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Nonlinear magneto-optical rotation with modulated light in tilted magnetic fields

S. Pustelny and W. Gawlik
Centrum Badań Magnetooptycznych, M. Smoluchowski Institute of Physics, Jagiellonian University, Reymonta 4, 30-059 Kraków, Poland

S. M. Rochester
Department of Physics, University of California at Berkeley, Berkeley, California 94720-7300, USA

D. F. Jackson Kimball
Department of Physics, California State University—East Bay, 25800 Carlos Bee Boulevard, Hayward, California 94542, USA

V. V. Yashchuk
Advanced Light Source Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

D. Budker
Department of Physics, University of California at Berkeley, Berkeley, California 94720-7300, USA and Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
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Larmor precession of laser-polarized atoms contained in antirelaxation-coated cells, detected via nonlinear magneto-optical rotation (NMOR), is a promising technique for a new generation of ultrasensitive atomic magnetometers. For magnetic fields directed along the light propagation direction, resonances in NMOR appear when linearly polarized light is frequency or amplitude modulated at twice the Larmor frequency. Because the frequency of these resonances depends on the magnitude but not the direction of the field, they are useful for scalar magnetometry. Additional NMOR resonances at the Larmor frequency appear when the magnetic field is tilted away from the light propagation direction in the plane defined by the light propagation and polarization vectors. These resonances, studied both experimentally and with a density matrix calculation in the present work, offer a convenient method of achieving additional information about a direction of the magnetic field.

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When linearly polarized light, frequency or amplitude modulated at Ω_m, resonantly interacts with an atomic vapor in the presence of a magnetic field, the polarization of the light can be observed to rotate synchronously with the modulation. This effect is known as nonlinear magneto-optical rotation with frequency-modulated light (FM NMOR) [1–3] or amplitude-modulated light (AMOR) [5,6]. It occurs when Ω_m is a subharmonic of the quantum beat frequency; the quantum beat frequency is at the first or second harmonic of the Larmor frequency Ω_L for the lowest-order effect discussed here. Higher-order effects involve quantum beat frequencies at other multiples of Ω_L [4,7]. The width of the resonance between Ω_m and Ω_L is given by the relaxation rate of the atomic ground-state coherences. When the atomic vapor is contained in a paraffin-coated cell, in which ground-state atomic coherences can survive for as long as on the order of a second, the widths of the resonances can be as small as 0.6 Hz [8]. These resonances allow extremely precise measurements of the magnetic field over a wide field range, with magnetometric sensitivities exceeding 10^{-11} G/$\sqrt{\mathrm{Hz}}$ for low fields [competitive with the best atomic magnetometers (see, for example, Ref. [9]) as well as superconducting quantum interference device magnetometers [10]] and reaching 6×10^{-10} G/$\sqrt{\mathrm{Hz}}$ for higher fields (up to 1 G) [11]. The FM NMOR method used in the present work, as well as AMOR, can be applied to studies of nuclear magnetic resonance and magnetic-resonance imaging [12,13], measurements of geophysical fields [11], and tests of fundamental symmetries [14]. The same approach can also be used in construction of chip-scale atomic magnetometers [15].

Although the magnetometric method based on FM NMOR enables sensitive measurements of the magnetic field, the measurements are scalar, i.e., the position of a given resonance depends only on the magnitude, and not the direction, of the magnetic field. However, the relative magnitudes of the FM NMOR resonances can depend on the magnetic field direction. Thus, a detailed analysis of the FM NMOR signal could give some information about the direc-

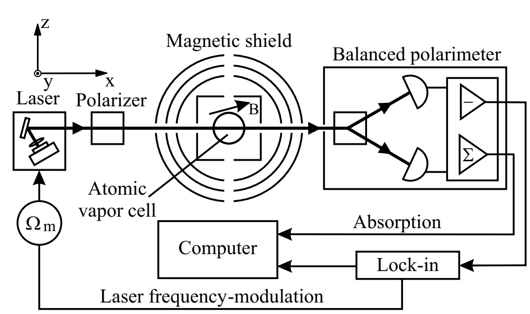

FIG. 1. Experimental setup. The magnetic field coils mounted inside the innermost shielding layer and used for creation of arbitrarily oriented magnetic field are not shown.

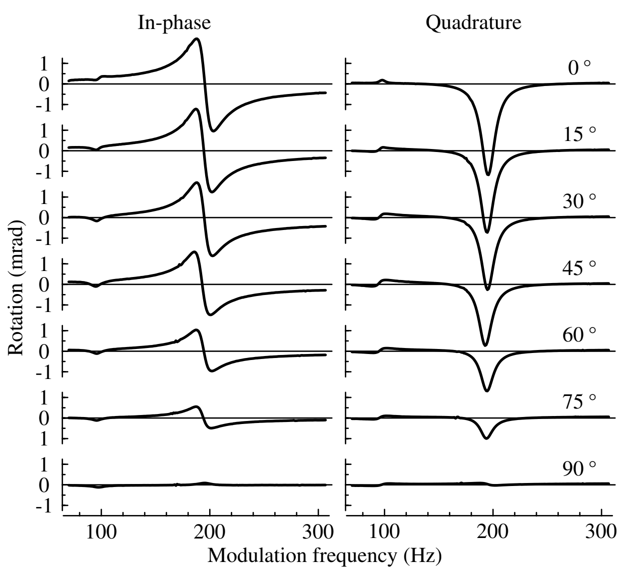

FIG. 2. The FM NMOR in-phase and quadrature signals vs modulation frequency recorded for various angles between the magnetic field and the light-propagation direction in the *xz* plane. A tiny residual signal observed for 90° is a result of a small misalignment in the experiment (the polarization of light is not completely parallel to the *y* axis).

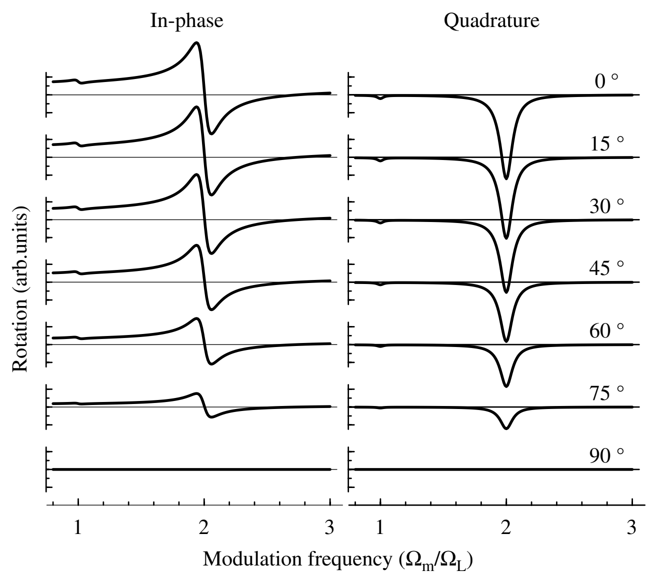

FIG. 4. Theoretical calculation of FM NMOR signal vs modulation frequency for an $F=1 \rightarrow F=0$ transition for various tilt angles of the magnetic field in the *xz* plane.

tion of the magnetic field. Toward this end, we study here the dependence of the FM NMOR signal on the magnetic field direction.

In the Faraday geometry, in which the magnetic field is along the light propagation direction, the main resonance occurs at $\Omega_m = 2\Omega_L$, because of the symmetry of the optically pumped state, as discussed below. We find that when the magnetic field direction is tilted in the plane perpendicular to the light-polarization axis, only this resonance at $2\Omega_L$ is observed, with its amplitude depending on the tilt angle. However, when the magnetic field is tilted toward the light polarization axis, a new resonance appears at Ω_L; the relative magnitudes of the resonances at Ω_L and $2\Omega_L$ depend on the tilt angle.

The scheme of the experiment is shown in Fig. 1. An

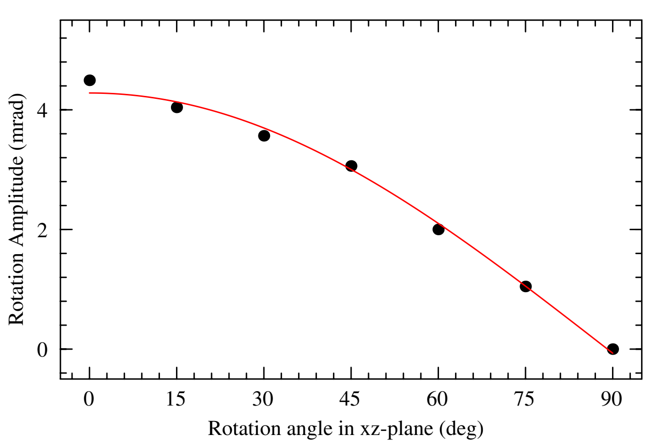

FIG. 3. (Color online) The amplitude of the FM NMOR signals recorded at $2\Omega_L$ vs the tilt angle of the magnetic field in the *xz* plane. The solid line is a cosine fit to the experimental points.

antirelaxation-coated buffer-gas-free vapor cell, containing isotopically enriched ^{85}Rb, was placed within a four-layer magnetic shield. The magnetic shield provided passive attenuation of dc magnetic fields by a factor of 10^6 for all directions [16]. A set of three mutually orthogonal magnetic field coils placed inside the innermost layer enabled compensation of the residual average magnetic field and first-order magnetic field gradients inside the shield (the influence of the magnetic-field inhomogeneities, especially first-order magnetic field gradients, on nonlinear magneto-optical resonances is studied in Ref. [17]). The coils were also used for generation of an arbitrarily oriented magnetic field inside the shield. The rubidium atoms interacted with an *x*-directed, 2-mm-diameter laser light beam, linearly polarized along the *y* axis. An external cavity diode laser was tuned to the rubidium D_1 line (795 nm) and its central frequency was stabilized with a dichroic atomic vapor laser lock [18,19] at the low-frequency wing of the $F=3 \rightarrow F'$ transition. The laser-modulation frequency ranged from 100 to 400 Hz with 300 MHz (peak to peak) modulation depth and the light power was $\sim 3\ \mu$W. Upon passing through the cell, the light was analyzed using a balanced polarimeter. The polarimetric signal was detected with a lock-in amplifier at the first harmonic of Ω_m and stored on a computer.

The FM NMOR signals were studied as the magnetic field direction was tilted in both the *xz* plane (perpendicular to the light-polarization axis) and the *xy* plane (containing the light-propagation vector and light-polarization axis). Figure 2 shows the signals with the magnetic field **B** in the *xz* plane at various angles to the light propagation direction. The strength of the magnetic field was equal for all the measurements ($\Omega_L = 98.5$ Hz). The main resonance occurs at $\Omega_m = 2\Omega_L$. The small resonance at Ω_L is a result of using the modulated pump beam also as a probe. For the frequency-modulated probe the time dependence of the light-

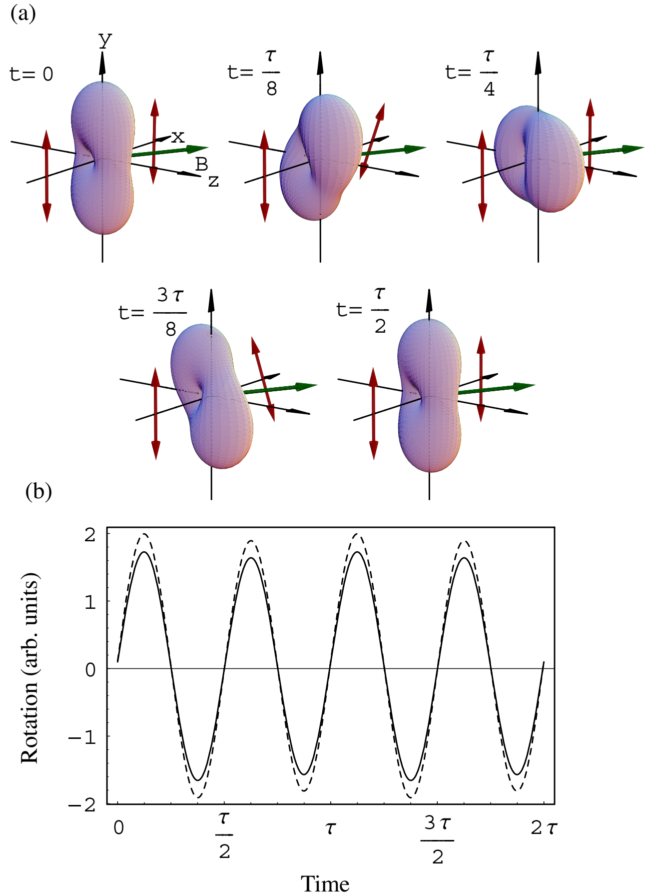

FIG. 5. (Color online) Illustration of the time dependence of FM NMOR for **B** in the xz plane at 30° to the light propagation direction $\hat{\mathbf{x}}$, using results of the calculation for an $F=1\rightarrow F'=0$ transition. (a) Angular momentum probability surfaces depicting the dynamic part of the atomic alignment at various times during the Larmor period τ. The alignment returns to its original position after time $\tau/2$. The double-headed arrows represent the light polarization before and after the medium. (b) Optical rotation as a function of time for the case shown in (a) (solid line), and for **B** along $\hat{\mathbf{x}}$ (dashed line). The dominant signal has period $\tau/2$ (frequency $2\Omega_L$), corresponding to the periodicity of the polarization state shown in (a). The effect of using the modulated pump as a probe beam can be seen here as a small additional modulation with period τ.

polarization plane is observed, even if the atomic polarization in the medium is static. This effect has been analyzed in detail in Ref. [7], Sec. IV. The amplitudes of the resonances decrease with increasing angle in the xz plane (Fig. 3). The experimental signals are in qualitative agreement with the predictions of a density-matrix calculation[1] of FM NMOR for a model $F=1\rightarrow F'=0$ transition (Fig. 4).

The observed dependence on the magnetic field direction

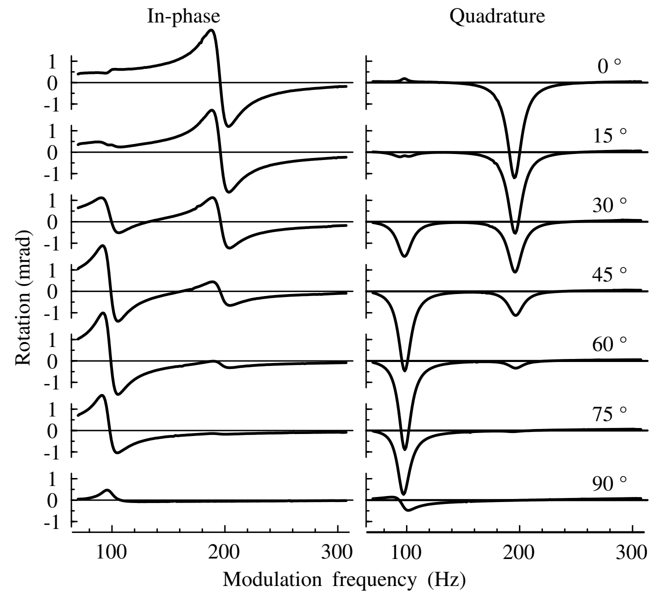

FIG. 6. The FM NMOR signal vs modulation frequency Ω_m recorded for different tilt angles of the magnetic field in the xy plane. We proved with theoretical simulations that the small signal observed at Ω_L for the magnetic field oriented along the y axis is due to the misalignment between the magnetic field and the light polarization direction.

can be understood by considering the time evolution of the atomic polarization. In the absence of a magnetic field, the linearly polarized light creates atomic alignment (polarization with a preferred axis, but no preferred direction) along the y axis. This alignment can be visualized with a method described in, for example, Ref. [20]: a surface is plotted whose radius in a given direction is equal to the probability of finding the maximum projection of angular momentum along that direction. When a magnetic field is applied, the alignment precesses around the magnetic-field direction at the Larmor frequency [Fig. 5(a)]. Since in this case **B** is perpendicular to the alignment axis, the polarization returns to its original state after a 180° rotation, i.e., in half a Larmor period.[2] Thus optical rotation, induced by the rotating linear dichroism, is periodic at twice the Larmor frequency [Fig. 5(b)]. Because only linear dichroism transverse to the light propagation direction (i.e., in the yz plane) can produce rotation, the amplitude of the rotation decreases as the cosine of the angle between **B** and the light-propagation direction, as seen experimentally in Fig. 3.

Figure 6 shows the FM NMOR signals for the magnetic field tilted in the xy plane at various angles to the light propagation direction; corresponding theoretical results for a $F=1\rightarrow F'=0$ transition are shown in Fig. 7. When the magnetic field is parallel to the light propagation direction a strong FM NMOR resonance is observed only at $\Omega_m=2\Omega_L$. However, when the magnetic field is tilted toward the y axis an additional resonance appears at $\Omega_m=\Omega_L$. The amplitude

[1]The optical Bloch equations are solved using the "matrix-continued-fraction method" in the manner of Ref. [21]: the density matrix is expanded in a Fourier series in harmonics of the modulation frequency, and the resulting matrix recursion relation for the Fourier coefficients is inverted as a matrix continued fraction, which is evaluated by assuming a high-harmonic cutoff.

[2]In this work the light power was sufficiently low to exclude effects such as alignment-to-orientation conversion [22] that cause more complicated polarization evolution.

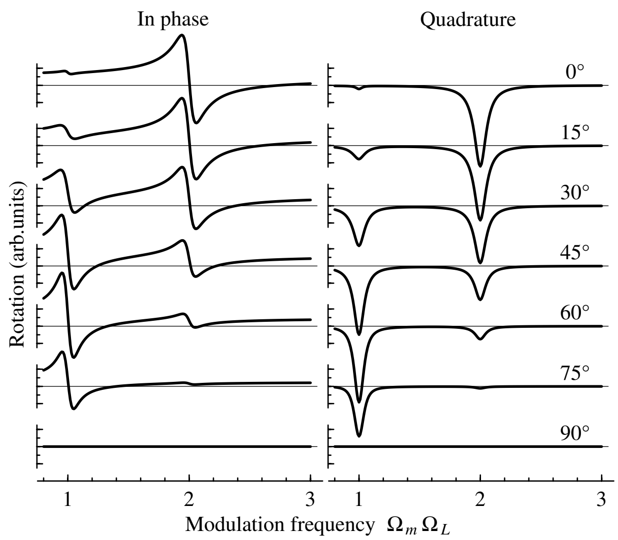

FIG. 7. Theoretical calculation of FM NMOR signal vs modulation frequency for an $F=1\rightarrow F=0$ transition for various tilt angles of the magnetic field in the xy plane.

of this resonance becomes more pronounced with increasing the angle, while the amplitude of the resonance at $2\Omega_L$ decreases (Fig. 8). However, when the magnetic field is tilted by more than 60° the amplitude of the resonance recorded at Ω_L also starts to decrease and reaches zero when the magnetic field is directed along the y axis.

Once again, these signals can be explained by considering the time evolution of atomic polarization [Fig. 9(a)]. Because the magnetic field is no longer perpendicular to the alignment axis, the polarization takes a full Larmor period to return to its original state. The anisotropy of the medium in the yz plane is modulated at two frequencies Ω_L and $2\Omega_L$, as reflected in the plot of time-dependent optical rotation [Fig. 9(b)]. The larger the tilt angle between **B** and the light propagation direction, the bigger the difference between the polarization states at the beginning and middle of the Larmor periods, increasing the signal at Ω_L. However, for large angles, the magnetic field direction is nearly along the alignment axis, reducing the effect of the field on the polarization and thus reducing the optical rotation.

The appearance of the FM NMOR resonance at $\Omega_m=\Omega_L$ can be used for determination of the tilt angle of the magnetic field from the light propagation direction. For small tilts (tilt angle less than 15°), the amplitude of the resonance depends linearly on the angle. The slope of the dependence obtained from the experimental data recorded with the conditions as in Fig. 6 is $37(2)\times10^{-6}$ rad/deg. For a light beam of 3 μW the photon shot-noise limit for the detection of the polarization plane rotation is $\sim1.4\times10^{-7}$ rad/$\sqrt{\mathrm{Hz}}$ one gets that the sensitivity of the tilt angle measurement with our unoptimized system is $3.8(3)\times10^{-3}$ deg/$\sqrt{\mathrm{Hz}}$.

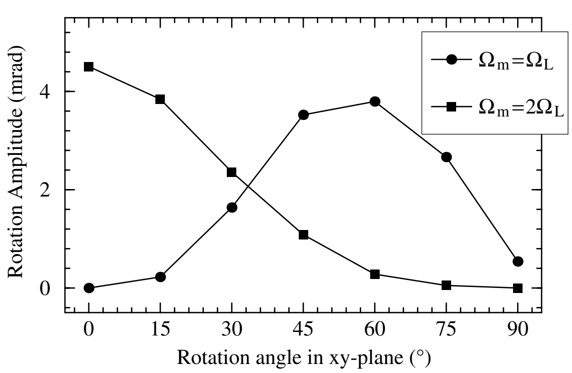

FIG. 8. The amplitude of the FM NMOR signals recorded at Ω_L and $2\Omega_L$ vs the tilt angle of the magnetic field in the xy plane.

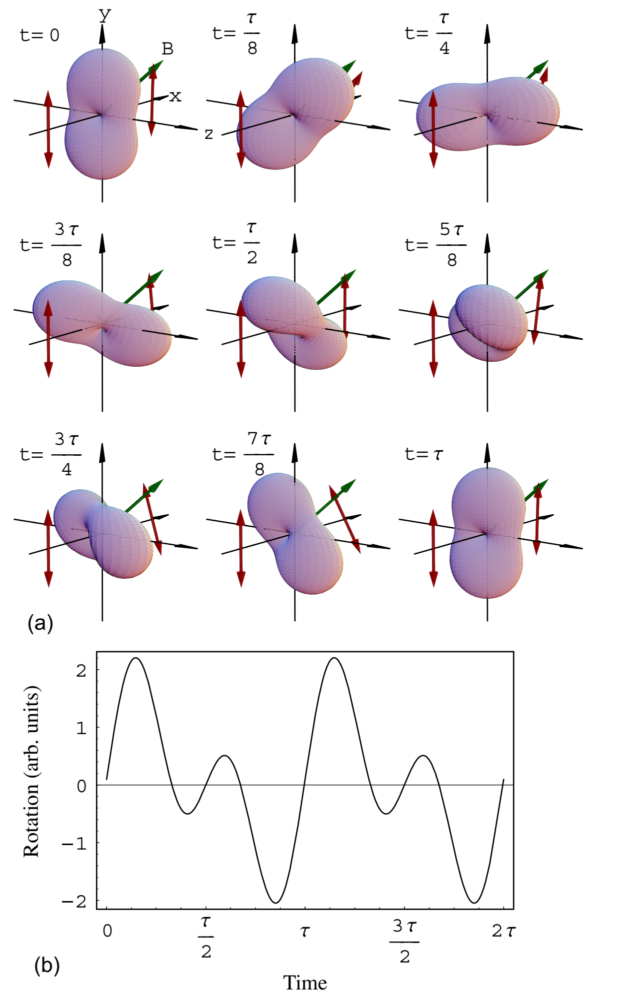

FIG. 9. (Color online) As Fig. 5, but with **B** in the xy plane at 17.5° to the light propagation direction $\hat{\mathbf{x}}$. (a) The alignment axis in this case is not perpendicular to the magnetic field, so it takes a full Larmor period τ to return to its original state. (b) The optical rotation. The optical anisotropy in the yz plane has components at Ω_L and $2\Omega_L$.

An advantage of our magnetometer is direct measurement of the tilt angle of the magnetic field. This is in contrast to the methods where angular information is extracted from independent measurements of three orthogonal components of the field [23,24]. In these methods, since measurement in each channel is performed with a given sensitivity the sensitivity of the measurement of the tilt angle changes with the total strength of the magnetic field. Our method, however, enables the tilt angle measurement with the sensitivity independent on the total strength of the magnetic field.[3]

The data presented in Figs. 2 and 6 can also be understood in the language of atomic coherences. When the magnetic field is in the xz plane, the light-polarization axis is perpendicular to the magnetic field. Thus, for the $F=1\rightarrow F=0$ model and the quantization axis along the magnetic field, the light can create coherence only between the $m=1$ and $m=-1$ Zeeman sublevels. The frequency splitting between these sublevels is $2\Omega_L$, so the resonance is observed at this frequency. However, when the magnetic field is tilted in the xy plane, the light is a linear superposition of polarizations parallel and perpendicular to the magnetic field. In this case, the light can create coherences between sublevels with $\Delta m=1$ and $\Delta m=2$, so resonances are observed at both Ω_L and $2\Omega_L$.

In conclusion, we have studied resonances in nonlinear magneto-optical rotation with frequency modulated light when the magnetic field is tilted at angles away from the direction of light-propagation. When the magnetic field is tilted in the plane defined by the light-polarization and light-propagation vectors, an additional FM NMOR resonance appears at the Larmor frequency. The amplitude of the resonance depends on the tilt angle and thus can be useful for vector magnetometry or for aligning a scalar magnetometer to reduce heading errors. When the magnetic field is tilted in the orthogonal plane, no additional resonance appears at the Larmor frequency. For tilt angles in both planes, the amplitude of the FM NMOR resonance at twice the Larmor frequency decreases with increasing tilt angle. The effects have been modeled with density matrix calculations, yielding good agreement with the experimental observations, and can be understood through visualization [20] of the time dependence of the atomic polarization moments.

[3]A possibility of vector magnetic field measurements similar to the method described in this paper has been recently proposed in Ref. [25].

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