Perfect teleportation and superdense coding with *W* states

Pankaj Agrawal* and Arun Pati†
Institute of Physics, Bhubaneswar-751005, Orissa, India
(Received 8 August 2006; published 26 December 2006)

True tripartite entanglement of the state of a system of three qubits can be classified on the basis of stochastic local operations and classical communications. Such states can be classified into two categories: GHZ states and *W* states. It is known that GHZ states can be used for teleportation and superdense coding, but the prototype *W* state cannot be. However, we show that there is a class of *W* states that can be used for perfect teleportation and superdense coding.

DOI: 10.1103/PhysRevA.74.062320 PACS number(s): 03.67.Hk, 03.65.Ud

I. INTRODUCTION

Quantum teleportation is a prime example of a quantum information processing task where an unknown state can be perfectly transported from one place to another using previously shared entanglement and classical communication between the sender and the receiver. This is a remarkable application of entangled states which has many ramifications in quantum information technology. The original protocol of Bennett *et al.* [1] involves teleportation of an unknown qubit using an Einstein-Podolsky-Rosen (EPR) pair and by sending two bits of classical information from Alice to Bob. They have also generalized the protocol for an unknown qubit using a maximally entangled state in $d \times d$ dimensional Hilbert space and by sending $2 \log_2 d$ bits of classical information. Then, quantum teleportation was generalized to the case where sender and receiver do not have a perfect EPR pair but a noisy channel [2]. Quantum teleportation is also possible for infinite dimensional Hilbert spaces, so called continuous variable quantum teleportation [3,4]. If we do not have a maximally entangled state then one cannot teleport a qubit with unit fidelity and unit probability. However, it is possible to have unit fidelity teleportation but with a probability less than a unit—called probabilistic quantum teleportation [5,6]. Using a nonmaximally entangled basis as a measurement basis this was shown to be possible. Subsequently, this probabilistic scheme has been generalized to teleport N qubits [7]. In the past several years quantum teleportation has been also experimentally verified by various groups [8–10].

Quantum entanglement lies at the heart of quantum teleportation and, in fact, at other host of information processing tasks. Since this refers to the property of multiparticle (two or more) quantum states, a natural question that comes to mind is that in addition to the existing two-particle entangled states whether one can exploit other multiparticle entangled states for quantum teleportation. That one can teleport an unknown qubit using a three particle GHZ state [11] and four particle GHZ state [12] was shown to be possible. Dealing with multiparticle entangled states is not always easy. In the case of three qubits, the class of entangled states have been well studied. There are two types of inequivalent entangled states for three qubits, one is a GHZ type and other is a *W* type. It has been shown by Duer *et al.* [13] that *W* states cannot be converted to GHZ states under stochastic local operation and classical communication (SLOCC). There is an interesting class of three qubit entangled states called zero sum amplitude (ZSA) states which are useful for creating universal entangled states [14]. These are not equivalent to GHZ states. It may be mentioned that the GHZ states are not robust against loss of particles (i.e., if we trace out one particle then there is no entanglement between the remaining two) while *W* states are. If we trace out any one particle from the *W* state, then there is some genuine entanglement between the remaining two. Therefore, in contrast to a GHZ state, a *W* state can still be used as a resource even after the loss of a particle. Recently, the *W* state has been used for quantum key distribution [15] and in illustrating the violation of local realism in an interesting way that is not seen in a GHZ state [16]. Considering the importance of *W* states, there have been various proposals to prepare *W* states in the literature [17–19].

Entangled states not only enhance our ability to do a variety of quantum information processing tasks but they also enhance classical information capacity. For example, in superdense coding if Alice and Bob share an EPR pair, then by sending a qubit Alice can send two classical bits [20]. More generally, if Alice and Bob share a maximally entangled state in a $d \times d$ dimensional Hilbert space, then by sending a qubit Alice can communicate $2 \log_2 d$ classical bits. Thus, a maximally entangled state doubles the classical information capacity of a channel. Similar to quantum teleportation, in recent years, people have exploited other classes of entangled states for dense coding. One can also use a nonmaximally entangled state for superdense coding either in a deterministic fashion [21] or in a probabilistic manner [22,23]. Recently, many people have explored various kinds of quantum channels for deterministic and unambiguous superdense coding [24–26].

The purpose of this paper is to show that there exists a class of *W* state which can be used for perfect quantum teleportation and superdense coding. Earlier, quantum teleportation protocol using the *W* state has been studied [27]. But it was shown that one needs to do a nonlocal operation to recover the unknown state. In another piece of work it has been shown that if one uses the *W* state then the teleportation protocol works with nonunit fidelity, i.e., it is not perfect [28]. Also, it is not possible to recover the unknown state

*Email address: agrawal@iopb.res.in
†Email address: akpati@iopb.res.in

using the W state as a channel. However, in the present work we show that by performing von Neumann projection on the three particles and by sending two bits of classical information one can teleport an unknown qubit perfectly using a class of W states. Furthermore, we show how one can perform superdense coding with the same class of W states. Interestingly, the quantum resource used in teleportation and dense coding protocols presented here with W states is again one bit of shared entanglement between Alice and Bob.

The organization of this paper is as follows. In Sec. II, we discuss a class of W states that are useful for quantum teleportation and dense coding. In Sec. III, we discuss the actual teleportation protocol. For the sake of completeness we also discuss the teleportation protocol using the GHZ state. In Sec. IV, we discuss superdense coding protocol using the W state. Then, the conclusion follows in Sec. V.

II. A CLASS OF *W* STATES

In this section we briefly discuss a class of W states that are useful for perfect quantum teleportation and superdense coding. It may be recalled that although the prototype GHZ state such as this

$$|\mathrm{GHZ}\rangle_{123} = \frac{1}{\sqrt{2}}(|000\rangle_{123} + |111\rangle_{123}) \tag{1}$$

is suitable for perfect teleportation and superdense coding, there are states in its category which are not suitable without the application of SLOCC. Similarly, although the prototype W state

$$|W\rangle_{123} = \frac{1}{\sqrt{3}}(|100\rangle_{123} + |010\rangle_{123} + |001\rangle_{123}) \tag{2}$$

may not be suitable for perfect teleportation and superdense coding, there may exist a class of states within the W states category which are suitable. Below, we give one such example.

The class of states $|W_n\rangle_{123}$ belongs to the category of W states that can be used as an entanglement resource. This is given by

$$|W_n\rangle_{123} = \frac{1}{\sqrt{2+2n}}(|100\rangle_{123} + \sqrt{n}e^{i\gamma}|010\rangle_{123} + \sqrt{n+1}e^{i\delta}|001\rangle_{123}), \tag{3}$$

where n is a real number and γ and δ are phases. As we shall see, this class of W states can be used for perfect teleportation and superdense coding. In particular, if we take $n=1$ for simplicity and set phases to zero, then we have the following W state.

$$|W_1\rangle_{123} = \frac{1}{2}(|100\rangle_{123} + |010\rangle_{123} + \sqrt{2}|001\rangle_{123}). \tag{4}$$

We can check that this state belongs to the category of W states using the criteria given in the Ref. [13]. This state has true tripartite entanglement. This is because the determinants of the reduced density matrices ρ_1,ρ_2, and ρ_3 of this state are not zero. For a state belonging to the GHZ category the three-tangle is nonzero; while it is zero for a state belonging to the W state category. For the state $|W_1\rangle_{123}$ (and $|W_n\rangle_{123}$), the three-tangle is zero. (The prescription to compute a three-tangle is given in Ref. [29]). We also note the concurrences $C_{12}=\frac{1}{2}$, $C_{13}=\frac{1}{\sqrt{2}}$, and $C_{1(23)}=\frac{\sqrt{3}}{2}$. This is because the concurrence $C_{ab}=2\sqrt{\mathrm{Det}\,\rho_a}$ for a pure state of the system "ab." So the inequality of the Ref.[29], $C_{12}^2+C_{13}^2\leqslant C_{1(23)}^2$ is saturated and there is no residual entanglement. This is a characteristic of a state belonging to the W state category.

Another way to argue that the W state is not equivalent to a GHZ state is to compute reduced density matrices ρ_{12},ρ_{23}, or ρ_{13}. We can check that the reduced density matrices obtained from $|W_1\rangle_{123}$ cannot be written as a linear combination of product states, while for the $|\mathrm{GHZ}\rangle_{123}$ they can be. For example, for the $|\mathrm{GHZ}\rangle_{123}$ state we have

$$\rho_{12} = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|), \tag{5}$$

while for the $|W_1\rangle_{123}$ state we have

$$\rho_{12} = \frac{1}{3}(|00\rangle\langle 00| + 2|\psi^+\rangle\langle\psi^+|), \tag{6}$$

where $|\psi^+\rangle=\frac{1}{2}(|00\rangle+|11\rangle)$ is a Bell state. Two such states cannot be equivalent under SLOCC.

We can use either the state given in (3) or (4) for the purpose of perfect quantum teleportation. In our scheme Alice will have two qubits and Bob will have one qubit. Then, by performing a suitable projection on the input qubit and the two qubits, Alice can convey the measurement outcome to Bob, who can then perform suitable local unitary transformation to convert his particle into the input state. One may ask how much entanglement is shared between Alice and Bob? Even though, the shared W state is a three qubit state, but with respect to Alice and Bob partitioning we can imagine this as a bipartite system. Then, we can calculate the von Neumann entropy of the either the reduced subsystem to know how much entanglement there is between the subsystem of Alice and the subsystem of Bob [30]. Now, if we trace the particle "1" and "2" then the reduced density matrix of particle "3" is random mixture for all values of n, i.e., $\rho_3=\mathrm{tr}_{12}(|W_n\rangle\langle W_n|)=I/2$. This shows that the von Neumann entropy of ρ_3 is just one. That is there is one ebit of entanglement shared between Alice and Bob when they use this class of W state. As we will see, by using one ebit of shared entanglement and communication of two classical bits Alice can send a qubit to Bob using the W state as a resource.

III. TELEPORTATION WITH *W* STATES

Let us consider a situation where Alice has particles 1 and 2 and Bob has the particle 3. Alice wishes to teleport the unknown state of a particle "a." She can make a measurement on the three particles "$a12$" and convey her results to Bob via classical communication. Then the question is: What are the states that Alice and Bob can share that can be used as a quantum resource to transfer the state of particle a to particle 3 using usual teleportation protocol?

Before discussing the protocol using a W state as a resource, let us describe how the protocol works for the prototype GHZ state. Suppose that Alice and Bob share a three qubit entangled state $|\mathrm{GHZ}\rangle_{123}$ given by

$$|\mathrm{GHZ}\rangle_{123}=\frac{1}{\sqrt{2}}(|000\rangle_{123}+|111\rangle_{123}). \tag{7}$$

Alice has particles 1 and 2, while Bob has the particle 3. Alice also has a particle a in the following unknown state:

$$|\psi\rangle_a=(\alpha|0\rangle_a+\beta|1\rangle_a). \tag{8}$$

Alice now wishes to transmit this state to Bob. To see how the protocol works, let us consider the combined input and entangled state and rewrite it as follows:

$$\begin{aligned}|\psi\rangle_a|\mathrm{GHZ}\rangle_{123}&=\frac{1}{\sqrt{2}}(\alpha|0\rangle_a+\beta|1\rangle_a)(|000\rangle_{123}+|111\rangle_{123})\\&=\frac{1}{2}[|\psi_1^+\rangle_{a12}(\alpha|0\rangle_3+\beta|1\rangle_3)+|\psi_1^-\rangle_{a12}(\alpha|0\rangle_3\\&\quad-\beta|1\rangle_3)+|\psi_2^+\rangle_{a12}(\beta|0\rangle_3+\alpha|1\rangle_3)\\&\quad+|\psi_2^-\rangle_{a12}(\beta|0\rangle_3-\alpha|1\rangle_3)].\end{aligned} \tag{9}$$

Alice can make a von Neumann type measurement using the states $\{|\psi_1^\pm\rangle,|\psi_2^\pm\rangle\}$, which are given by

$$|\psi_1^\pm\rangle=\frac{1}{\sqrt{2}}(|000\rangle\pm|111\rangle),$$

$$|\psi_2^\pm\rangle=\frac{1}{\sqrt{2}}(|100\rangle\pm|011\rangle). \tag{10}$$

After performing a three-particle measurement, Alice can convey her results to Bob by sending two classical bits of information. Bob can then convert the state of his particle to that of particle a by applying appropriate unitary transformations.

Let us examine the category of W states. The prototype example of this category is given by

$$|W\rangle_{123}=\frac{1}{\sqrt{3}}(|100\rangle_{123}+|010\rangle_{123}+|001\rangle_{123}). \tag{11}$$

This state is not useful as a quantum resource for the usual teleportation protocol [18]. But it turns out that we can use a class of states of the W state category for teleportation. An example of this class of states is given by

$$|W_1\rangle_{123}=\frac{1}{2}(|100\rangle_{123}+|010\rangle_{123}+\sqrt{2}|001\rangle_{123}). \tag{12}$$

Let us suppose that Alice and Bob share a three qubit entangled state $|W_1\rangle_{123}$. Alice has particles 1 and 2, while Bob has the particle 3. Alice also has a particle a in the unknown state (8). Alice now wishes to teleport this state to Bob. To examine its possibility, let us consider the combined input and entangled state and rewrite this as follows:

$$\begin{aligned}|\psi\rangle_a|W_1\rangle_{123}&=\frac{1}{2}(\alpha|0\rangle_a+\beta|1\rangle_a)(|100\rangle_{123}+|010\rangle_{123}\\&\quad+\sqrt{2}|001\rangle_{123})\\&=\frac{1}{2}[\alpha|010\rangle_{a12}|0\rangle_3+\alpha|001\rangle_{a12}|0\rangle_3\\&\quad+\sqrt{2}\alpha|000\rangle_{a12}|1\rangle_3+\beta|110\rangle_{a12}|0\rangle_3\\&\quad+\beta|101\rangle_{a12}|0\rangle_3+\sqrt{2}\beta|100\rangle_{a12}|1\rangle_3]\\&=\frac{1}{2}[\alpha(|010\rangle_{a12}+|001\rangle_{a12})|0\rangle_3+\sqrt{2}\alpha|000\rangle_{a12}|1\rangle_3\\&\quad+\beta(|110\rangle_{a12}+|101\rangle_{a12})|0\rangle_3+\sqrt{2}\beta|100\rangle_{a12}|1\rangle_3]\\&=\frac{1}{2}[|\eta_1^+\rangle_{a12}(\alpha|0\rangle_3+\beta|1\rangle_3)+|\eta_1^-\rangle_{a12}(\alpha|0\rangle_3\\&\quad-\beta|1\rangle_3)+|\xi_1^+\rangle_{a12}(\beta|0\rangle_3+\alpha|1\rangle_3)+|\xi_1^-\rangle_{a12}(\beta|0\rangle_3\\&\quad-\alpha|1\rangle_3)]=\frac{1}{2}[|\eta_1^+\rangle_{a12}\otimes|\psi\rangle_3+|\eta_1^-\rangle_{a12}\otimes\sigma_3|\psi\rangle_3\\&\quad+|\xi_1^+\rangle_{a12}\otimes\sigma_1|\psi\rangle_3+|\xi_1^-\rangle_{a12}\otimes(-i\sigma_2)|\psi\rangle_3].\end{aligned} \tag{13}$$

Here $|\eta_1^\pm\rangle$ and $|\xi_1^\pm\rangle$ are a set of orthogonal states in the W state category given by

$$|\eta_1^\pm\rangle=\frac{1}{2}(|010\rangle+|001\rangle\pm\sqrt{2}|100\rangle), \tag{14}$$

$$|\xi_1^\pm\rangle=\frac{1}{2}(|110\rangle+|101\rangle\pm\sqrt{2}|000\rangle). \tag{15}$$

Alice can now make a von Neumann measurement in a basis that includes the states $\{|\eta_1^\pm\rangle,|\xi_1^\pm\rangle\}$ on the combined system of three particles $a12$. She then sends the result of her measurements using two classical bits to Bob who can apply one of the unitary transformations $\{1,\sigma_1,i\sigma_2,\sigma_3\}$ to convert the state of his particle 3 to that of particle a. For example, if the outcome is $|\xi_1^+\rangle_{a12}$, then Bob has to apply σ_1 to get the desired state $|\psi\rangle_3$. This completes the teleportation protocol using the W state. This protocol consumes one ebit of shared entanglement and communication of two classical bits of information between Alice and Bob.

As mentioned before, the state $|W_1\rangle_{123}$ belongs to the following class of W states that can also be used as an entanglement resource

$$\begin{aligned}|W_n\rangle_{123}=\frac{1}{\sqrt{2+2n}}(&|100\rangle_{123}+\sqrt{n}e^{i\gamma}|010\rangle_{123}\\&+\sqrt{n+1}e^{i\delta}|001\rangle_{123}).\end{aligned} \tag{16}$$

Alice then can use a set of basis vectors for the measurement that includes the vectors:

$$|\eta_n^\pm\rangle=\frac{1}{\sqrt{2+2n}}(|010\rangle+\sqrt{n}e^{i\gamma}|001\rangle\pm\sqrt{n+1}e^{i\delta}|100\rangle), \tag{17}$$

$$|\xi_n^{\pm}\rangle = \frac{1}{\sqrt{2+2n}}(|110\rangle + \sqrt{n}e^{i\gamma}|101\rangle \pm \sqrt{n+1}e^{i\delta}|000\rangle). \tag{18}$$

Using these orthogonal states Alice can carry out the quantum teleportation protocol as given above.

As a remark, the GHZ state, given in (7), cannot only be used for the above situation, but for other scenarios also. We can envisage another situation. Alice, instead of making a three-particle measurement, may wish to make a two particle measurement, followed by a one-particle measurement. Or, there can be three parties: Alice, Bob, and Charlie, each with a qubit. Then Alice makes a two particle measurement on "$a1$" and Bob makes a one particle measurement on the particle 2 and the state of the particle a is teleported to Charlie who makes appropriate unitary transformations on his qubit to change the state of his particle 3 to that of the particle a. The above GHZ state can be used in this scenario as a resource [11,12]; but the above $|W_n\rangle_{123}$ state may not work for this scenario in a straightforward manner.

IV. SUPERDENSE CODING AND *W* STATES

In the original superdense coding scenario, Alice can transmit two classical bits to Bob by sending a qubit if they share a Bell state [20]. First, Alice encodes her message by making suitable unitary transformation on her qubit and then sends back it to Bob. Bob has two qubits at his disposal and can perform von Neumann projections to distinguish four Bell states and retrieve the two classical bits.

In this section we briefly discuss how a GHZ state can be used for superdense coding. If Alice and Bob share such a state then Alice can transmit two classical bits by sending one qubit. We can see this as follows. The prototype GHZ state (7) is same as the state $|\psi_1^+\rangle$. Alice has particle 1 and Bob has the particles 2 and 3. Alice can apply $\{I,\sigma_1,i\sigma_2,\sigma_3\}$ transformations on her qubit

$$\begin{aligned}
|\psi_1^+\rangle &\rightarrow I\otimes I\otimes I|\psi_1^+\rangle = |\psi_1^+\rangle,\\
|\psi_1^+\rangle &\rightarrow \sigma_1\otimes I\otimes I|\psi_1^+\rangle = |\psi_2^+\rangle,\\
|\psi_1^+\rangle &\rightarrow i\sigma_2\otimes I\otimes I|\psi_1^+\rangle = |\psi_2^-\rangle,\\
|\psi_1^+\rangle &\rightarrow \sigma_3\otimes I\otimes I|\psi_1^+\rangle = |\psi_1^-\rangle.
\end{aligned} \tag{19}$$

These transformations convert the original state to a set of four orthogonal states: $\{|\psi_1^{\pm}\rangle,|\psi_2^{\pm}\rangle\}$. After receiving Alice's qubit, Bob can make a three-particle measurement using these orthogonal states to recover two classical bits. However, using the $|W\rangle_{123}$ state Alice cannot transmit two classical bits by sending one qubit by using conventional superdense coding protocol.

The advantage of the state $|W_n\rangle_{123}$ is that using a scheme analogous to the usual superdense coding protocol, Alice can transmit two classical bits to Bob by sending one qubit. For this state, the protocol works as follows: Alice has the qubit 1, while Bob has the qubits 2 and 3. For simplicity, let Alice and Bob share $|\eta_+\rangle_{123}$, which is a $|W_n\rangle_{123}$ type state. Alice can now apply $\{I,\sigma_1,i\sigma_2,\sigma_3\}$ on her qubit 1. This will lead to the set of four orthogonal states as given below:

$$\begin{aligned}
|\eta_1^+\rangle &\rightarrow I\otimes I\otimes I|\eta_1^+\rangle = |\eta_1^+\rangle,\\
|\eta_1^+\rangle &\rightarrow \sigma_1\otimes I\otimes I|\eta_1^+\rangle = |\xi_1^+\rangle,\\
|\eta_1^+\rangle &\rightarrow i\sigma_2\otimes I\otimes I|\eta_1^+\rangle = |\xi_1^-\rangle,\\
|\eta_1^+\rangle &\rightarrow \sigma_3\otimes I\otimes I|\eta_1^+\rangle = |\eta_1^-\rangle,
\end{aligned} \tag{20}$$

where the set of four mutually orthogonal states are $\{|\eta_1^{\pm}\rangle,|\xi_1^{\pm}\rangle\}$. These are defined in (14) and (15).

Now Alice can send her qubit to Bob who makes a three-particle von Neumann measurement using only the orthogonal states $\{|\eta_1^{\pm}\rangle,|\xi_1^{\pm}\rangle\}$. Since these are orthogonal, Bob can perfectly distinguish what operation Alice has applied. In this way he can recover two classical bits of information. Here, it is necessary that Bob knows the shared state in advance; then he will know about the orthogonal states to use for the measurement. More generally, if Alice and Bob have shared the entangled state $|W_n\rangle$ given in (3), then they can use the orthogonal W states $\{|\eta_n^{\pm}\rangle,|\xi_n^{\pm}\rangle\}$ for superdense coding. The superdense coding protocol with W state also uses one ebit of shared entanglement.

The GHZ state (7) can also be used to transmit three classical bits by sending two qubits. This works as follows. Alice has now particles 1 and 2, while Bob has the particle 3. Alice can make unitary transformation on both her particles. Then apart from the transformations given in (19), she can apply the following transformations:

$$\begin{aligned}
|\psi_1^+\rangle &\rightarrow I\otimes \sigma_1\otimes I|\psi_1^+\rangle = |\psi_3^+\rangle,\\
|\psi_1^+\rangle &\rightarrow I\otimes i\sigma_2\otimes I|\psi_1^+\rangle = |\psi_3^-\rangle,\\
|\psi_1^+\rangle &\rightarrow \sigma_1\otimes \sigma_1\otimes I|\psi_1^+\rangle = |\psi_4^+\rangle,\\
|\psi_1^+\rangle &\rightarrow \sigma_1\otimes i\sigma_2\otimes I|\psi_1^+\rangle = |\psi_4^-\rangle,
\end{aligned} \tag{21}$$

where $\{|\psi_3^{\pm}\rangle,|\psi_4^{\pm}\rangle\}$ are given by

$$\begin{aligned}
|\psi_3^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|010\rangle \pm |101\rangle),\\
|\psi_4^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|110\rangle \pm |001\rangle).
\end{aligned} \tag{22}$$

Now Alice can send her two qubits to Bob who makes a von Neumann measurement using the orthogonal set $\{|\psi_1^{\pm}\rangle,|\psi_2^{\pm}\rangle,|\psi_3^{\pm}\rangle,|\psi_4^{\pm}\rangle\}$. This measurement will yield three classical bits of information. Unfortunately, it may not be possible to use the W states $|W_n\rangle_{123}$ to transmit three classical bits of information. But, it is possible that some other class of W states exist which can be used for such a task.

As earlier, we would note that $|W_n\rangle_{123}$ may not be the only class of W states that allow superdense coding. If one explores further, specifically those W states that are of a linear superposition of more than three states, one may find W states which may even allow Alice to transmit three classical bits by sending two qubits. This is worth exploring in future.

V. CONCLUSIONS

In this paper we have shown that there exists a class of W states which are useful for quantum teleportation. The scheme presented here works analogously to the original protocol. The only difference is that Alice needs to carry out a three-qubit von Neumann projection instead of a Bell-state measurement. Interestingly, the resource used for teleporting an unknown qubit using the W state is one shared ebit and communication of two classical bits. This reassures one that we cannot outperform quantum teleportation of a qubit with less resource. The original protocol also uses one shared ebit and two classical bits. Furthermore, we have shown that one can perform a standard superdense coding scheme using a W state. Again, here by sharing one ebit and sending a qubit Alice can communicate two classical bits to Bob. So instead of using an EPR pair for quantum teleportation and dense coding one can use a W_n state as well for the above purpose. In the future, one can investigate if there are other classes of W states useful for quantum teleportation and superdense coding. One can also study if there are classes of W states that can be used to send three classical bits by sending two qubits. One can also investigate the possibility of teleporting an unknown state by making two-qubit and one-qubit measurements instead of a three-qubit measurement with a class of W states as a resource.

[1] C. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wooters, Phys. Rev. Lett. **70**, 1895 (1993).

[2] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A **53**, 2046 (1996).

[3] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).

[4] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).

[5] P. Agrawal and A. K. Pati, Phys. Lett. A **305**, 12 (2002).

[6] A. K. Pati and P. Agrawal, J. Opt. B: Quantum Semiclassical Opt. **6**, S844 (2004).

[7] G. Gordon and G. Rigolin, Phys. Rev. A **73**, 042309 (2006).

[8] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).

[9] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).

[10] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).

[11] A. Karlsson and M. Bourennane, Phys. Rev. A **58**, 4394 (1998).

[12] A. K. Pati, Phys. Rev. A **61**, 022308 (2000).

[13] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).

[14] A. K. Pati, Pramana, J. Phys. **59**, 217 (2002).

[15] J. Joo, J. Lee, J. Jang, and Y-J. Park, e-print quant-ph/0204003.

[16] A. Cabello, Phys. Rev. A **65**, 032108 (2002).

[17] G.-C. Guo and Y.-S. Zhang, Phys. Rev. A **65**, 054302 (2002).

[18] V. N. Gorbachev, A. I. Trubilko, and A. A. Rodichkina, Phys. Lett. A **314**, 267 (2003).

[19] A. Biswas and G. S. Agarwal, J. Mod. Opt. **51**, 1627 (2004).

[20] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).

[21] P. Hausladen, R. Jozsa, B. Schumacher, M. Westmoreland, and W. K. Wootters, Phys. Rev. A **54**, 1869 (1996).

[22] J. C. Hao, C. F. Li, and G. C. Guo, Phys. Lett. A **278**, 113 (2000).

[23] A. K. Pati, P. Parashar, and P. Agrawal, Phys. Rev. A **72**, 012329 (2005).

[24] S. Wu, S. M. Cohen, Y. Sun, and R. B. Griffiths, Phys. Rev. A **73**, 042311 (2006).

[25] Q. B. Fan and S. Zhang, Phys. Lett. A **348**, 160 (2006).

[26] Y. Feng, R. Duan, and Z. Ji, Phys. Rev. A **74**, 012310 (2006).

[27] V. N. Gorbachev, A. A. Rodichkina, and A. I. Trubilko, Phys. Lett. A **310**, 339 (2003).

[28] J. Joo, Y-J. Park, S. Oh, and J. Kim, New J. Phys. **5**, 136 (2003).

[29] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).

[30] S. Popescu and D. Rohrlich, Phys. Rev. A **56**, R3319 (1997).