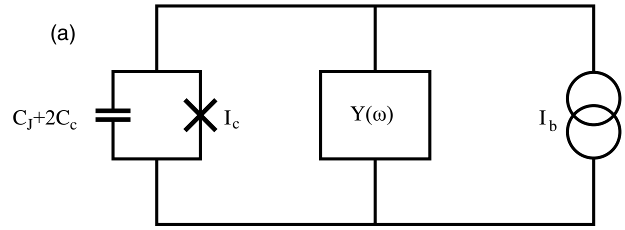
$$V_R(x) = \sqrt{\hbar\,\omega/Lc}(b^\dagger + b)\cos(kx+\delta), \tag{2b}$$

where the small phase shift δ satisfies the condition $\tan\delta = 2\pi\epsilon_2$.

The CBJJ. The equivalent circuit representation of a large Josephson junction biased by external current is sketched in Fig. 3(a). From this capacitance-shunted-junction (CSJ) diagram [21] the CBJJ can be modelled by a fictitious particle of mass $m^*=(\hbar/2e)^2(C_J+2C_c)$ moving in a tilted washboard potential $U(\varphi)=-\hbar I_b\varphi/2e-E_J\cos\varphi$ where C_J is the junction capacitance; $E_J=(\hbar/2e)I_c$ is the magnitude of maximum Josephson coupling energy; I_c is the critical current of the junction; I_b is the bias current, and φ is the gauge invariant phase difference of the superconducting order parameter across the junction. The Hamiltonian of a single CBJJ has the form

$$H_{\text{CBJJ}} = \frac{P^2}{2m^*} - \hbar\,I_b\varphi/2e - E_J\cos\varphi, \tag{3}$$

where $P=m^* d\varphi/dt$ is the canonical conjugate of the operator φ. When I_b is close to the critical bias I_c, there exist only a few levels in each washboard well. The bound states $|n\rangle_{\text{CBJJ}}$ with energy E_n can be observed spectroscopically by resonantly inducing transitions with microwaves at frequencies $\omega_{mn}=(E_m-E_n)/\hbar$. We consider a large junction with a bias such that there are only three such levels, as shown in Fig. 3(b). The reduced Hamiltonian in the three-level quantum system is



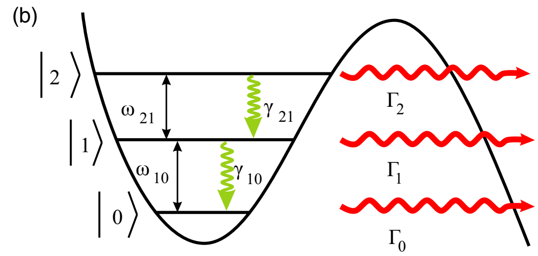


FIG. 3. (Color online) (a) Electronic model of a current-biased Josephson junction. The Josephson junction can be described by a tunnel junction shunted by its plate capacitor, while the current bias is modelled by an ideal current source in parallel with a finite admittance $Y(\omega)$. (b) Diagram of the tilted washboard potential. From the capacitance-shunted-junction model the dynamic behavior of the CBJJ is modelled by a particle in a washboardlike potential. By modulating the bias current we can construct a three-level quantum system in a metastable well. Decoherence processes in this system include quantum tunneling of each bound state to the continuum and the spontaneous emission caused by the bias fluctuation.

$$H^{\text{eff}}_{\text{CBJJ}} = \hbar\,\omega_{10}|1\rangle_{\text{CBJJ}}\langle 1| + \hbar\,(\omega_{10}+\omega_{21})|2\rangle_{\text{CBJJ}}\langle 2|, \tag{4}$$

where the energy of $|0\rangle_{\text{CBJJ}}$ is chosen as energy zero point. Near the metastable minimum, $U(\varphi)$ can be well approximated by a cubic potential, therefore the junction acts as a tunable anharmonic oscillator, whose level structure could be modulated by the external bias I_b. Anharmonicity is crucial here as it guarantees the nonuniform level spacing in the potential well. For typical experimental parameters [18,19], the lowest resonant transition frequency ω_{10} is on the order of 10 GHz and the separation of the two lowest resonant frequencies is $\Delta=|\omega_{21}-\omega_{10}|\approx 0.1\omega_{10}$.

The combined system. The interaction between the TLRs and the CBJJ can be written as

$$H_{\text{int}} = C_c\sqrt{\frac{\hbar\,\omega}{Lc}}\frac{\hbar}{2em^*}P[(a^\dagger + a) + (b^\dagger + b)]\cos\delta. \tag{5}$$

This term originates from the capacitive coupling $C_c[V_L(0^-)+V_R(0^-)]V$ where V is the voltage across the CBJJ. If ω_{10} is near resonant with ω, the TLRs' full wave modes are effectively coupled to the CBJJ's lowest two states $\{|0\rangle_{\text{CBJJ}},|1\rangle_{\text{CBJJ}}\}$. The higher state $|2\rangle_{\text{CBJJ}}$ can be neglected due to the very large detuning $|\omega_{21}-\omega_{10}|$ [22,23,16]. Therefore the CBJJ effectively acts as a two-level atom. Expanding the momentum operator P in the two-level subspace $\{|0\rangle_{\text{CBJJ}},|1\rangle_{\text{CBJJ}}\}$ and using rotating-wave approximation (RWA), we get the two-mode JC Hamiltonian describing TLR-CBJJ-TLR coupling

$$H_{\text{int}}^{\text{eff}} = \hbar g[(a+b)\Sigma^+ + (a^\dagger + b^\dagger)\Sigma^-], \quad (6)$$

where $g=[2Lc(C_J+2C_c)]^{-1/2}\omega C_c \cos\delta$ is the coupling factor and $\Sigma^+=|1\rangle\langle 0|$, $\Sigma^-=|0\rangle\langle 1|$ are the raising and lowering operators of the CBJJ, respectively.

The coupling factor g can be estimated from experimental data [18,19,24]. We set the length L of the TLRs to 12 mm, which leads to a full wave frequency $\omega/2\pi=12$ GHz and the capacitance of one TLR to the ground section $Lc=1.6$ pF. The parameters of the CBJJ are $C_J=5.8$ pF, $I_c=140$ μA, and $I_b\approx 0.99I_c$. This results in a three-level quantum system in the metastable potential well with transition frequency $\omega_{10}=12$ GHz between two lowest states. We further choose the coupling capacitance $C_c=6$ fF. With the above parameters we can arrive at $g/2\pi=17$ MHz, which is on the same order of the observed qubit-photon vacuum Rabi splitting [6]. We can go back to test the two-level approximation we have made in deriving the interaction Hamiltonian Eq. (6). The large separation $\Delta=|\omega_{21}-\omega_{10}|$ suppresses the leakage probability of quantum state population from $\{|0\rangle_{\text{CBJJ}},|1\rangle_{\text{CBJJ}}\}$ to $|2\rangle_{\text{CBJJ}}$ to $P\sim O[g^2/(g^2+\Delta^2)]$. With previous chosen parameters we have $P\leq 10^{-5}$, i.e., the two-level approximation is reasonable.

By modulating the bias current I_b, the TLR-CBJJ coupling can be effectively tuned off. When we switch I_b to $0.985I_c$, there are more than 14 energy levels in the potential well and the TLR-CBJJ detuning $|\omega-\omega_{10}|$ becomes as large as 6 GHz. Without loss of generality we focus on the subspace M_1 spanned by states with a single total excitation. On basis $\{|1\rangle_L|0\rangle_{\text{CBJJ}}|0\rangle_R,\ |0\rangle_L|1\rangle_{\text{CBJJ}}|0\rangle_R,\ |0\rangle_{\text{L}}|0\rangle_{\text{CBJJ}}|1\rangle_R\}$ the eigenvectors of Hamiltonian $H=H_{\text{cavity}}+H_{\text{CBJJ}}^{\text{eff}}+H_{\text{int}}^{\text{eff}}$ are $\{(0.003,0.999,0.003),(0.707,-0.003,0.707),(0.707,0,-0.707)\}$, from which we can see the CBJJ is only weakly entangled with the two TLRs, i.e., the CBJJ and the TLRs are effectively separable.

Several important intercavity quantum operations could be realized in this coupled system. First we show how to generate the inter-TLR maximal entangled state. Initially we prepare the combined system to its ground state $|0\rangle_L|0\rangle_{\text{CBJJ}}|0\rangle_R$ and tune the CBJJ far off-resonant with the TLRs. We first pump the CBJJ to the excited state $|1\rangle_{\text{CBJJ}}$ by classical microwave and then adiabatically tune the CBJJ into resonance with the TLRs. Due to the TLR-CBJJ-TLR coupling the energy of CBJJ begins to diffuse into the left and right TLRs symmetrically. The reduced interaction Hamiltonian in M_1 reads

$$H_{\text{int}}^{(1)} = \hbar g\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (7)$$

After a time $\tau_1=\pi/(2\sqrt{2}g)$, the initial state $(0,1,0)$ evolves to $(1,0,1)/\sqrt{2}$, in which the CBJJ is separable with the maximal entangled TLRs in state $(|0\rangle_L|1\rangle_R+|1\rangle_L|0\rangle_R)/\sqrt{2}$. The required adiabatical tuning of ω_{10} is feasible since there is a large gap between the coupling strength g (of the order 10 MHz) and the level space ω_{10} (of the order 10 GHz), which allow us to change the washboard potential at a speed much slower than ω_{10} but much faster than g.

Now we discuss the decoherence process resulting from environmental and systematic errors, which is the main obstacle of implementing quantum computing in solid state system. The dissipation of a single TLR occurs mainly through coupling to the external leads. In general the magnitude of this process can be described by the decay factor $\kappa=\omega/Q$, where Q is the quality factor of the TLRs [20]. In the reported high-finesse TLR cavity with $Q=5\times 10^5$, the κ factor is suppressed to an order of 20 kHz [24]. For the CBJJ, as shown in Fig. 3(b), each state $|n\rangle_{\text{CBJJ}}$ in the potential well has a quantum tunneling rate Γ_n to the outside, which are of the magnitude $\Gamma_0\approx 100$ Hz and $\Gamma_n/\Gamma_{n-1}\sim 1000$ [18]. In addition, the finite admittance of the current bias causes the fluctuation of I_b, which induces dissipation and dephasing to the CBJJ, as shown in Fig. 3(a). The admittance could be modelled as a bosonic bath [25,26] and treated based on the Bloch-Redfield formalism [27–30]. In the Bloch-Redfield formalism, the influence of this bosonic bath on the dynamical evolution of a CBJJ could be represented by the decay factor γ_{10} and dephasing factor γ_φ [19], which are related to the spectral density function of the bath (i.e., the admittance of the current bias) and can be revised by design techniques of external circuit [18]. The spontaneous emission rate from $|1\rangle_{\text{CBJJ}}$ to $|0\rangle_{\text{CBJJ}}$ is $\gamma_{10}\approx \text{Re}[Y(\omega_{10})]/C_J$. Long coherence time requires an environment with sufficiently high impedance, engineered to be $1/\text{Re}[Y(\omega)]=560$ kΩ in Ref. [18] instead of the standard 100 Ω at microwave frequencies. With this $Y(\omega)$ and previous parameters for the CBJJ, $T_{1,2}$ of the CBJJ are on the order of 6 μs. We can see the coupling strength of the bosonic bath to the CBJJ is much weaker compared with that of the two TLRs, since the decoherence rate of the CBJJ is of the order 0.2 MHz, while the TLR-CBJJ-TLR coupling strength is of the order 10 MHz. Therefore, in treating the dynamics of the TLR-CBJJ-TLR coupling, we believe it is reliable to incorporate the bosonic bath into the master equation by using exponential decay factor γ_{10} and γ_φ of the single CBJJ.

Following the standard quantum theory of damping, we investigate the combined influence of all the above decoherence processes on the coupled system. After tracing out the reservoir degrees of freedom and performing the Markov approximation, we obtain the master equation for the reduced density matrix ρ of the three-party system

$$\frac{d\rho}{dt} = -i[H_{\text{int}}^{\text{eff}},\rho] + \kappa\left(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a\right) + \kappa\left(b\rho b^\dagger - \frac{1}{2}b^\dagger b\rho - \frac{1}{2}\rho b^\dagger b\right) + \frac{\gamma_\varphi}{2}(\Sigma_z\rho\Sigma_z - \rho) + \left(\frac{\gamma_{10}+\Gamma_1}{4}\right)\left(\Sigma^-\rho\Sigma^\dagger - \frac{1}{2}\Sigma^\dagger\Sigma^-\rho - \frac{1}{2}\rho\Sigma^\dagger\Sigma^-\right),$$

where γ_φ is the pure dephasing rate of the CBJJ. Choosing $\gamma_\varphi/2\pi=0.1$ MHz and $\Gamma_1/2\pi=0.1$ MHz, we calculate the error probability D of the entanglement generation versus γ_{10} and κ, as shown in Fig. 4. Results imply that in the region of already reported parameters $\gamma_{10}/2\pi=0.2$ MHz and $\kappa/2\pi=50$ kHz, D is suppressed to lower than 1%.

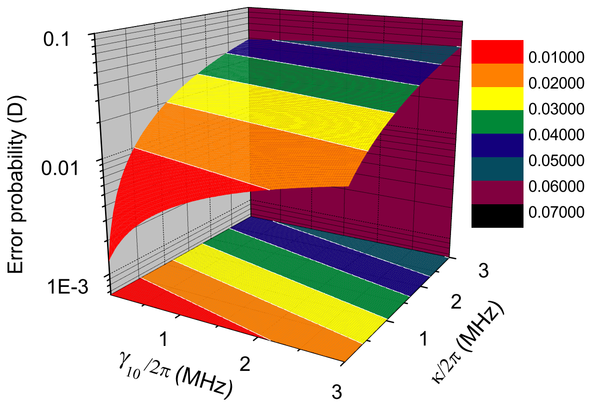


FIG. 4. (Color online) Dependence of the error probability D in intercavity entanglement generation on dissipation factor γ_{10} and κ. In this evaluation we assume the left and right TLRs have the same decay factor. The pure dephasing rate and the quantum tunnel rate of state $|1\rangle_{\mathrm{CBJJ}}$ are set to $\gamma_{\varphi}/2\pi=0.1$ MHz and $\Gamma_1/2\pi=0.1$ MHz.

We can also coherently transmit the single-photon state from one TLR cavity to the other using the CBJJ as a tunable databus. Suppose the initial TLRs' state is $(\alpha|1\rangle+\beta|0\rangle)_L|0\rangle_R$ and the CBJJ is in its ground state, we tune the CBJJ to resonance for time $\tau_2=\pi/(\sqrt{2}g)$, then we have the CBJJ separable with the TLRs and the TLRs evolve to $|0\rangle_L(\alpha|1\rangle+\beta|0\rangle)_R$. This process could also be performed with high fidelity since it is governed by the same interaction Hamiltonian Eq. (7) in entanglement generation.

A natural generalization is using the CBJJ to couple two TLRs with different eigenfrequencies. If the CBJJ is tuned in resonance with the left TLR it is effectively decoupled with the right one, so the CBJJ acts as a tunable data bus to transfer information between the two TLRs. We can also consider coupled TLRs array in which each TLR is connected to the previous and the next TLR by CBJJ. This is very similar to the problem of quantum state transfer on one-dimensional spin chain [31]. In this coupled cavity array structure we can switch on and off each interaction branch at will by tuning the corresponding CBJJ. Therefore many multiparty quantum gates can be implemented. If we further combine these intercavity coupling with the already achieved qubit-photon strong coupling, we can realize various quantum gates between qubits placed in different TLRs. This may make improvement to a variety of applications including quantum teleportation, quantum purification, and distributed quantum computing.

In conclusion, in this paper we propose a tunable coupling scheme between two TLR cavities in which a CBJJ is used as databus. We show that important quantum operations such as entanglement generation and coherent single photon state transfer is achievable in this system. All the parameters we use are already demonstrated in previous experiments. With present long lifetime of both the TLRs and the CBJJ we further show that performing quantum information processes in this setup is highly reliable.

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