

Preparation of ultralow atomic velocities by transforming bound states into tunneling resonancesF. Delgado,^{1,2,*} J. G. Muga,^{2,†} and A. Ruschhaupt^{3,‡}¹*Departamento de Física Básica, Universidad de La Laguna, La Laguna, Tenerife, Spain*²*Departamento de Química-Física, UPV-EHU, Apartado 644, 48080 Bilbao, Spain*³*Institut für Mathematische Physik, TU Braunschweig, Mendelssohnstrasse 3, 38106 Braunschweig, Germany*

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A procedure is proposed to prepare the average and width of the velocity distribution of ultracold atoms. The atoms are set initially in the ground state of an optical trap formed by an inner red-detuned-laser well and an outer blue-detuned-laser barrier. Then the well and barrier parameters are changed until the ground state becomes a Breit-Wigner tunneling resonance. An optimal time dependence of the switching process, between the sudden and adiabatic limits, adjusts the final translational energies of the leaking atoms to the Lorentzian distribution of the resonance state.

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I. INTRODUCTION

With the advent of laser cooling techniques, the traditional atomic velocity selection or preparation methods [1] have to be substituted or modified, due to the increasing importance of gravity or recoil effects, and the quantum nature of translational motion. Thus, the standard classical-mechanical analysis of mechanical velocity-selection methods becomes invalid for small-time temporal slits and large wavelengths, because of diffraction and a quantum momentum spread in agreement with a time-energy uncertainty principle [2]. In general, the concept of “velocity selection” is used with quite different meanings; in a broad sense, cooling or velocity measurements may be interpreted as velocity-selection processes [3]. The selection may be passive (filtering), if a particular component of the initial distribution is retained and the rest discarded, or active (preparation) when the momentum distribution is transformed and shaped to the desired objective. The distinction is blurred in some cases: for example, a passive method may in principle be incorporated into an active cyclic scheme in which selection is followed and preceded by a step to refill the depleted velocity domain in the original (source) sample, by collisions or other means, so as to produce more atoms within the desired velocity range [4]; it is also possible to select a given initial momentum by imparting a momentum transfer so that the outgoing atoms have a well defined velocity, but different from the selected initial velocity range, as in Bragg-diffraction or Raman techniques [5].

There are many different applications of velocity selection which determine in part the best choice and type of approach: examples relevant for cold or ultracold atoms are the measurement of momentum distributions [5], cooling [3], outcoupling mechanisms in atom lasers [6], and atomic preparation for scattering experiments [7], for atomic clocks [8], precise measurement of fundamental constants [9–11], lithography [12], or any other velocity-dependent processes.

Velocity-selection methods may also be classified according to the coherence or incoherence of the resulting state and according to the physical phenomena on which they are based: The Doppler effect, in particular, plays a dominant role in many of the optical methods. As early as 1927 the Doppler shift of the light frequency was proposed to provide a suitable way to probe and select velocities [13]. This is even more clear today because of the monochromaticity of laser light: the resonance condition, which reflects energy and momentum conservation of a laser-induced transition, is satisfied only for a narrow velocity range depending on the level width; the Doppler-shifted absorption technique has thus become a routine method for monitoring velocity distributions in supersonic beams of atoms and molecules in individual quantum states [1]. In this high-velocity scenario the momentum transferred in the absorption and emission of photons is of no concern and relatively negligible; this has changed dramatically for ultracold atoms and several Doppler-based techniques have been developed or proposed in which the transferred momentum is of fundamental importance: The Zeeman slower and Doppler cooling are obvious examples. Coherent techniques based on stimulated two-photon Raman transitions achieve very narrow, subrecoil velocity widths proportional to the sum of the two laser frequencies involved [14,15]. Bragg scattering of atoms from a light grating formed by two different lasers (which may also be regarded as recoil-induced resonances [16] or stimulated optical Compton scattering [17]) similarly uses the Doppler effect for subrecoil selectivity, but only one internal state. Bragg spectroscopy has thus been used to measure momentum distributions of condensates [5] and quasicondensates [18], and the same principle is applied to build atom interferometers [19] or an outcoupling mechanism in atom lasers [20]. One more Doppler based method proposal makes use of quantum interference in the two-photon ionization through two quasidegenerate intermediate levels [21,22]. Also, velocity-selective coherent population transfer (VSCPT) is an interference-based cooling method that makes use of the Doppler shift in a somewhat subtle way [23,24]: for a Λ level system with degenerate ground states, spontaneous transitions lead after sufficiently long times to a dark state combining the two ground states with plus and minus recoil momentum. In this process the Doppler shift acts as a mo-

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tional coupling that favors decay for large atomic momenta and hinders it for small ones. In the original formulation this leads to a two-peaked velocity distribution around zero, but other velocities may be achieved, e.g., by applying a magnetic field [25].

Alternative methods not based on the Doppler effect are also worth exploring, since the physical limits and requirements may be very different. In particular, these methods may be quite independent of specific atomic level structures. Examples are a recent mechanical rotor to slow down supersonic beams of noble gases [26], or a moving magnetic mirror to slow down rubidium atoms [27]. In the domain of optical methods, Fabry-Pérot (FP) matter-wave interferometers, basically double barriers realized with detuned lasers or microwave cavities, have been proposed to provide coherent atomic velocity selection [28–32], or an atom laser [33]. In a recent paper [29], we have investigated the fundamental limits of a matter-wave Fabry-Pérot optical device made of two blue-detuned-laser barriers and a red-detuned-laser well, for selecting both the average and the width of the atomic velocity distribution according to a scattering resonance. The basic control knob in [29] was the well depth, which lets us modify the resonance energy and thus the velocity window. It was theoretically and numerically demonstrated that this method may produce arbitrarily small velocities but, since it is based on filtering the incident velocity distribution with a resonance peak of the transmission probability, it is a purely passive approach, so that the resulting fraction of transmitted atoms may be very small and will depend strongly on the incident state. This conservation of the weight of each velocity slice also occurs, e.g., for the stimulated Raman transition method, but at variance with it, the FP approach does not impart any momentum.

A method with the goal of increasing the number of atoms with a desired velocity, used for example in atomic fountains [8], is based on moving optical molasses [34]. The velocity width that can be achieved by this method is clearly limited by the recoil velocity. This could be avoided without photon absorption or emission, as in the approach presented here.

This paper describes a method to efficiently prepare and control the average and width of the final velocity distribution of the atomic cloud. In common with the FP transmission resonance method, it may also be implemented with red- and blue-detuned lasers, a red-detuned laser forming an inner well and a blue-detuned laser for an outer barrier. Note that the implementation of well and barrier is the only step in the method where “some” internal structure of the atoms is required (see, however, the final discussion for a more accurate analysis). This approach also uses a tunneling resonance as in the FP approach, but instead of being based on a full scattering process for filtering out a velocity slice, it relies on a half-scattering process where virtually all atoms decay within a narrow velocity range. A given velocity average and width—possibly in the subrecoil regime—are actively prepared, in contrast to other passive methods such as in Raman, Bragg, or double-barrier FP selection. The basic setting is in Fig. 1: initially the atoms are prepared in the ground state of the well (and its only bound state). In the next step, we change well depth and barrier height to a configuration which has no bound states but instead a first Breit-Wigner

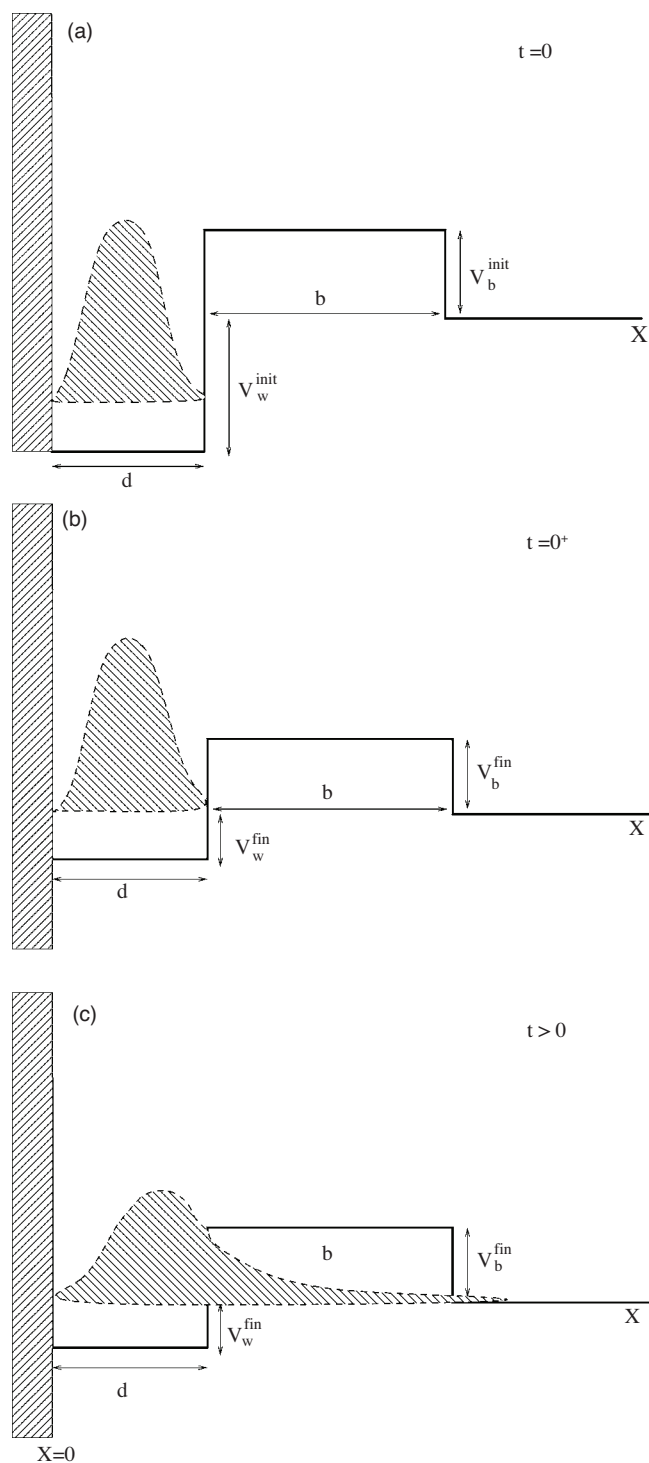


FIG. 1. Schematic representation of the velocity preparation method. The left barrier represents an “infinite wall.”

resonance corresponding to the desired atomic velocity average and width. After this switching process, the former ground state becomes a resonance state and the atoms will leak out of the trap. We will show that by choosing an optimal time dependence of the switching process, the asymptotic velocity distribution of the atoms will be determined by the Lorentzian energy distribution of the resonance state.

In Sec. II, this procedure will be described in more detail. Some basic properties of resonance states will be reviewed in Sec. III where we shall also examine the resonances of the basic configuration of a well and barrier. In Sec. IV we shall give a numerical example of this velocity preparation method, using different switching times between the sudden and adiabatic limits. Conclusions and comments will be finally provided in Sec. V.

II. BOUND-STATE TO RESONANCE TRANSFORMATION

For simplicity, we shall assume a one-dimensional (1D) model corresponding to the effective 1D atomic motion in a narrow waveguide, and a “square” shape for the intensities of the detuned lasers and therefore for the effective potentials (see Fig. 1), which takes the initial and final forms

$$V^{init/fin}(x) = \begin{cases} \infty, & x \leq 0, \\ -V_w^{init/fin}, & 0 < x \leq d, \\ V_b^{init/fin}, & d < x \leq d+b, \\ 0, & x > d+b. \end{cases}$$

Similar results can be achieved for realistic smoother profiles as in [29]. The infinite wall at the origin is also a simplifying feature of the model, but it is not strictly necessary. In particular, One could also use two finite barriers, one at each side of the well [29], to represent the radial potential profile of a cylindrical confinement with free atomic motion or weak confinement in the axial direction.

At this stage we also assume that the atoms are independent and can be described by the Schrödinger equation, disregarding nonlinear effects that could be incorporated within a mean-field treatment as in [29].

The starting point of the velocity preparation process is a laser configuration which holds only one bound state. It is assumed that the atom can be prepared in this ground state [see Fig. 1(a)]. Several possibilities exist to prepare that initial state: for example, the original trap could hold more than one bound state; in that case an arbitrary trapped atomic state overlaps with several of them, but the trap may be modified to hold one bound state only so that the wave component in the continuum subspace is eliminated by its evolution away from the interaction region. More sophisticated and efficient methods without losing atoms may be based on ground-state cooling using resolved-sideband transitions [35]. Pushing up the potential well later on, the ground state will eventually become the only bound state, thus realizing our starting point objective.

Once the initial state of Fig. 1(a) is formed, the potential well is moved upward by decreasing the intensity of the red-detuned laser, i.e., V_w is decreased. In addition, the intensity of the blue-detuned laser could also be changed for further control. This is represented in Fig. 1(b), where the potential switch has been performed suddenly with respect to other relevant time scales. The consequence is that the bound state becomes, for a final well depth shallower than a threshold value, a “resonance state.” As is well known, resonances may be regarded as quasibound states associated with poles of the S matrix in the lower half-momentum plane; they can

be linked continuously with bound states (poles on the positive imaginary axis) by varying the potential parameters. An important difference though, is that bound states are in Hilbert space and normalizable, while Gamow (resonance) states are not, since they increase exponentially at large distances from the potential center. The normalized state achieved by shifting the well bottom, as in Fig. 1(b), is thus not a true Gamow state, but it will share approximately some of its properties, in particular its decay rate, the basic Lorentzian shape in energy space (neglecting threshold effects) and its coordinate-space form in the potential region. We insist that this agreement is necessarily a partial one.

After the switching process, the atom will leak out [see Fig. 1(c)] having a given (total) energy distribution. Note that the energy distribution calculated at the end of the switching process, i.e., at a time when the atom is still interacting with the trap, is equal to the kinetic energy distribution of the released atoms at asymptotically large time, as follows from energy conservation. Therefore, at a sufficiently large time, the atom will move with the velocity distribution determined by the Lorentzian shape, in energy space, of the Gamow state.

Two limits consisting on sudden or infinitely slow well switching may be considered. (a) A sudden well shift produces a state with contributions from higher resonances. They will lead to perturbations with respect to the ideal velocity distribution which will affect the short time decay behavior. (b) The opposite limit of very slow switching implies a different problem: since the pole motion in the complex momentum plane up to the final resonance position is slow, a continuum of intermediate resonances are excited. They will have a decay time larger than the one desired, thus inducing a deviation with respect to the exponential decay rate, in this case due to a bias towards slow components. We will see in Sec. IV that an adjustment of the switching time may avoid the perturbations of the fast and the slow processes and produce an excellent agreement with the velocity-transformed Lorentzian shape.

III. CONFIGURATIONS AND CORRESPONDING RESONANCES

An implementation of the proposed velocity preparation method will require the knowledge of the resonances in the final configuration. In this section we will review how the resonance positions can be found and we will give an example of the dependence between resonant position and the laser configuration, i.e., the well and barrier parameters.

In the final configuration [see Fig. 1(b)], the stationary states of a single ultracold atom moving along the x direction will satisfy

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V^{fin}(x) \right) \psi_k(x) = E_k \psi_k(x),$$

where $E_k = \hbar^2 k^2 / 2m$ (for the rest of this section we omit the superscript *fin*). For the calculations we use the mass of ^{23}Na . The scattering states have the form

$$\psi_k(x) = \frac{1}{\sqrt{2\pi}} \begin{cases} C_1 e^{iqx} + C_2 e^{-iqx}, & 0 \leq x \leq d, \\ C_3 e^{iq'x} + C_4 e^{-iq'x}, & d \leq x \leq d+b, \\ e^{-ikx} - S(k)e^{ikx}, & x \geq d+b, \end{cases}$$

where $q = \sqrt{k^2 + 2mV_w/\hbar^2}$ and $q' = \sqrt{k^2 - 2mV_b/\hbar^2}$. q and q' have a branch cut in the p plane joining the two branch points at $\pm i\sqrt{2mV_w/\hbar^2}$ and $\pm\sqrt{2mV_b/\hbar^2}$, respectively. The different coefficients are obtained from the matching conditions at $x=0$, $x=d$, and $x=d+b$. The resonances and bound states can be calculated from the poles of the S matrix in the complex k plane. They are solutions of the equation

$$-(k - q')[q + e^{2idq}(q - q') + q'] + e^{2ibq'}(k + q')[q - q' + e^{2idq}(q + q')] = 0.$$

The corresponding roots in the upper half-imaginary axis are the bound states of the system, while the roots in the fourth and third quadrants are resonances and antiresonances, respectively. An alternative way to find the resonances is to look for jumps of the phase shift $\delta(k)$,

$$\delta(k) = \frac{1}{2i} \ln[S(k)],$$

or the peaks of the Wigner delay time Δt ,

$$\Delta t(k) = 2\hbar \frac{\partial \delta(E_k)}{\partial E_k} = \frac{2m}{\hbar k} \frac{\partial \delta(k)}{\partial k}. \quad (1)$$

In the Breit-Wigner regime of isolated and sharp resonances, we can fit this delay time to a modified Lorentzian $(2m/\hbar k)k_2/[(k-k_1)^2 + k_2^2]$ and obtain the position of the resonance $k_{res} = k_1 - ik_2$ in the complex k plane. This may be easier than determining the poles by analytical continuation of $S(k)$ and this procedure can be also applied to general potentials if an analytic expression for the S matrix is not available.

The corresponding resonance position in energy space is given by $E_{res} = E_R - i\Gamma/2$ with

$$E_R = \frac{\hbar^2}{2m}(k_1^2 - k_2^2), \quad \Gamma = \frac{2\hbar^2}{m}k_1k_2.$$

The energy distribution of the resonance in the Breit-Wigner regime is given approximately by a Lorentzian

$$p_R(E) = \frac{\Gamma}{2\pi} \frac{1}{(E - E_R)^2 + (\Gamma/2)^2}. \quad (2)$$

For later convenience, we define a velocity-transformed distribution of (2) with $v = \sqrt{2E/m}$,

$$p_R(v) = \frac{dE}{dv} p(E) = \frac{2v_R \Delta v_R}{\pi} \frac{v}{(v^2 - v_R^2)^2 + (v_R \Delta v_R)^2}, \quad (3)$$

where the resonance velocity is $v_R := \sqrt{2E_R/m}$ and its velocity width $\Delta v_R := \Gamma/\sqrt{2mE_R}$. If $\Delta v_R/v_R \ll 1$, v_R is approximately at the maximum and Δv_R is approximately the full width at half maximum of the probability distribution $p_R(v)$ (3). Note that this transformation to velocities will be meaningful at asymptotic times when the total energy consists solely of kinetic energy.

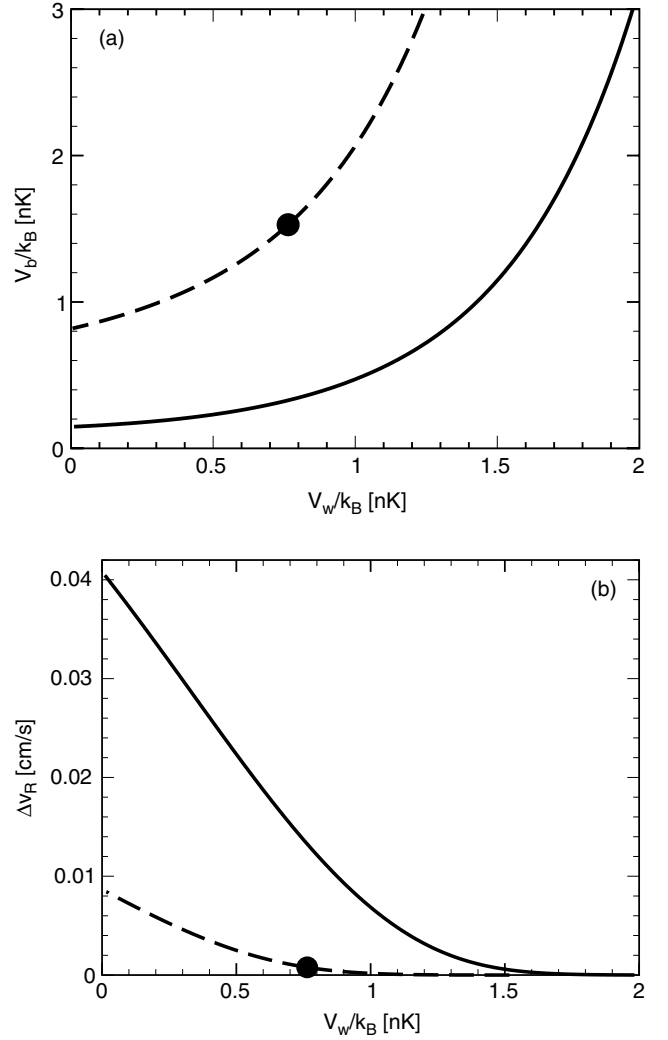


FIG. 2. (a) Combinations of barrier height V_b and well depth V_w resulting in resonance velocities $v_R = 0.05431$ (solid line) and 0.08621 cm/s (dashed line); (b) velocity width Δv_R versus well depth V_w , for a given V_w the corresponding barrier height V_b is chosen as in (a) such that the resonance velocity is $v_R = 0.05431$ (solid line) and 0.08621 cm/s (dashed line), respectively; $b = 10 \mu\text{m}$, $d = 5 \mu\text{m}$; the dots mark the combination which is used as the final configuration in Figs. 3 and 4.

Let us present an example for the dependence between resonance and laser configuration. Note that different combinations of well depth V_w and barrier height V_b can lead to the same resonant velocity v_R . In Fig. 2(a), we have plotted curves of combinations for two resonant velocities v_R with $b = 10 \mu\text{m}$, and $d = 5 \mu\text{m}$. Along a curve for a fixed v_R , the velocity width Δv_R changes, this change is plotted in Fig. 2(b). For a fixed velocity v_R , we can—in principle—find combinations of V_w and V_b which make the velocity width Δv_R arbitrary small.

IV. RESULTS FOR DIFFERENT SWITCHING TIMES

Now we return back to our velocity preparation procedure. Let $\varphi_0(x)$ be the ground state in the initial potential

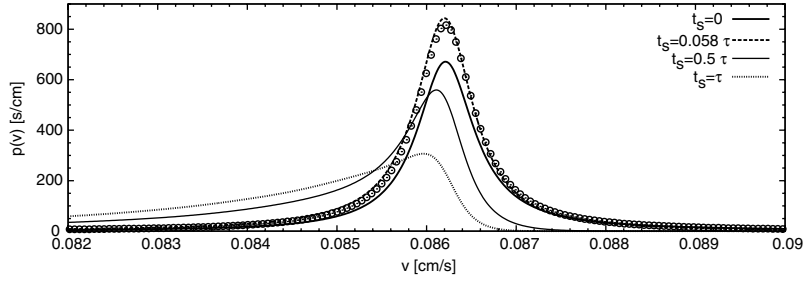


FIG. 3. Asymptotic velocity distribution $p(v)$ for different switching times t_s ; the initial configuration is $V_w^{init}=k_B \times 2.673$ nK, $V_b^{init}=k_B \times 3.055$ nK; the final configuration is $V_w^{fin}=k_B \times 0.764$ nK, $V_b^{fin}=k_B \times 1.528$ nK; $d=5$ μm , $b=10$ μm ; the dots correspond to $p_R(v)$ [see Eq. (3)] with $v_R=0.08621$ cm/s, $\Delta v_R=0.00078$ cm/s.

configuration of Fig. 1(a) with $V_w^{init}=k_B \times 2.673$ nK, $V_b^{init}=k_B \times 3.055$ nK. Note that the velocity width at half maximum of the ground state is $\Delta v_0=0.23$ cm/s. For the destination resonance in Fig. 1(b), we choose the lowest one corresponding to $V_w^{fin}=k_B \times 0.764$ nK and $V_b^{fin}=k_B \times 1.528$ nK. In this case, the resonance complex energy is $E_{res}=(134.509 - i1.217)\hbar/s = k_B \times (1.027 - i9.294 \times 10^{-3})$ nK, thus its velocity is $v_R=0.08621$ cm/s and the velocity width is $\Delta v_R=0.00078$ cm/s $\ll \Delta v_0$ such that $v_R/\Delta v_R \approx 100$. These are extremely small and monochromatic velocities chosen for illustrating the potential capabilities of the approach. A broader and faster distribution is easy to achieve by playing with barrier and well parameters.

If we move suddenly the bottom of the well making it shallower such that the initial bound state overlaps strongly with the desired resonance [Fig. 1(b)], the wave function will evolve in time, and the atoms will leak out through the barrier [Fig. 1(c)] according to

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V^{fin}(x) \right) \psi(x,t), \quad (4)$$

where $\psi(x,0)=\varphi_0(x)$. Our main objective is to achieve an asymptotic velocity distribution as close as possible to that distribution related to the Lorentzian shape of the chosen Gamow resonance of the final potential configuration [Figs. 1(b) and 1(c)]. The resulting total energy distribution of the wave packet in the new potential configuration is given by $p(E)=\sqrt{m/2\hbar^2 E} |\langle \psi_{k(E)} | \varphi_0 \rangle|^2$. $p(E)$ coincides with the kinetic energy distribution when the packet moves away from the potential region.¹ Therefore the asymptotic velocity distribution is $p(v)=(m/\hbar) |\langle \psi_{k=mv/\hbar} | \varphi_0 \rangle|^2$ which is shown in Fig. 3 (case $t_s=0$). After such a sudden process, several resonances are excited as it may be seen in different ways: note in particular that the velocity distribution $p(v)$ has approximately the same shape as the velocity-transformed Lorentzian $p_R(v)$ (which is also plotted in Fig. 3) but it is lower in magnitude. This means that part of the norm is in higher resonances, and a consequence is the fast decay of the nonescape probability

¹This is easily derived from the intertwining relation of scattering theory, the isometry of Moller operators, and the assumption that there is no bound state in the final configuration.

$$P_W(t) = \int_0^d dx |\psi(x,t)|^2$$

at short times in Fig. 4 (solid line). In other words, with the sudden switching, a significant fraction of atoms is released at early times with too much energy. Of course, if we discard the early, fast atoms, the decay occurs finally with the desired rate and energy distribution, see again Fig. 4. However, we may try to produce an ensemble without undesired high-velocity components. This can be achieved by a progressive, rather than abrupt, switching of the potentials.

Let us assume that the potential profile changes in time according to the smooth function

$$V(t,x) = [V^{fin}(x) - V^{init}(x)](1 - e^{-t/t_s}) + V^{init}(x). \quad (5)$$

The sudden change corresponds to $t_s=0$ and the infinitely slow change to $t_s=\infty$. In Fig. 4 we show the decay of the nonescape probability $P_W(t)$ for different t_s . The lifetime of the first resonance (calculated from the pole of the S matrix) is $\tau=0.411$ s, in perfect agreement with the fitting to the exponential decay that dominates after the early transients, independent of t_s . This occurs because the final potential configuration is common to all cases so that the resonance

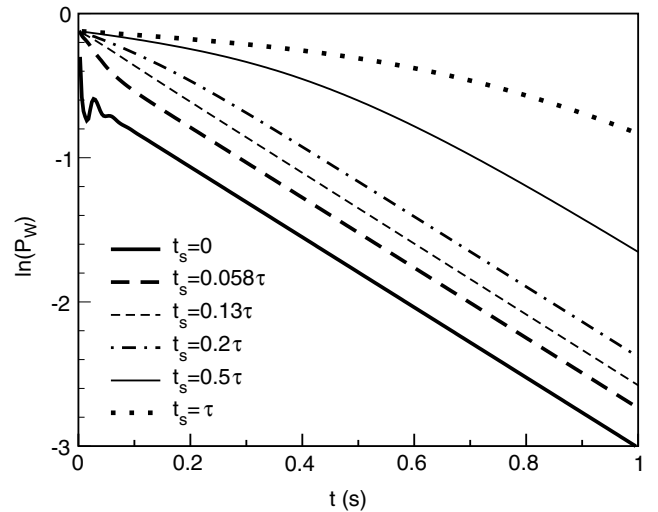


FIG. 4. Decay of the probability $P_W(t)$ to find the atom in the well versus time for different values of t_s [see Eq. (5)]; the initial and final potential configurations are given in Fig. 3.

with the longest lifetime is the same in all cases. Nevertheless, the first transient regime varies substantially with t_s , and for $t_s \approx \tau$ the initial decay is slowed down considerably. The best fit to the purely exponential decay is found for $t_s \approx 0.13\tau$.

Our interest is in the asymptotic and stationary velocity distribution at large time, $t_\infty \gg t_s$, when the total energy is purely kinetic energy. In Fig. 3 we have plotted the asymptotic velocity distribution for different switching times t_s . As discussed above, the velocity distribution for $t_s=0$ (sudden change) has a shape corresponding to $p_R(v)$ but its height is reduced because of the excitation of higher resonances. Increasing t_s , an optimal value is found so that the energy distribution fits even in magnitude to $p_R(v)$, i.e., to the velocity-transformed Lorentzian shape. For the case studied in Fig. 3, the best fit to $p_R(v)$ of the selected resonance corresponds to $t_s \approx 0.058\tau$. This optimal value of t_s is different from the one that provides the best fit to the purely exponential decay ($t_s \approx 0.13\tau$) (compare Figs. 3 and 4). As t_s is increased further, the distribution is deformed, the symmetry is lost, and a distortion favoring lower energies is observed.

V. DISCUSSION

We have proposed a method to prepare states with well-defined average velocity and velocity width. The basic setting consists of a trap formed by an inner well and an outer barrier which, in atom optics, may be realized with lasers detuned from an atomic transition. After the atoms are initially prepared in the ground state of the well, the depth of the well and the barrier height are changed until the ground state becomes a chosen Breit-Wigner resonance state and the atoms leak out. By choosing a special time dependence of the potential change, we have shown that their asymptotic velocity distribution is determined by the Lorentzian energy shape of the chosen resonance.

This method does not require a special internal atomic structure and the velocity width is not limited by the recoil limit. In addition, the resonance width and therefore the asymptotic velocity width can—in principle—be made arbitrarily small by choosing the proper barrier and well configuration. The final momentum is also quite arbitrary, within ultracold temperatures, since the well bottom can be put above the asymptotic zero-energy level for free motion with blue laser detuning. (An upper bound for the velocities that can be prepared in this fashion is set by the available barrier heights.) For atom-laser outcoupling, this could possibly be an advantage with respect to the low efficiency of high momentum transfers (involving several recoils) using Bragg diffraction [20]. Also, at variance with that method, in the present one an adiabatic expansion of the trap is not needed to narrow the momentum distribution so as to impart the desired recoil momentum to the whole atomic cloud. In fact in our case the momentum distribution of the trapped atoms may be quite broader than the final one.

We have illustrated the method in a numerical example with ^{23}Na ($v_{\text{recoil}} \approx 2.9$ cm/s, $T_{\text{recoil}} \approx 2.4$ μK). Starting with a ground state with a velocity width at half maximum of $\Delta v_0 = 0.23$ cm/s, our method results in the preparation of the

velocity $v_R = 0.086$ cm/s with a velocity width $\Delta v_R \approx v_R/100 \ll \Delta v_0$. Note that even if we have focused on an atom-optical implementation the method may be also applied to electrons in semiconductor heterostructures, where the well depth can be modified by potential gate voltages [36,37].

Manipulations of laser-induced potentials similar to the ones required for converting a bound state into a resonance have been realized for optical trap engineering to prepare number states [38,39]. Thus, a realization of the proposed method may be challenging but not totally out of reach with current technology. A practical implementation and a realistic comparison with existing velocity preparation methods will require to consider other factors not taken into account in the present, preliminary study. For example, fluctuations of the potentials may tend to broaden or blur quantum resonances. Nevertheless, stabilized lasers provide effective constant intensities in the time scale of τ and t_s (~ 0.01 – 1 s) [40–42] so that our analysis would apply to the effective, time-averaged potentials. Atom-atom interactions should also be considered: in a Bose-Einstein condensate they may be modeled by a mean-field theory, and the resonances and bound levels will suffer shifts [37]. Resonance decay from double barriers holding an inner well has been studied for nonlinear Schrödinger equations in the context of semiconductor structures and quantum dots, and similar techniques may be applied for a Gross-Pitaevski equation for atoms. Another limiting factor may be the residual absorption that takes place if a detuned two level transition is used to implement the well and barrier. In the case of large detuning Δ and assuming $\Delta \gg \gamma$ (where γ is the Einstein coefficient), the absorption rate is $\Gamma_{\text{eff}} = \gamma\Omega^2/(4\Delta^2)$ (where Ω is the Rabi frequency) while the real potential is given by $V = \hbar\Omega^2/(4\Delta)$, such that $\Gamma_{\text{eff}} = \gamma V/(\Delta\hbar)$. The absorption rate may of course be made very small by increasing Δ , but since the real potential should not change, it is necessary to increase Ω too, keeping the ratio Ω^2/Δ constant. In practice this change may be limited by other transitions or by the needed laser intensity so that a study of the role of this effect is required in specific applications. For ^{23}Na , red- and blue-detuned optical traps have been demonstrated with effective lifetimes beyond 1 s [43,44]. For the D transition in ^{23}Na at 589 nm, $\gamma/(2\pi) = 10$ MHz. For the barrier height $k_b \times 3$ nK of the numerical example, an effective lifetime $\Gamma_{\text{eff}}^{-1} \approx 1$ s can be achieved with rather moderate values of detuning and Rabi frequency: $\Delta/(2\pi) \approx 4$ GHz, and $\Omega/(2\pi) \approx 1$ MHz, respectively.

In closing, we believe that the present results provide the motivation for further theoretical and experimental work to prepare ultracold velocities by transforming trap bound states into resonances.

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