Few-cycle nonlinear optics of multicomponent media

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Using Maxwell-Bloch equations, we analyze the response of a two-component medium of two-level atoms driven by a two-cycle optical pulse beyond the traditional approach of slowly varying amplitudes and phases. We show that the notions of carrier, envelope, phase, and group velocities can be generalized to this situation. For optical pulses of a given duration, we show that the optical field can form a temporal soliton.

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I. INTRODUCTION

The recent success in solid-state mode-locked lasers has resulted in the generation of two-cycle optical pulses [1,2]. These pulses have been immediately exploited to generate both unipolar single-cycle electromagnetic pulses and even shorter, attosecond pulses in extreme UV in a variety of nonlinear media [3,4] where their wide spectra and intense electric fields raised concerns of an adequate description of a few-cycle pulse (FCP) laser-matter interaction within the slowly varying envelope approximation (SVEA) operating with a quasimonochromatic field [5–12]. Considerable progress has been made in obtaining a description of both resonant [5,6,9,17] and nonresonant [7,8,11–23] spatiotemporal dynamics of the FCP beyond the SVEA.

In the Maxwell-Bloch formulation [8–11,16,17], the dynamic response of a medium is modeled by truncating the density-matrix equation either using a long- or short-wave approximation. This enables us to arrive at an integrable nonlinear evolution equation and build up a two-soliton solution that can be treated as a correct envelopeless representation of a single-cycle optical pulse [11,12,17]. More recent numerical solutions of its natural extension onto the $(2+1)$-D propagation [14,18] have exhibited noticeable departures from the Brabec-Krausz results [10], which were obtained within the SVEA. However all this research in the FCP phenomenology has been elaborated for a single-component medium that can be either a two-level resonant system or nonresonant nonlinear matrix alone.

In this paper, we consider a more general case of the two-component medium where we can derive a nonlinear evolution equation, and its respective two-cycle solution. In the integrable case of the FCP plane-wave propagation, an adequate interpretation of the breather solution allows the demonstration of the physically meaningful quantities of carrier frequency, envelope, and phase and group velocity, which emerge self-consistently outside the limitations of the SVEA. In the nonintegrable case, we show that the FCP dynamics are extremely sensitive to the relative strength of the two qualitatively different optical nonlinearities and third-order dispersion. Moreover, when analyzing the impact of the coherent absorption and cubic nonlinearity on a two-cycle optical pulse a remarkable feature is distinguished: a stabilization of carrier-envelope phase.

This article is organized as follows. In Sec. II, we recall the nonlinear evolution equation for the electric field and its derivation from the Maxwell-Bloch equations for the two-component medium. In Sec. III, the reinterpretation of the breather solution is presented whereas in Sec. IV we show numerically that the propagation of the two-cycle soliton and envelope-phase stabilization also occurs in the nonintegrable case.

II. TWO-CYCLE OPTICAL PULSE IN TWO-COMPONENT NONLINEAR MEDIUM—A MODEL

Citing the examples of ions embedded in a crystal host, multiple bands in semiconductors, and defects generated in a guiding structure, we note that optically nonlinear condensed matter often contains more than one polarizable component, even though only one may be of primary interest. Here we formulate the evolution beyond the SVEA approximation for optically nonlinear materials, which have more than one polarizable component. The equation governing the evolution of an optical FCP in a two-component medium was first derived in Ref. [21]. We recall briefly the derivation below.

We consider the time-dependent propagation of a two-dimensional femtosecond pulse through a two-component medium. The response of the medium upon interaction with the femtosecond electromagnetic field $E(x,z,t)$ is described by the total macroscopic polarization P obtained by summing over all components, $P=\mathrm{Re}\{\Sigma_{j=1}^{2}N_j d_j R_j\}$, d_j being the dipole transition matrix element and N_j the atomic density of the jth component. The time dependence of the off-diagonal density-matrix elements $\rho_{21}^{(j)}\equiv R_j$ is given by the Maxwell-Bloch equations,

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$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E=\frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2}, \tag{1}$$

$$\frac{\partial R_j}{\partial t}=-i\omega_j R_j-2d_j\hbar^{-1}EW_j, \tag{2}$$

$$\frac{\partial W_j}{\partial t}=2d_j\hbar^{-1}E\,\mathrm{Re}\{R_j\}, \tag{3}$$

where W_j is the inversion population $(-1/2\leqslant W_j\leqslant +1/2)$ and ω_j is the atomic transition frequency of the jth component. It is assumed that the pulse duration τ_p is much shorter than any of the decay times T_1, T_2 of either of the two-level components, which places an upper bound on τ_p. It is convenient to rewrite Eqs. (2) and (3) in terms of the quantity U_j such that $\mathrm{Re}\{R_j\}=\omega_j^{-1}\partial U_j/\partial t$.

We can see that by knowing a functional relationship between E, R_j, and W_j, by using Eqs. (2) and (3) we can find the dependence $P_j=P_j(E)$ and, by substituting this relation into the wave equation (1), we obtain a nonlinear equation in terms of E alone. We assume that the FCP duration τ_p is such that

$$\varepsilon\sim\omega_1\tau_p\ll 1 \tag{4}$$

for the first component, and

$$\varepsilon^{-1}\sim\omega_2\tau_p\gg 1 \tag{5}$$

for the second component, correspondingly. In terms of the underlying physics, these quite controversial requirements can be met when a two-cycle pulse of a Ti:sapphire laser at 780 nm traverses, for example, a Yb-doped $KGd(WO_4)_2$ crystal where condition (4) is satisfied for the dopant and condition (5) for the wolframate matrix. Notice that the two-component medium under assumptions (4) and (5) can be considered as a simplified model of a general transparent dielectric. Let us consider indeed the lossless propagation of a wave with frequency ω in some medium: the latter must be transparent at this frequency, and hence all transitions of the medium are assumed to be far off from ω. Consequently, some of the transition frequencies ω_j are much smaller than ω, and all others much larger. In the case of the FCP, the inverse of the pulse duration τ_p is assumed to have the same order of magnitude as the central frequency ω of the spectrum, and this statement coincides with assumptions (4) and (5). From this standpoint, the model we suggest here simply consists in reducing the lower and the upper absorption bands to two single transitions.

Then for the first component where condition (4) is valid, exploiting a short-wave approximation yields the following solution of the Bloch equations (2) and (3):

$$W_1(t)=W_1(-\infty)\cos\theta,\quad R_1=-W_1(-\infty)\sin\theta,$$

$$\theta=2d_1\hbar^{-1}\int_{-\infty}^{t}Edt', \tag{6}$$

where $W_j(-\infty)$ is the initial population difference [e.g., in the case of a medium in the ground state $W_j(-\infty)=-1/2$]. Correspondingly, the FCP interaction with the second component is described within a long-wave approximation so that the level population renders almost intact,

$$U_2(t)=-\frac{2d_2E}{\hbar\omega_2}W_2(t)+\frac{2d_2W_2(-\infty)}{\hbar\omega_2^3}\frac{\partial^2E}{\partial t^2},$$

$$W_2(t)=W_2(-\infty)\left[1-2\left(\frac{d_2E}{\hbar\omega_2}\right)^2\right]. \tag{7}$$

Making use of Eqs. (6) and (7) and eliminating the inversion W_j enables one to rewrite the wave equation (1) in terms of the pulse area θ. We then introduce a local time $\tau=t-zn_0/c$ and a "slow" propagation variable $\zeta=\varepsilon z$. In the second order of the small parameter ε, and returning back to the physical propagation variable z, we arrive at the nonlinear propagation equation,

$$\frac{\partial^2\theta}{\partial z\partial\tau}+c_1\sin\theta-c_2\frac{\partial}{\partial\tau}\left(\frac{\partial\theta}{\partial\tau}\right)^3-c_3\frac{\partial^4\theta}{\partial\tau^4}=0, \tag{8}$$

where $c_1=-8\pi d_1^2\omega_1N_1W_1(-\infty)/(n_0\hbar c)$, $c_2=d_2^2c_3/(2d_1^2)$, $c_3=-8\pi d_2^2N_2W_2(-\infty)/(n_0\hbar c\omega_2^3)$. The linear refractive index of the medium is $n_0=[1-16\pi d_2^2N_2W_2(-\infty)/(\hbar\omega_2)]^{1/2}$.

If we neglect the "resonant" term in Eq. (8), i.e., if we set $c_1=0$, this reduces to the modified Korteweg-de Vries (mKdV) equation. In turn, setting $c_2=c_3=0$ transforms Eq. (8) into a sine-Gordon (sG) equation by cutting off the "nonresonant" nonlinearity and dispersion, respectively. Both equations are completely integrable by the inverse scattering transform (IST) method and have been extensively studied as a lowest-order approximation to the complete set of Maxwell-Bloch equations beyond the SVEA [11,17,18]. Then, it seems only natural that the spatiotemporal evolution of the FCP having a pulse width such that $\omega_1\ll 1/\tau_p\ll\omega_2$ will obey Eq. (8), which is the superposition of the mKdV and sG equations. This equation has already appeared in the dynamics of anharmonic crystals with dislocations. If $c_2=c_3/2$, then Eq. (8) becomes integrable by the IST method [24,25]. Physically, this requirement in our case reads as $d_1=d_2$. Although this imposes severe restrictions to the medium composition, this model can still be recognized as a reliable tool for the propagation of the FCP to be assessed.

Before proceeding to our results it is worth noticing another interesting effect provided by Eq. (8). This is the dynamics of the third-harmonic signal $3\omega_p$ that is generated by an intense FCP of carrier frequency ω_p. If the spectral width τ_p^{-1} of the FCP is comparable with ω_p, then one should expect the appearance of even harmonics, the second harmonic as a low-frequency sideband, and the fourth harmonic as a high-frequency one, even for materials with zero quadratic nonlinearity. The phenomenon of the low-frequency sideband generation denoted as "a third-harmonic generation in disguise of a second-harmonic generation" has already been addressed in the literature [26–28] but, to the best of our knowledge, no theoretical treatment relevant to the optics of few-cycle pulses has been given so far. The other example worth noticing is a transverse stability of superluminal pulse propagation [29,30].

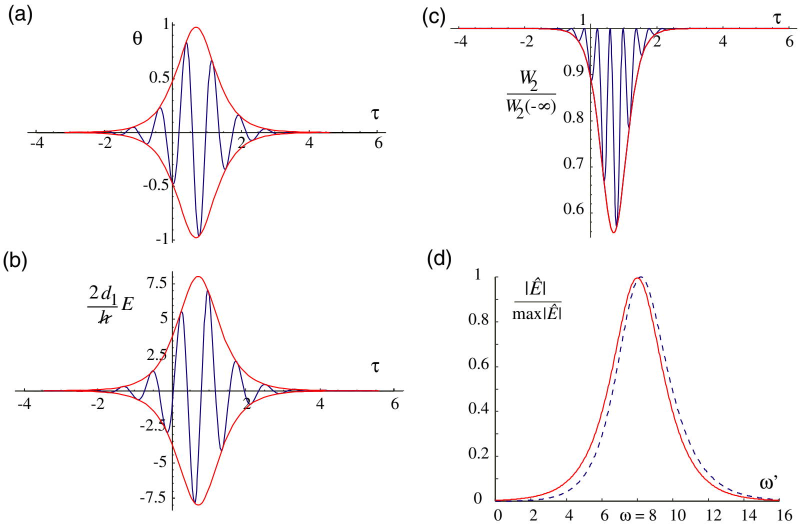

FIG. 1. (Color online) (a) The analytical two-cycle solution to Eq. (9) in the integrable case, and its envelope. (b) The electric field and its envelope. (c) The "resonant" population inversion and its envelope. (d) The optical spectrum (dashed) compared to the spectrum of the field envelope shifted for the carrier frequency ω (solid). Parameters are $C=1$, $p=2$, $\omega=8$, $\varphi=0$ (arbitrary units).

III. INTEGRABLE CASE: ENVELOPE, PHASE AND GROUP VELOCITIES IN THE TWO-CYCLE REGIME

Setting $Z=-c_3 z$ reduces Eq. (8) to

$$\frac{\partial^2\theta}{\partial Z\partial\tau} - a\sin\theta + 3b\left(\frac{\partial\theta}{\partial\tau}\right)^2\frac{\partial^2\theta}{\partial\tau^2} + \frac{\partial^4\theta}{\partial\tau^4} = 0, \qquad (9)$$

with $a=c_1/c_3$ and $b=c_2/c_3$ and which in turn is integrable at $b=1/2$, i.e., as seen above, at $d_1=d_2$, which we assume to be satisfied throughout this section. Following Ref. [24] we can write the two-soliton solution of Eq. (9) as

$$\theta = -4\tan^{-1}[Q(e^{-s_1}, e^{-s_2})], \qquad (10)$$

where

$$Q(X,Y) = \left(\frac{c_{10}}{2\eta_1}X + \frac{c_{20}}{2\eta_2}Y\right) \times \left(1 - c_{10}c_{20}\frac{(\eta_1-\eta_2)^2}{4\eta_1\eta_2(\eta_1+\eta_2)^2}XY\right)^{-1}, \qquad (11)$$

$$s_j = 2A_{j0}Z + 2\eta_j\tau, \quad A_{j0} = -4\eta_j^3 + \frac{a}{4\eta_j}. \qquad (12)$$

Here $i\eta_1$ and $i\eta_2$ are the discrete eigenvalues, and c_{10} and c_{20} are the corresponding initial scattering data. Within the IST framework, c_{10} and c_{20} must be real and η_1 and η_2 must be real positive. However, the solution [(11) and (12)] is an exact explicit solution of Eq. (9), and this statement remains true for any complex value of η_1, η_2, c_{10}, and c_{20}. Since θ must be real, we must have $\eta_2=\eta_1^*$ and $c_{20}=c_{10}^*$. Let us now set $\eta_1=(p+i\omega)/2$ and $c_{10}/2\eta_1=Ce^{i\varphi}$. Then $s_2=s_1^*$ and $s_1=\Psi+i(\Phi+\varphi)$, where p, q, C, φ, Ψ, and Φ are real. This yields the breather solution of Eq. (9), as already given in Ref. [21]. We show below that this breather can be decomposed into a carrier and an envelope, not only in an approximate way at the SVEA limit, but also exactly, for a FCP, hence generalizing these notions beyond the SVEA. Indeed, the property of Q being a rational expression let us see that $Q(e^{-s_1}, e^{-s_2}) = P(e^{-\Psi}, \cos\Phi)$, where P is another rational expression, that is,

$$P(X,Y) = 2CXY \Big/ \left(1 + C^2\frac{p^2}{\omega^2}X^2\right). \qquad (13)$$

This way appears explicitly a carrier wave $\cos\Phi$, and an envelope

$$\theta_e = -4\tan^{-1}P(e^{-\Psi},1) = -4\tan^{-1}\left[2Ce^{-\Psi}\Big/\left(1 + C^2\frac{p^2}{\omega^2}e^{-2\Psi}\right)\right]. \qquad (14)$$

The coincidence of the envelope given this way with the extrema of the pulse is seen from Fig. 1(a), whereas Figs. 1(b) and 1(c) present the corresponding electric field and "resonant" population inversion. The wave spectrum can be evaluated numerically from the above formulas. It differs slightly from the spectrum of the envelope (shifted for the carrier frequency ω), as shown in Fig. 1(d). This stems from the fact that the relation between the envelope, carrier, and the complete expression of the wave are much more complicated than within the SVEA.

The expression of Ψ and Φ are

$$\Psi = p\left(\tau - \frac{Z}{V_g}\right), \quad \Phi = \omega\left(\tau - \frac{Z}{V_\varphi}\right) - \varphi, \qquad (15)$$

with

$$\frac{1}{V_g} = p^2 - 3\omega^2 - \frac{a}{p^2+\omega^2}, \quad \frac{1}{V_\varphi} = 3p^2 - \omega^2 + \frac{a}{p^2+\omega^2}. \qquad (16)$$

ω and p are thus the wave pulsation and inverse of pulse duration, respectively. The important point is that the expressions (16) of the relative dimensionless velocities V_g and V_φ, already given in Ref. [21], allow us to define a phase velocity v_φ and group velocity v_g for the FCP, which so far have been meaningful only within the SVEA. The velocities are related to each other through the relation

$$\frac{1}{v_{g,\varphi}} = \frac{n_0}{c} - \frac{c_3}{V_{g,\varphi}}. \tag{17}$$

Having the wave vector defined as $k=\omega/\nu_\varphi$, we can compute the derivative $d\omega/dk$ and compare it to the group velocity ν_g [or equivalently compare $d(\omega/V_\varphi)/d\omega$ to $1/V_g$]. We find that $d\omega/dk$ is not equal to ν_g, except in the limit $p\to 0$, which corresponds to the SVEA. This important result could be expected, since the relation $\nu_g=d\omega/dk$ is always derived within the framework of the SVEA, and there is no reason for it to remain valid beyond this approximation.

Notice that the relations (16) are obtained elsewhere by the method of analytically continuing the dispersion relation to the complex plane [12,21,22]. Let us give further comment on this method. The existence of an analytic soliton solution allows us to derive a "dispersion relation" valid for purely imaginary values $k=i\kappa$ of the wave vector k. The integrability through the IST method implies the evolution of the scattering data to be determined by the linearized variant of the integrable system [31], and consequently, it implies the relation satisfied by $k=i\kappa$ to be the analytic continuation of the linear dispersion relation $k=F(\omega)$. Replacing $\omega\to\omega+ip$, with $p=1/\tau_p$, and $k\to k+i\kappa$, we obtain a complex dispersion relation $k+i\kappa=F(\omega+ip)$. Its validity on the whole complex half plane is ensured by the existence of the breather. Notice that it is not always the case: e.g., the solution to the KdV equation obtained by the same procedure is singular, and hence no breather exists. Then the total phase becomes

$$(k+i\kappa)Z-(\omega+ip)t = k(Z-V_\varphi t)+i\kappa(z-V_\varphi t), \tag{18}$$

with $V_\varphi=\omega/k$ and $V_g=p/k$. The latter expression is equivalent to

$$\frac{1}{V_g} = \frac{F(\omega+ip)-F(\omega-ip)}{2ip}, \tag{19}$$

which, taking into account the analyticity of the dispersion relation, allows us to recover the well-known relation for the group and phase velocity of quasimonochromatic pulses $V_g=d\omega/dk$ under the condition $p\to 0$. Linearizing Eq. (9) and substituting $\theta\sim\exp[i(\omega t-kZ)]$, arrives at the dispersion relation and then relations (16) can be recovered using the above procedure.

The expression of the electric field of the FCP is obtained from the corresponding expression for the two-soliton solution derived in Ref. [25]. We deduce then the expression of the envelope of the electric field, by setting $\Phi=\pi/2$. It yields

$$E_{env} = \frac{2\hbar}{d_1}p\,\mathrm{sech}\left(\Psi - \ln\frac{p}{C\omega}\right), \tag{20}$$

[see Fig. 1(b)]. It has been shown in Ref. [21] that, within the SVEA, Eq. (9) can be reduced to the nonlinear Schrödinger (NLS) equation,

$$i\frac{\partial A}{\partial Z} + \mu\frac{\partial^2 A}{\partial T^2} + \eta A|A|^2 = 0, \tag{21}$$

with $\mu=-3\omega+a/\omega^3$, $\eta=3b\mu d_1^2/\hbar^2$, and $T=\tau+Z(3\omega^2+a/\omega^2)$, and $E\approx Ae^{i\omega(\tau-Z/\nu_\varphi)}$+c.c. (c.c. stands for complex conjugate). The soliton solution of the NLS equation (21) is [31]

$$A = 2q\,\mathrm{sech}\,2q\left(\sqrt{\frac{\eta}{2\mu}}(T-T_0)+2\lambda\eta Z\right) \times\exp\left(-i\left[2\lambda\sqrt{\frac{\eta}{2\mu}}T+2\eta(\lambda^2-q^2)Z\right]\right), \tag{22}$$

with λ and q being arbitrary real parameters. Since λ represents a shift between the central frequency of the pulse and that of the envelope, it must be set here to $\lambda=0$. We identify $q=p\hbar/d_1$, and Eq. (22) reduces to

$$A = E_{env}e^{ip^2(a/\omega^3-3\omega)Z}. \tag{23}$$

Hence we have exactly $E_{env}=|A|$, i.e., the envelope of the FCP pulse coincides with the envelope of the NLS soliton not only in the SVEA limit $p\to 0$, as was already noticed in Ref. [21], but also in the two-cycle regime where the SVEA is not valid and the envelope soliton is merely meaningless. The exponential factor in Eq. (23) is the first correction to the limit $p\to 0$ of the expression of the carrier phase Φ, as can be easily checked by expanding Φ in a power series of p. Expression (20), and also the complete expression of the field (omitted here), remain valid within the jurisdiction of the mKdV equation. Only the velocities must be modified by setting $a=0$ in Eq. (16). The group and phase velocities for the pure sG model can also be derived in the same way as above, and also can be retrieved from Eq. (16), having the term without a neglected. An analogous computation proves that the relation $\nu_g=d\omega/dk$ is neither satisfied for these two models, except within the SVEA limit.

It is also possible to obtain a pulse with constant carrier-envelope phase. The relative phase of the envelope and of the carrier is of importance for a FCP. This relative phase is constant if the group and phase velocities are equal. From relations (16), we see that this happens when

$$(p^2+\omega^2)^2 = -a. \tag{24}$$

This requires a negative value of a. Coming back to the physical variables and making use of the expressions of coefficients given in Sec. II, this condition is transformed into $W_1(-\infty)W_2(-\infty)<0$. That is, an initial population inversion must be reached for one of the two transitions only. The pulse duration $\tau_p=1/p$ is then given by

$$\frac{1}{\tau_p} = \sqrt{\sqrt{\frac{-W_1(-\infty)}{W_2(-\infty)}\omega_1\omega_2{}^3} - \omega^2}. \tag{25}$$

This implies the carrier frequency ω of the same order of magnitude as the combination $(\omega_1\omega_2{}^3)^{1/4}$ of the two resonance frequencies, which is consistent with the assumption $\omega_1\ll\omega\sim\tau_p^{-1}\ll\omega_2$.

IV. TWO-CYCLE OPTICAL SOLITON OF THE NONINTEGRABLE mKdV sG EQUATION

In the general, nonintegrable case, the existence of two-cycle dispersion-free pulses in the two-cycle regime must be proven numerically [notice however that, as far as we know, the nonintegrability of Eq. (9) for $b \neq 1/2$ and $a \neq 0$ has not yet been rigorously proven]. We do not intend here to give an exhaustive analysis of the behavior of the solutions of Eq. (8), but to show that a two-cycle pulse can still propagate without being destroyed by the dispersion. We exploit the exact breather of the mKdV equation as an input signal [32]. Using the reduced form (9) of the equation, this solution is seen to be valid for $a=0$, but $b \neq 1/2$, and can be written as

$$\frac{\partial\theta}{\partial\tau} = \frac{2d_1}{\hbar}E = \frac{2}{\sqrt{2b}}\left[e^{\eta_1} + e^{\eta_2} + \left(\frac{p_1-p_2}{p_1+p_2}\right)^2\left(\frac{e^{\eta_1}}{4p_1^2} + \frac{e^{\eta_2}}{4p_2^2}\right)e^{\eta_1+\eta_2}\right] \times\left[1 + \frac{e^{\eta_1}}{4p_1^2} + \frac{e^{\eta_2}}{4p_2^2} + \frac{2}{(p_1+p_2)^2}e^{\eta_1+\eta_2} + \left(\frac{p_1-p_2}{p_1+p_2}\right)^4\frac{e^{2\eta_1+2\eta_2}}{16p_1^2p_2^2}\right]^{-1}, \tag{26}$$

where

$$\eta_i = p_i\tau - p_i^3 z - \gamma_i, \quad p_2 = p_1^*, \quad \text{and } \gamma_2 = -\gamma_1, \tag{27}$$

which exactly coincides with the derivative of Eq. (14), apart from a factor $1/\sqrt{2b}$ in Eq. (26) and the absence of the term that includes the parameter a in the expression (27) of the phase. We consider a fixed value of $b \neq 1/2$, and negative values of the parameter a, which accounts for the strength of the resonant term, and increases progressively $|a|$. We exploit the so-called "exponential time differencing method" [33] along with absorbing boundary conditions introduced to avoid numerical instability of the background.

Figure 2 refers to the moderate contribution from the "resonant" interaction when the dynamics of the FCP pulse is mainly contributed from the "nonresonant" component (7). The cubic nonlinearity and third-order dispersion cause fast propagation of the FCP. The relative inverse group velocity of the breather solution of the mKdV equation is given by

$$V_{g0}^{-1} = [(\text{Re}\, p_1)^2 - 3(\text{Im}\, p_1)^2]. \tag{28}$$

The pulse velocity can be determined numerically. If a is not zero but not too large (see Fig. 2), the velocity remains close to the velocity (28) of the mKdV breather. This pulse evolution is in sharp contrast with the linear case, $a=b=0$, depicted in Fig. 3, where the third-order dispersion spreads the pulse envelope out.

Figure 4 summarizes the propagation of the two-cycle pulse (26) in the medium with the concentration of the "resonant" atoms ten times higher than in Fig. 2. The velocity departs considerably from that of the mKdV breather. For the particular set of the parameters chosen in Fig. 4, it is numerically determined in arbitrary units as $V_g^{-1}=-32.25$ instead of $V_{g0}^{-1}=-47$ for the mKdV. Since

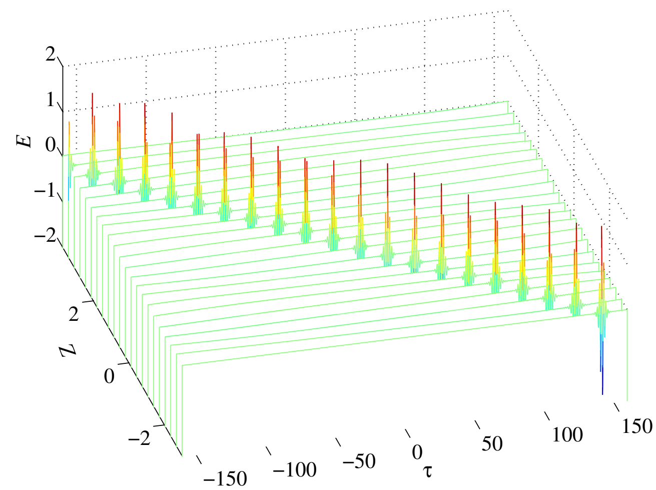

FIG. 2. (Color online) Two-cycle solution to Eq. (9) for a weak resonant term. It is very close to the breather solution of the mKdV equation. Parameters are $p_1=1+4i$, $a=-50$, $b=2$, the vertical scale corresponds to $2d_1/\hbar=1$ (arbitrary units).

$$V = \left(\frac{n_0}{c} + \frac{1}{V_g}\right)^{-1} \approx \frac{c}{n_0} - \frac{c^2}{n_0^2 V_g}, \tag{29}$$

this variation corresponds to a decrease of V relative to the mKdV case. Hence, increasing the contribution brought by the term $\sim\sin\theta$ results in appreciably slowing down the two-cycle optical soliton. In order to show the stability of the pulse, the computation in Fig. 4 has been performed in a frame moving at the inverse speed V_g^{-1}. It is clear that as the pulse penetrates deeper into the medium, its velocity remains constant and we see a stable two-cycle optical pulse traversing the two-component medium.

The above computation shows that, at least for the particular value of parameters we have considered, a dispersion-free two-cycle regime pulse can exist also in the nonintegrable case. Furthermore, the computation proves that it is

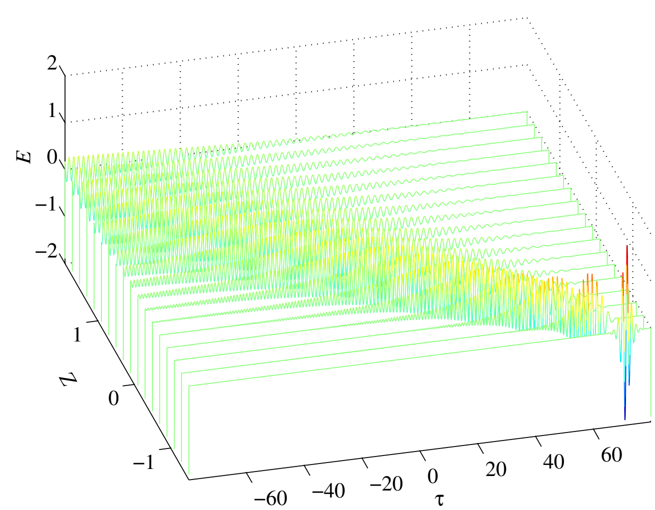

FIG. 3. (Color online) Two-cycle pulse evolution in the linear case: dispersion. Parameters are the same as in Fig. 2 except that $a=b=0$ (arbitrary units).

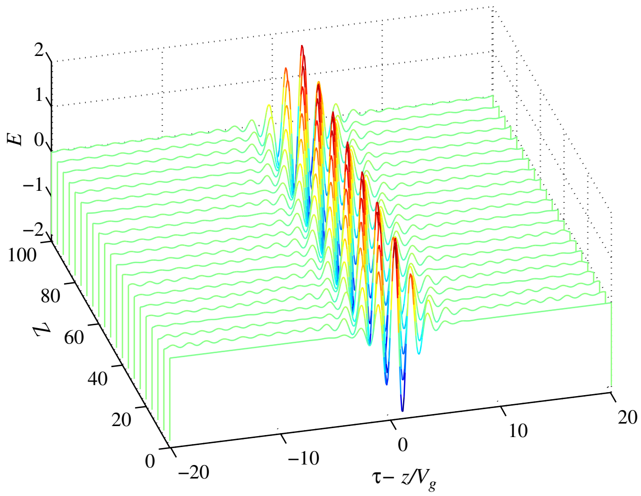

FIG. 4. (Color online) Two-cycle optical pulse in a case where the resonant term is large: dispersion-free propagation is observed. Parameters are the same as in Fig. 2 except that $a=-500$ (arbitrary units).

stable. Notice that the stability of the two-cycle pulse in the integrable case is not a straightforward consequence of the IST. Indeed, the IST method implies the stability of the solitons, which corresponds to purely imaginary eigenvalues, i.e., η_1, η_2 are the real positive within the notations of [25]. The issue of the stability must be considered numerically. We can expect that two-cycle pulses are more stable in the integrable case, in which an infinite set of conservation laws exists, than in the nonintegrable one. Thus the propagation is very likely stable.

In the above computations we considered negative values of the parameter a only. For positive a, that is, both of the initial population differences $W_j(-\infty)$ to be of the same sign, the situation is somehow different. An example of computation with $a=+500$ is shown in Fig. 5. The initial pulse given by Eq. (26) still propagates free from dispersion, but its velocity is not constant: it tends very slowly towards some constant value. This means that a stable FCP soliton still exists in this case, but appreciably differs from the mKdV breather, as given by Eq. (26). In the inset of Fig. 6, both an input and output pulse, which is close to the soliton, are plotted. There is little difference between the two pulse shapes.

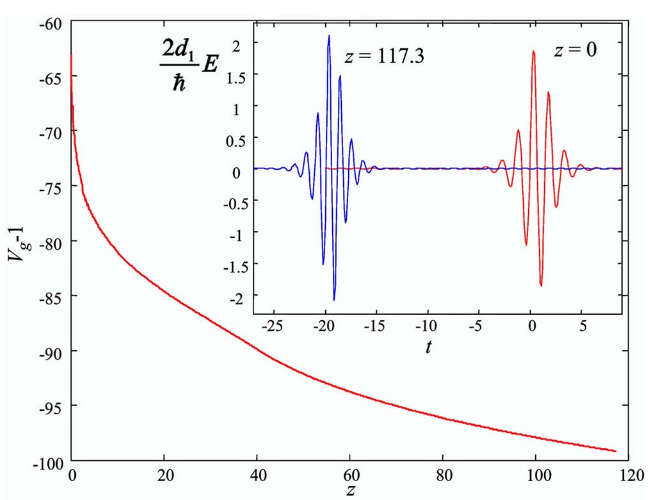

FIG. 5. (Color online) Evolution of the two-cycle pulse speed for $a=+500$. Inset: the two-cycle pulse profile at the beginning and at the end of the simulation. The other parameters are the same as in Fig. 2 (arbitrary units).

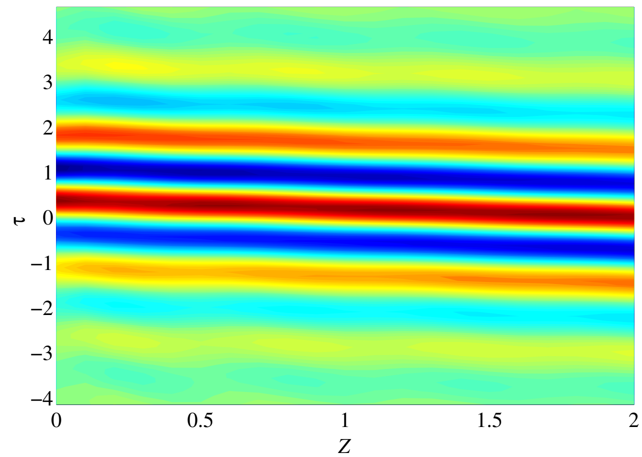

FIG. 6. (Color online) Quasistabilization of the carrier-envelope phase of the two-cycle optical pulse when the condition derived in the integrable case is satisfied. Parameters are the same as in Fig. 2 except that $a=-289$ (arbitrary units).

Further, condition (24) predicts the stabilization of the carrier-envelope phase for a given value of a in the integrable case. With the magnitudes of p and ω used in Figs. 2–5, this can be expected at $a=-289$. Figure 6 shows the phase is almost stabilized for this value of a, especially as compared to the huge variations of the carrier-envelope phase observed for the value $a=-500$ considered above (see Fig. 7). A more accurate analysis allows us to get closer to perfect stabilization, as shown in Fig. 8, which is computed for $a=-292.55$. Indeed, the lines are parallel up to the precision of the computation. Obviously, the optimal value of a could be refined with longer computation. In conclusion, the

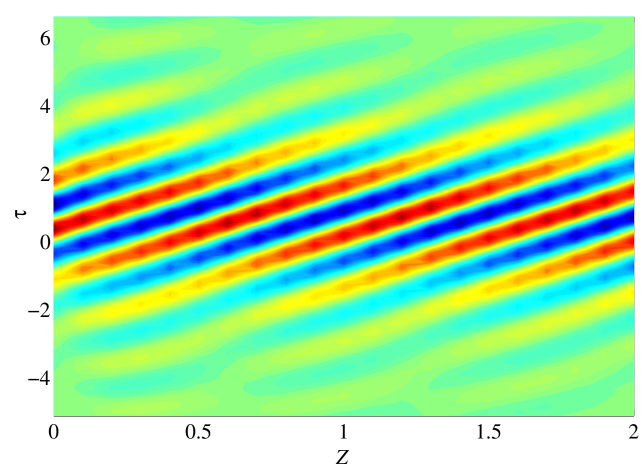

FIG. 7. (Color online) Density plot of the FCP soliton of Fig. 4 showing the huge variations of the carrier-envelope phase for comparison with Fig. 6.

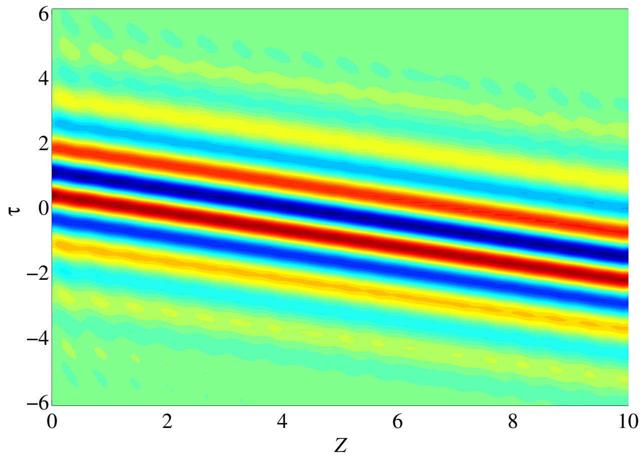

FIG. 8. (Color online) Stabilization of the carrier-envelope phase of the two-cycle optical pulse. Parameters are the same as in Fig. 2 except that $a=-292.55$ (arbitrary units).

carrier-envelope phase stabilization also takes place in the nonintegrable case, and for a value of the parameters very close to the one predicted in the integrable case.

V. CONCLUSIONS

We have shown the existence of two-cycle optical solitons in a one-dimensional two-composite dielectric material. A rich variety of solitons is proven to exist under the circumstance where the incident light-cycle duration is set properly to the resonant frequency of the components and corresponding relaxation time. Specifically, in the integrable case, a two-cycle analytical solution is available and enables us to generalize the notion of envelope and carrier to a FCP, without resorting to the SVEA, which is by no means valid in this case. Group and phase velocities have been determined, showing that the usual relation $v_g=d\omega/dk$ is not valid for a FCP. On the other hand, the envelope of the FCP soliton has exactly the sech shape of the envelope soliton of the NLS equation. We demonstrate that in the integrable case there is a family of the two-cycle solitons with a stable carrier-envelope phase. The soliton depends strongly on the dopant's matrix elements as well as on the relative population inversion of the components. In the general case, the existence of two-cycle solitons and the stabilization of the carrier-envelope phase have been demonstrated numerically. The two-component approximation therefore provides a valuable starting point for the assessment of few-cycle optical pulses and the estimation of the carrier-envelope phase, which has become an emerging issue in photonics research.

We have restricted our analysis to a special and illustrative case, which can be recognized as a few-cycle pulse propagating through a wide-gap dielectric with a system of impurities or defects generated there. An obvious generalization of our model is therefore to include a nonsteady-state density of the resonant centers and near-field effects that drastically change the frequency-pulse bandwidth hierarchy. In reality, the transverse propagation effects must also be taken into consideration in order to understand the pulse stability. It is also useful to analyze how to excite these pulses with a stable carrier-envelope phase in a finite medium. It is our hope that our simple model analysis will motivate much more comprehensive treatment, which should explore the full parameter space of the two-cycle soliton solution. This may in turn lead to the application of few-cycle pulses for ultrafast-laser modification of materials for nanophotonics.

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