Suppression of Zeno effect for distant detectors

F. Delgado,[1,2,*] J. G. Muga,[2,†] and G. García-Calderón[2,‡]
[1]*Departamento de Física Básica, Universidad de La Laguna, La Laguna, Tenerife, Spain*
[2]*Departamento de Química-Física, Universidad del País Vasco, Apdo. 644, 48080 Bilbao, Spain*
(Received 26 July 2006; published 5 December 2006)

We describe the influence of continuous measurement in a decaying system and the role of the distance from the detector to the initial location of the system. The detector is modeled first by a step absorbing potential. For a close and strong detector, the decay rate of the system is reduced; weaker detectors do not modify the exponential decay rate but suppress the long-time deviations above a coupling threshold. Nevertheless, these perturbing effects of measurement disappear by increasing the distance between the initial state and the detector, as well as by improving the efficiency of the detector.

DOI: 10.1103/PhysRevA.74.062102 PACS number(s): 03.65.Xp, 03.65.Ta, 03.65.Nk

I. INTRODUCTION

The decay of unstable quantum states is an ubiquitous process in virtually all fields of physics and energy ranges, from particle and nuclear physics to condensed matter, or atomic and molecular science. The exponential decay, by far the most common type, is surrounded by deviations at short and long times [1,2]. The short-time deviations have been much discussed, in particular in connection with the Zeno effect [3–5] and the anti-zeno effect [6–9]. Experimental observations of short [10,11] and long [12] time deviations are very recent. A difficulty for the experimental verification of long-time deviations has been the weakness of the decaying signal [13], but also the measurement itself may be responsible, because of the suppression of the initial state reconstruction [2,14].

It was soon recognized that the measurement could perturb in an important way the dynamics, not only at long times but also at short times, and that even the rate of exponential decay in the intermediate regime could be affected [2]. A related and interesting issue is the *quantum zeno paradox*: repeated instantaneous measurements over a decaying system freeze the decay as the period tends to zero, if the projection postulate is applied. The same conclusions hold in some limits when "quantum measurement theory," which incorporates the measuring apparatus or part of it in the theoretical model [15–18], is used for describing generalized (noninstantaneous and nonideal) measurements. Several works have analyzed the conditions for the existence of zeno and antizeno effects in unstable systems from the point of view of the spectral properties of the response function of the detector [18–20]. For a detailed discussion about quantum zeno and antizeno effects in a generalized sense (i.e., the slow down or speed up of the decay for generalized measurements including continuous ones), see a complete review by Koshino and Shimizu [18].

A still controversial and rather crucial question is: how is the decay affected by the distance between detector and system in indirect measurements [18,21–28]? Home and Whitaker in their conceptual analysis of the zeno effect [21], considered that the only really paradoxical point is that the system is predicted to have its decay affected by a detector at a macroscopic distance. Indeed, a common sense expectation is that separating the detector from the initial location of the system will soften the perturbing effects of measurement, making the zeno effect eventually irrelevant, but the results based on some measurement models showing this fact have been disputed [24,26,27], and the need for more work has been stated to arrive at more definite conclusions [18].

In the present paper the decaying particle is initially localized within an interaction potential region, and the effect of the "continuous" detection is modeled by an imaginary absorbing potential which accounts for the passage from the initial channel to some other channels which are not represented explicitly [29]. A physical system that may be represented in this way is an atom detected by the fluorescence induced by an on-resonance laser beam [30]. In this case, the wave function describes undetected atoms in the ground state, and every detected atom (by the first spontaneous photon detection) ceases to be part of the statistical ensemble associated with the wave function. Moreover, the rate of norm loss becomes equal to the detection rate [32]. Two helpful approximations for an analytical treatment are a sharply defined beam edge, and the substitution of the actual beam width by a semi-infinite potential [30]. The physical validity of this approximation was studied in Ref. [31], and requires that the penetration length of the undetected atom amplitude be smaller than the laser beam width. This condition and the production of sharp borders are well within the scope of current ultracold atom experiments.

We shall describe the main effects of the absorption (or, as discussed previously, detection), in the decay process of a quantum system. In particular, we will show that the slow down of the decay, i.e., the generalized zeno effect, disappears when the distance to the detector is increased. We shall also analyze the suppression of the deviations from exponential decay at long times as a function of the strength and quality of the absorber and the influence of the "observation distance."

*Electronic address: qfbdeacf@lg.ehu.es
†Electronic address: jg.muga@ehu.es
‡Permanent address: Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20 364, 01000 México, D.F., México. Electronic address: gaston@fisica.unam.mx

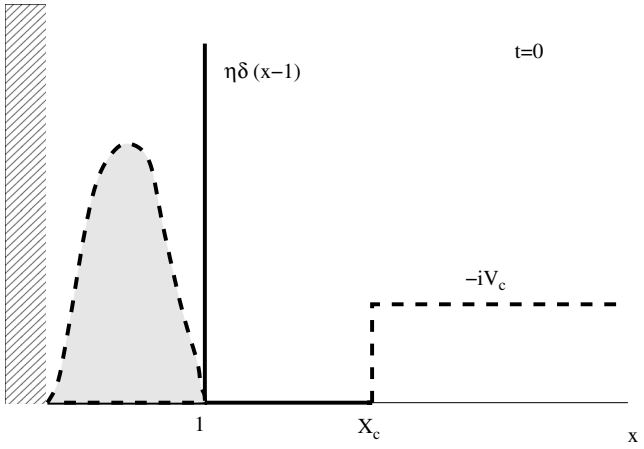

FIG. 1. Schematic representation of the model including a hard wall at $x=0$, a delta function potential at $x=1$ and a detector for $x \geqslant X_c$. The wave packet is initially confined into the inner region [0, 1].

II. MODEL

Our model represents an atom in one dimension which is initially located between an infinite wall at $x=0$ and a delta barrier at $x=1$ of "strength" $\eta > 0$. In absence of the detector, the wave function will leak out and decay exponentially except for short and long time deviations. The dimensionless Hamiltonian including the detector is

$$H = -\frac{\partial^2}{\partial x^2} + \eta\delta(x-1) - iV_c\Theta(x-X_c), \quad x \geqslant 0, \tag{1}$$

where Θ is the Heaviside function, X_c is the position of the detector edge, and $V_c \geqslant 0$ as corresponds to absorption. The norm of a wave function put in the absorbing region, for the atom at rest, would decay with a lifetime, or response time of the detector, $1/(2V_c)$. Because of the position dependence, however, a very large V_c leads to reflection without detection. Boundary effects can be to a large extent avoided, as we shall see below, with an adequate shaping of the potential [29].

We assume that the initial state at $t=0$ is the ground state of an infinite well between $x=0$ and $x=1$, namely,

$$\psi(x,0) = \sqrt{2}\sin(\pi x)\Theta(x)\Theta(1-x), \tag{2}$$

see Fig. 1 for a schematic representation. Hence, the system evolves with time according to the non-Hermitian Hamiltonian of Eq. (1), $\psi(t) = e^{-iHt}\psi(0)$. The survival amplitude, defined as $A(t) = \langle\psi(0)|\psi(t)\rangle$, may be written in terms of the eigenstates forming a biorthogonal basis [33],

$$A(t) = \sum_{l=1}^{N_{\text{loc}}} \mathcal{C}_l\hat{\mathcal{C}}_l e^{-iE_l t} + \int_0^\infty f(q)e^{-i(q^2-iV_c)t}dq, \tag{3}$$

where $\mathcal{C}_l = \langle\psi_0|u_l\rangle$, $\hat{\mathcal{C}}_l = \langle\hat{u}_l|\psi_0\rangle$, $f(q) = \langle\psi_0|\phi_q\rangle\langle\hat{\phi}_q|\psi_0\rangle$, and $|u_l\rangle$ and $|\hat{u}_l\rangle$ are, respectively, right and left localized eigenstates obeying

$$H|u_l\rangle = E_l|u_l\rangle = k_l^2|u_l\rangle, \tag{4}$$

$$\langle\hat{u}_l|H = E_l\langle\hat{u}_l| = k_l^2\langle\hat{u}_l|, \tag{5}$$

$$\langle u_l|\hat{u}_j\rangle = \delta_{l,j}, \tag{6}$$

$\delta_{l,j}$ being the Kronecker delta and N_{loc} the total number of localized, (Kronecker) normalizable states. The continuum eigenstates appearing above, $|\phi_q\rangle$ and $|\hat{\phi}_q\rangle$, satisfy

$$H|\phi_q\rangle = E_q|\phi_q\rangle = (q^2 - iV_c)|\phi_q\rangle, \tag{7}$$

$$\langle\hat{\phi}_q|H = E_q\langle\hat{\phi}_q| = (q^2 - iV_c)\langle\hat{\phi}_q|, \tag{8}$$

$$\langle\phi_q|\hat{\phi}_{q'}\rangle = \delta(q-q'). \tag{9}$$

Note that $|\phi_q\rangle$ and its corresponding biorthogonal partner are not usual scattering states because the exterior region is not free from interaction [$V(x) \neq 0$ when $x \to \infty$]. However, the potential is constant there and this enables us to write the solution in the external region in terms of a $\mathcal{S}$ matrix,

$$\phi_q(x) = \frac{1}{(2\pi)^{1/2}}\begin{cases} C_1 \sin kx, & 0 \leqslant x \leqslant 1, \\ Ae^{ikx} + Be^{-ikx}, & 1 \leqslant x \leqslant X_c, \\ e^{-iqx} - \mathcal{S}(q)e^{iqx}, & x \geqslant X_c, \end{cases} \tag{10}$$

where $k = (q^2 - iV_c)^{1/2}$ is the wave number inside, q the wave number outside, and C_1, A, B, and $\mathcal{S}$ are obtained from the matching conditions at $x=1$ and $x=X_c$. For scatteringlike solutions, q is positive. Note the two branch points of k in the complex q plane. We shall take the branch cut joining these points. Similarly, the root in $q = (k^2 + iV_c)^{1/2}$ is defined with a branch cut joining the two branch points in the k plane. In contrast to scattering-like states of the continuum, localized states are characterized by a complex q with positive imaginary part.

III. RESULTS

We shall start analyzing the effects of detection when the detector is placed close to the unstable system, at $X_c = 1$. We have chosen $\eta = 5$ to facilitate the calculations since small values of η lead to smaller lifetimes and a badly defined exponential regime whereas, by contrast, very large values of η are associated with long lifetimes and well-defined (narrow and isolated) resonances, but the numerical integration of Eq. (3) becomes much more complicated. The quantity investigated here and later for other values of X_c is the survival probability $S(t) = |A(t)|^2$ [36]. Experimentally, the probability $P_{[0,1]}$ to find the particle in the initial region [0, 1] could be more accessible, but $S(t)$ is much easier to compute and the differences with $P_{[0,1]}$ are generally very minor.

In Figs. 2 and 3, we have plotted the logarithm of the survival probability versus time for different values of the complex absorbing potential V_c. For increasing V_c, below a threshold, there is a continuous shift to higher values of the transition time, t_{trans}, which marks the passage from the exponential dominated regime to the final nonexponential decay. Beyond the threshold value, $V_c^{\text{thre}} \approx 0.926$, the decay does not present apparently any deviation from the exponential decay.

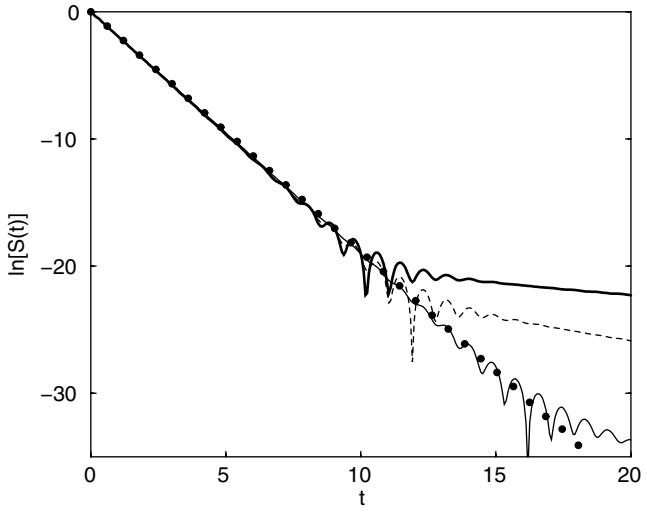

FIG. 2. $\ln[S(t)]$ for different absorptive step potentials, see Eq. (1). $V_c=0$ (thick solid line), 0.1 (dashed line), 0.3 (thin solid line), and 0.5 (dots). $X_c=1$ and $\eta=5$.

A quantitative approximation to $S(t)$ helps to understand these effects: Let q_r be the resonance with the longest lifetime and let us assume that it is narrow and isolated. If the rest of the resonances have already decayed, for weak enough absorption, i.e., $N_{\mathrm{loc}}=0$, the integral of Eq. (3) can be approximated, using contour deformation in the complex q plane, by the residue corresponding to the first resonance plus a saddle contribution,

$$A(t) \approx -2\pi i \,\mathrm{Res}[f(q)]_{q=q_r} e^{-i\mathcal{E}_r t} e^{-\Gamma_r t/2} - \frac{\sqrt{\pi i}}{8} \ddot{f}(0) e^{-V_c t} \frac{1}{t^{3/2}}, \tag{11}$$

where $\mathcal{E}_r$ represents the energy of the decaying particle, Γ_r the corresponding decaying width and $\ddot{f}(0)=[d^2 f(q)/dq^2]_{q=0}$.

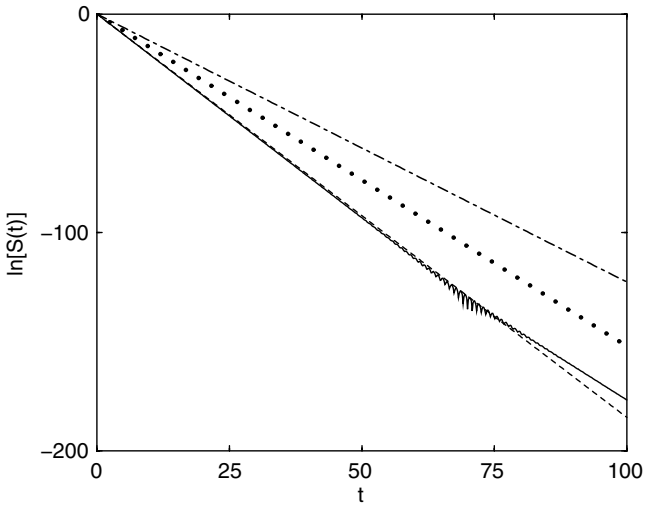

FIG. 3. $\ln[S(t)]$ for stronger absorptive potentials (compared to Fig. 2): $V_c=0.75$ (solid line, still with a visible long-time deviation), 1 (dashed line), 10 (dots), and 100 (dotted-dashed line). $X_c=1$ and $\eta=5$, as in Fig. 2.

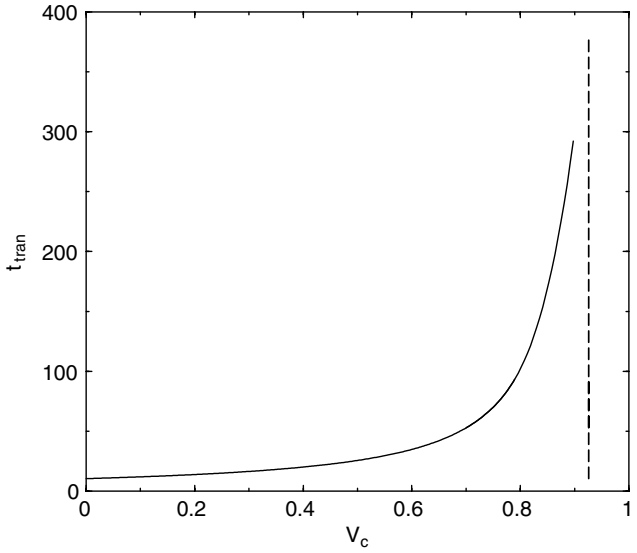

FIG. 4. Transition time t_{tran} versus V_c for $\eta=5$ and $X_c=1$. The critical value V_c^{thre} is marked with a vertical dashed line.

The second term is responsible for the deviation from the exponential decay in $S(t)$. The difference with respect to the nonabsorption case, $V_c=0$, is that the deviation is not given by a purely algebraic term: the usual algebraic dependence is multiplied by the exponentially decaying factor $\exp(-V_c t)$. By increasing V_c, the deviation term decays more and more rapidly until, at threshold, i.e., $\Gamma_r=V_c$, the deviation decays faster than the residue term. This threshold value corresponds exactly to the passage from a resonance to a localized, normalizable state with purely exponential decay. While for $V_c<V_c^{\mathrm{thre}}$, the dominant term at long times is the saddle contribution [proportional to $\exp(-V_c t)t^{-3/2}$], in the opposite case, $V_c>V_c^{\mathrm{thre}}$, the decay is purely exponential, see Fig. 3, and the dominant contribution comes from the discrete part of the spectrum. This peculiar behavior can be also observed in the divergence of t_{tran} versus V_c at V_c^{thre}, see Fig. 4. If the absorption is increased further, see Fig. 3, the decay rate decreases, an evidence of a generalized zeno effect.

We shall next examine how the above perturbing effects of measurement are affected by the distance to the absorber. In Fig. 5 we show the dependence of $\ln[S(t)]$ with X_c for $V_c=100$, compared to the reference case $V_c=0$. For $X_c=1$, the decay is slowed down with respect to the exponential decay for $V_c=0$, as in the cases shown in Fig. 2. A small increase of the distance to $X_c=2$ leads, perhaps counterintuitively, to an even slower exponential decay, but only after an oscillatory transient at short times. This is explained by the approach of a second pole and their mutual interference. By increasing X_c the poles move closer and closer to each other and a simple analysis in terms of one or few poles becomes soon impossible. For larger values of X_c the zeno effect disappears, namely, the exponential decay rate is, at least initially, the same as for $V_c=0$, and the transition time t_{tran} separating the exponential decay region and the long-time deviation increases with X_c, and approaches the transition time for $V_c=0$. Beyond the time region in which the curves with and without absorption agree, there are some

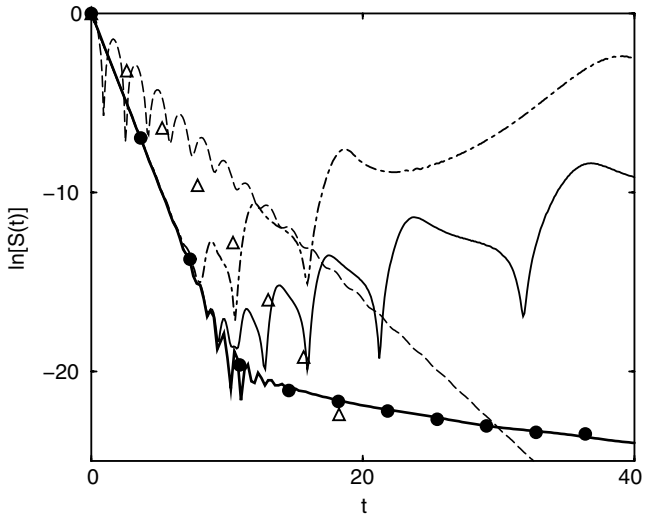

FIG. 5. $\ln[S(t)]$ versus time for $\eta=5$, $V_c=100$, and different values of X_c: 1 (triangles), 2 (thick dashed line), 100 (dotted-dashed line), and 200 (thin solid line). The reference curve for $V_c=0$ (thick solid line) is also shown. The dots correspond to a better detector placed at $X_c=100$, see Eq. (12) and related text, with $E_{\min}=0.003$ and $L=100$.

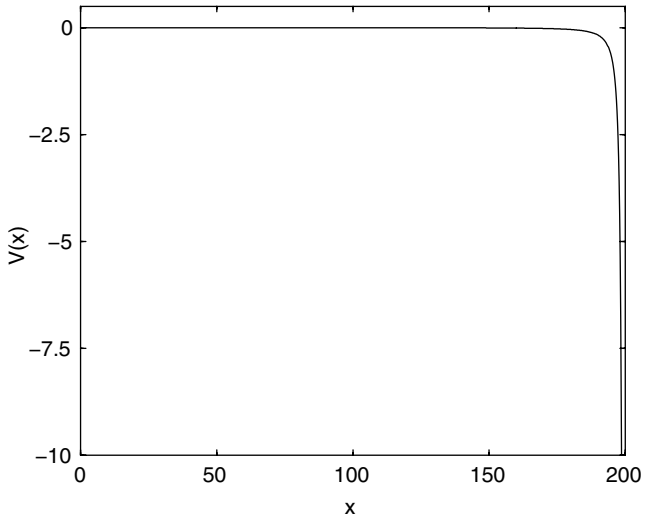

FIG. 6. Manolopoulos potential for $X_c=100$, see Eq. (12) and related text, with $E_{\min}=0.003$ and $L=100$.

important oscillations in $\ln[S(t)]$, and a return to the initial state due to the reflectivity at low energies of the sharply edged detector model. To show the importance of the detector quality we have also used a better absorber, namely, the potential proposed by Manolopoulos [29,34],

$$V^M(x) = -iE_{\min}\Theta(x-X_c)\vartheta\left(\frac{c(x-X_c)}{L}\right), \tag{12}$$

where L is the absorption width and

$$\vartheta(y) = ay - by^3 + \frac{4}{(c-y)^2} - \frac{4}{(c+y)^2},$$

with $a \approx 0.112\,45$, $b \approx 8.287\,72 \times 10^{-3}$, and $c \approx 2.622\,06$. For this new model, the survival probability curves (an example is shown in Fig. 5, see the form of the potential in Fig. 6), fit to the unperturbed curve for a longer time and the back-reaction of the measurement apparatus on the system is much reduced.

Absence of antizeno effect

The exponential decay for the values of V_c chosen in Figs. 3 and 5 is slower than for $V_c=0$. We have carried out a more systematic calculation, sweeping continuously over V_c: Fig. 7 shows the lifetime of the dominant exponential decay versus V_c and clearly no antizeno effect is observed. This may appear contradictory with the fact that the survival probability for the case $V_c=0$ decays indeed faster than purely exponential decay in several time spans, see Fig. 8. Continuous measurements are usually related to repeated instantaneous measurements of period δt by means of Schulman's relation [35]

$$\delta t = 4\tau_0, \tag{13}$$

where τ_0 is the response time of the apparatus. Since in the time regions of faster decay, the antizeno effect occurs with repeated instantaneous measurements [9], this relation suggests that the continuous measurement should lead to an antizeno effect for some value of the interaction. That this is not the case shows that Eq. (13) is not directly applicable in this case. The model Hamiltonian considered by Schulman [35] and ours have different forms and parameters, see the Appendix, in particular there is no x, X_c or η dependency in Ref. [35], where a multichannel model is considered, whereas ours refer to a one-channel treatment and, as stated in Ref. [35], "the forms taken by continuous observation are many, so that specific conclusions depend on the model of observation."

IV. CONCLUDING REMARKS

We have studied the influence of the detector, modeled by a "step" negative imaginary potential, in an unstable decay-

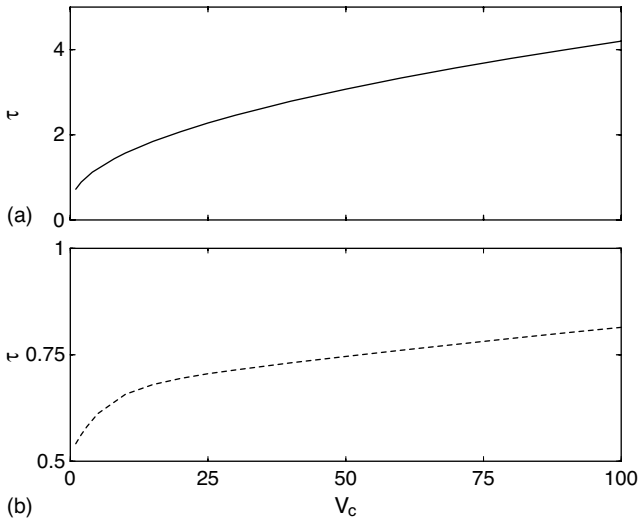

FIG. 7. Monotonic increase of the lifetime versus V_c for $\eta=5$ and two detector distances: $X_c=3$ (a); $X_c=1$ (b).

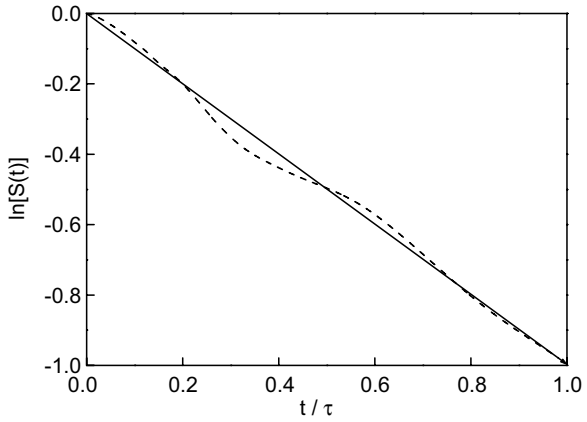

FIG. 8. Behavior of the survival probability versus time in units of the lifetime τ at short times for $\eta=5$ and $V_c=0$ (dashed line). The continuous line exhibits, for comparison, pure exponential decay.

ing system. One of the effects of absorption is the suppression of the deviations from exponential decay at long times, but only above a critical value of the absorption potential and if it occurs close to the system. The slow down of the decay rate for strong absorption (zeno effect) and its dependence with the distance to the absorber have also been examined: the change in the decay rate due to the continuous measurement is washed out by increasing the distance to the detector. The perturbing effects of measurement are also reduced by more efficient, reflectionless detectors.

ACKNOWLEDGMENTS

The authors thank L. Schulman for reading and commenting on the paper. This work has been supported by Ministerio de Educación y Ciencia (BFM2003-01003), and UPV-EHU (00039.310-15968/2004). One of the authors (G. G.-C.) acknowledges partial financial support from El Ministerio de Educación y Ciencia, Spain, under Grant No. SAB2004-0010.

APPENDIX: CONTINUOUS VERSUS PULSED OBSERVATIONS

To see possible relations and differences between Ref. [35] and our model let us briefly review the origin of Eq. (13) in the simplest model of Ref. [35]. Assume a Hamiltonian of the form

$$H_S = \frac{1}{2}\begin{pmatrix} 0 & \Omega \\ \Omega & -i\gamma \end{pmatrix} \tag{14}$$

which may be interpreted as an effective Hamiltonian describing the evolution of undetected two-level atoms (with levels $|1\rangle$ and $|2\rangle$) in a laser field with Rabi frequency Ω and Einstein coefficient γ, equal to the inverse lifetime τ_0 of state $|2\rangle$. As in the main text all quantities are dimensionless. Because of the laser driving, the effective decay time is actually $\tau_{EC}=\gamma/\Omega^2$ in the weak coupling ("driving") limit with large γ/Ω. (Note that in the strong coupling limit the effective decay time is very different, $\tau_{EC}=2/\gamma$, see, e.g., Ref. [32].) A zeno time $\tau_Z=2/\Omega$ is also introduced as the coefficient for quadratic time dependence that arises in the short time expansion of the probability of state $|1\rangle$ evolved with H_S,

$$P_1 = 1 - t^2/\tau_Z^2 + \ldots . \tag{15}$$

With the above definitions, the effective lifetime for continuous observation becomes, for weak coupling, $\tau_{EC}=\tau_Z^2/4\tau_0$.

The next step is to consider a pulsed, rather than continuous, measurement at time intervals δt with $\gamma=0$, represented by projections

$$|\psi\rangle\langle\psi| \to |1\rangle\langle 1|\psi\rangle\langle\psi|1\rangle\langle 1|. \tag{16}$$

The probability of finding state $|1\rangle$ at δt is $P_1(\delta t)=1-(\delta t/\tau_Z)^2$, and after successive pulses an effective exponential decay will take approximately the form $\exp(-t/\tau_{EP})$, with $\tau_{EP}=\tau_Z^2/\delta t$ being the effective lifetime for pulsed observations. Comparing with the decay of the continuous case and setting $\tau_{EC}=\tau_{EP}$, we get Schulman's relation

$$\delta t = 4\tau_0. \tag{17}$$

One may first try a rough fit of our model into the form of the Hamiltonian (14) by identifying state $|1\rangle$ with the initial state, and $|2\rangle$ with a coupled decay product state with lifetime $\tau_0=1/(2V_c)$. Presumably Ω would depend on η in such a way that a weak coupling corresponds now to an intense, delta barrier, i.e., a large η. A difficulty to establish this parallelism is that the distance X_c-1 between the initial state and the detector introduces a further time scale, the time required to travel up to the detector. Limiting the analysis to the case $X_c=1$, in which the edge of the state and the complex potential touch at $x=1$, is not enough for a simple translation of results: first because the assumed short time quadratic dependence does not hold at very short times, since the second moment of the energy does not exist for our truncated-sine initial state (2); and second because a relatively small value for the delta strength such as $\eta=5$ is not in a weak coupling regime, again one of the premises in Ref. [35]. This may be clearly seen combining Figs. 7(b) and 8: weak coupling is characterized by a linear dependence of τ with V_c, and this corresponds, approximately to $V_c>25$ in Fig. 7(b), but according to Fig. 8, the times required to see antizeno are between 0.1 and 0.2 (note that $\tau\approx 0.5$), which, assuming $\tau_0=1/(2V_c)$ and Eq. (17) gives $10<V_c<20$, a range of values below the weak coupling regime.

One could of course start anew and compare continuous (complex potential) and pulsed measurements in our model without any reference to the above two-channel Hamiltonian. This is done below, and the result confirms that continuous and pulsed observations cannot be accurately related here by Eq. (13).

We shall first define the following pulsed observation for our delta-potential model: assume a "chopping process" which amounts to a periodic projection of the wave function onto the $x<X_c$ region at instants separated by a time interval δt. If we denote by $\psi(x,t_{j-})$ the wave function immediately before, and by $\psi(x,t_{j+})$ the one immediately after the projection at the instant t_j,

$$\psi(x,t_{j+}) = \psi(x,t_{j-})\Theta(X_c - x). \tag{18}$$

A rapid cancellation of the $x > X_c$ part may also be achieved with the "kicked" imaginary (absorbing) and time-dependent potential

$$V_k = V\delta t F_{\delta t}(t), \tag{19}$$

where the subscript "k" stands for "kicked" and

$$F_{\delta t}(t) = \sum_{j=-\infty}^{\infty} \delta[t - (2j+1)\delta t/2], \tag{20}$$

$$V = -iV_c\Theta(x - X_c), \tag{21}$$

provided

$$V_c\delta t \gg 1. \tag{22}$$

The general (and exact) evolution operator between intermediate times is obtained by repetition of the basic unit

$$U_k(0,\delta t) = e^{-iH_0\delta t}e^{-iV\delta t}, \tag{23}$$

where $H_0 = -\partial^2/\partial x^2 + \eta\delta(x-1)$ is the kinetic energy operator plus the delta barrier. The reason for having introduced δt (and no other quantity with dimensions of time) in Eq. (19) should now become clear. Compare Eq. (23) with the evolution under the *continuous* (time independent) imaginary potential (21),

$$U(0,\delta t) = e^{-i(H_0+V)\delta t/\hbar} = e^{-iH_0\delta t}e^{-iV\delta t} + O(\delta t^2[V,H_0]/\hbar^2). \tag{24}$$

The approximation based on the first term in (24) will generally fail if $V_c\delta t \gg 1$, but this is precisely the condition imposed in Eq. (22). Thus for chopping intervals below $1/V_c$ chopping and complex potential models will generally disagree, whereas for chopping intervals above $1/V_c$ the two complex potential models (kicked and continuous) will also disagree. In either case the chain connecting the continuous and pulsed observations is broken. Setting $\delta t \sim 1/V_c$ and in particular $\delta t \approx 2/V_c$, as suggested by Eq. (13), may be a reasonable compromise to get a similar evolution, but clearly one should not expect this relation to provide an accurate connection between pulsed and continuous observations since neither the conditions necessary for the validity of the approximation $U(0,\delta t) \approx U_k(0,\delta t)$ or for the rapid absorption in (22) are well satisfied. Further details are clearly out of the scope and aim of the present paper and will be presented elsewhere.

[1] L. A. Khalfin, Zh. Eksp. Teor. Fiz. **33**, 1371 (1957) [Sov. Phys. JETP **6** 1053 (1958)].
[2] L. Fonda and G. C. Ghirardi, Nuovo Cimento Soc. Ital. Fis., A **7A**, 180 (1972).
[3] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1977).
[4] C. B. Chiu, E. C. G. Sudarshan, and B. Misra, Phys. Rev. D **16**, 520 (1977).
[5] J. J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, Reading, MA, 1994).
[6] W. C. Schieve, L. P. Horwitz, and J. Levitan, Phys. Lett. A **136**, 264 (1989).
[7] A. G. Kofman and G. Kurizki, Phys. Rev. A **54**, R3750 (1996).
[8] A. G. Kofman and G. Kurizki, Nature (London) **405**, 546 (2000).
[9] P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. **86**, 2699 (2001).
[10] S. R. Wilkinson, C. F. Bharucha, M. C. Fischer, K. W. Madison, P. R. Morrow, Q. Niu, B. Sundaram, and M. G. Raizen, Nature (London) **387**, 575 (1997).
[11] M. C. Fischer, B. Gutiérrez-Medina, and M. G. Raizen, Phys. Rev. Lett. **87**, 040402 (2001).
[12] C. Rothe, S. I. Hintschich, and A. P. Monkman, Phys. Rev. Lett. **96**, 163601 (2006).
[13] G. García-Calderón, V. Riquer, and R. Romo, J. Phys. A **34**, 4155 (2001).
[14] J. G. Muga, F. Delgado, A. del Campo, and G. García-Calderón, Phys. Rev. A **73**, 052112 (2006).
[15] J. von Neumann, *Die Mathematishe Grundlangen der Quantenmechanik* (Springer, Berlin, 1932).
[16] L. D. Landau and R. Peierls, Z. Phys. **69**, 56 (1931).
[17] M. Ozawa, Phys. Rev. A **67**, 042105 (2003).
[18] K. Koshino and A. Shimizu, Phys. Rep. **412**, 191 (2005).
[19] A. Shaji, J. Phys. A **37**, 11285 (2004).
[20] B. Elattari and S. A. Gurvitz, Phys. Rev. A **62**, 032102 (2000).
[21] D. Home and A. Whitaker, Ann. Phys. **258**, 237 (1997).
[22] M. Hotta and M. Morikawa, Phys. Rev. A **69**, 052114 (2004).
[23] M. G. Makris and P. Lambropoulos, Phys. Rev. A **70**, 044101 (2004).
[24] S. Wallentowitz and P. E. Toschek, Phys. Rev. A **72**, 046101 (2005).
[25] M. Ozawa, e-print quant-ph/0603030.
[26] B. Crosigni and D. di Porto, Nuovo Cimento Soc. Ital. Fis., B **109**, 555 (1994).
[27] B. Crosigni and D. di Porto, Europhys. Lett. **35**, 165 (1996); **39**, 233 (1997).
[28] L. C. Ryff, Europhys. Lett. **39**, 231 (1997).
[29] J. G. Muga, J. P. Palao, B. Navarro, and I. L. Eguzquiza, Phys. Rep. **395**, 357 (2004).
[30] A. Ruschhaupt, J. A. Damborenea, B. Navarro, J. G. Muga, and G. C. Hegerfeldt, Europhys. Lett. **67**, 1 (2004).
[31] J. A. Damborenea, I. Egusquiza, G. C. Hegerfeldt, and J. G. Muga, J. Phys. B **36**, 2657 (2003).
[32] J. A. Damborenea, I. L. Egusquiza, G. C. Hegerfeldt, and J. G. Muga, Phys. Rev. A **66**, 052104 (2002).
[33] L. Fonda, G. C. Ghirardi, and T. Weber, J. Math. Phys. **7**, 1643 (1966); J. Wong, *ibid.* **8**, 2039 (1967).
[34] D. E. Manolopoulos, J. Chem. Phys. **117**, 9552 (2002).
[35] L. S. Schulman, Phys. Rev. A **57**, 1509 (1998).
[36] This differs from other works, such as R. G. Newton, Ann. Phys. **14**, 333 (1961), in which the effect of the "observation distance" is analyzed at the level of the wave function value at some distant point, and no detector is explicitly included in the model.