PHYSICAL REVIEW A **74**, 061801(R) (2006)

Multimode squeezing of frequency combs

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(Received 3 May 2006; published 8 December 2006)

We have developed a full multimode theory of a synchronously pumped type-I optical parametric oscillator. We calculate the output quantum fluctuations of the device and find that, in the degenerate case (coincident signal and idler set of frequencies), significant squeezing is obtained when one approaches threshold from below for a set of well-defined "supermodes," or frequency combs, consisting of a coherent linear superposition of signal modes of different frequencies which are resonant in the cavity.

DOI: 10.1103/PhysRevA.74.061801 PACS number(s): 42.50.Dv, 42.65.Re, 42.65.Yj

Optical parametric oscillators (OPO's) are among the best sources of squeezed [1], correlated [2], and entangled [3] light in the so-called continuous-variable regime. They have allowed physicists to successfully implement demonstration experiments for high-sensitivity optical measurements and quantum information protocols. In order to maximize the quantum effects, one needs to optimize the parametric down-conversion process. This has been achieved so far by using either intense pump lasers or resonant cavities. Having in mind that the parametric process is an almost instantaneous one, femtosecond mode-locked lasers are the best pump sources in this respect, as they generate very high peak optical powers with high coherence properties. Furthermore, they minimize the thermal effects in the linear crystal which often hamper the normal operation of parametric devices. Mode-locked lasers have been already used extensively to generate nonclassical light, by either pumping a parametric crystal [4,5] or an optical fiber [6]. However, in such single-path configurations, perfect quantum properties are only obtained when the pump power goes to infinity. This is the reason why mode locking is often associated with Q switching and pulse amplification [7] in order to reach even higher peak powers, at the expense of a loss in the coherence properties between the successive pump pulses. In contrast, intracavity devices produce perfect quantum properties for a finite power: namely, the oscillation threshold of the device. It is therefore tempting to consider devices in which one takes advantage of the beneficial effects of both high peak powers and resonant cavity buildup. Such devices exist: they are the so-called synchronously pumped OPO's (SPOPO's). In a SPOPO the cavity round-trip time is equal to that of the pumping mode-locked laser, so that the effects of the successive intense pump pulses add coherently, thus reducing considerably its oscillation threshold. Such SPOPO's have already been implemented as efficient sources of tunable ultrashort pulses [8–13], and their temporal properties have been theoretically investigated [14–16]. Let us mention that mode-locked OPO's have also been developed: in such devices, the cavity is resonant only for the signal modes and idler modes, and the pump pulses are not recirculating. Degenerate mode-locked OPO's have been used to generate pulsed squeezed light in the picosecond regime [5].

In this Rapid Communication, we make a complete multimode quantum analysis of SPOPO's and show theoretically that these devices are very efficient to produce squeezed states. Squeezing is effective not in a single-frequency mode, as usual, but instead in a whole set of "supermodes," which are well-defined linear combinations of signal modes of different frequencies. Similar supermodes have been independently introduced by Wasilewski *et al.* [17] in the different context of transient, degenerate down-conversion in a single-pass, single-pulse configuration. In their case, the supermodes are continuous linear superpositions of the annihilation operators in free space, whereas in our case, because of the resonant cavity, they are a discrete combination of modes.

Let us first specify the model that we use (Fig. 1). We consider a ring cavity of optical length L containing a type-I parametric crystal of thickness l. Degenerate phase matching is assumed, meaning that the phase-matching condition is fulfilled for frequencies $2\omega_0$ and ω_0. This amounts to saying that $n(2\omega_0)=n(\omega_0)\equiv n_0$, $n(\omega)$ being the refractive index of the crystal at frequency ω. The mode-locked pump laser, having a repetition rate $\Omega/2\pi=c/L$, is tuned so that the frequency of one of its modes is equal to $2\omega_0$. The electric field generated by the pump mode-locked laser can be expressed as

$$E_{\mathrm{ext}}(t)=\left(\frac{P}{2\varepsilon_0 c}\right)^{1/2}\sum_m i\alpha_m e^{-i(2\omega_0+m\Omega)t}+\text{c.c.}, \tag{1}$$

where P is the average laser power per unit area, α_m the normalized $(\Sigma_m|\alpha_m|^2=1)$ complex spectral component of longitudinal mode labeled by the integer index m, and $m=0$ corresponds to the phase-matched mode. For the sake of simplicity in this first approach of the problem, we will take the modal coefficients α_m as real numbers, thus excluding chirped pump pulses. As already mentioned, the SPOPO cavity length is adjusted so that its free spectral range coincides with that of the pumping laser. In the nonlinear crystal,

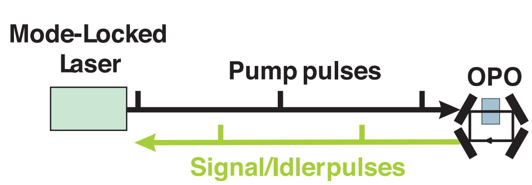

FIG. 1. (Color online) Synchronously pumped OPO.

pump photons belonging to all the different longitudinal pump modes are converted into signal and idler photons via the parametric interaction. In addition we will assume here that we are in the ideal case of *doubly resonant degenerate operation*, meaning that among all the OPO cavity resonant frequencies, there are all the pump mode frequencies $\omega_{p,m}=2\omega_0+m\Omega$ but also all the frequencies $\omega_{s,q}=\omega_0+q\Omega$ around the phase-matched subharmonic frequency ω_0. The intracavity electric field generated by the parametric interaction will then be a superposition of fields oscillating at frequencies $\omega_{s,q}$. We will finally call γ_p and γ_s the cavity damping rates for the pump and signal modes. Note that the free spectral range Ω is assumed to be the same in the pump and in the signal spectral regions. This is necessary for an efficient intracavity parametric down-conversion and requires, from the experimental viewpoint, the use of extra dispersive elements inside the cavity that compensate for the dispersion of the crystal. At the quantum level, the signal field, taken at the middle of the crystal, is represented by the quantum operator $\hat{E}_s$ which can be written as

$$\hat{E}_s(t)=\sum_q i\mathcal{E}_{s,q}\hat{s}_q(t)e^{-i\omega_{s,q}t}+\text{H.c.}, \tag{2}$$

where $\hat{s}_q$ is the annihilation operator for the qth signal mode in the interaction picture. $\mathcal{E}_{s,q}$ is the single-photon field amplitude, equal to $\sqrt{\hbar\omega_{s,q}/2\varepsilon_0 n(\omega_{s,q})AL}$, and A its effective transverse area.

The following Heisenberg equations for the field operators can be derived using the standard methods. The details of the derivation will be given in a forthcoming publication [18]. Below threshold and in the linearized regime for the pump fluctuations, they read

$$\frac{d\hat{s}_m}{dt}=-\gamma_s\hat{s}_m+\gamma_s\sigma\sum_q\mathcal{L}_{m,q}\hat{s}_q^{\dagger}+\sqrt{2\gamma_s}\hat{s}_{\text{in},m}, \tag{3}$$

where σ is the normalized pump amplitude,

$$\sigma=\sqrt{P/P_0}, \tag{4}$$

in which P_0 is the single-mode continuous wave (cw) oscillation threshold:

$$P_0=2\gamma_s^2\gamma_p n_0^3 c^3\varepsilon_0/(4\sqrt{2}\chi l\omega_0)^2, \tag{5}$$

with χ the crystal nonlinear susceptibility. $\mathcal{L}_{m,q}$ is the product of a phase-mismatch factor by the pump spectral normalized amplitude α_{m+q},

$$\mathcal{L}_{m,q}=\frac{\sin\phi_{m,q}}{\phi_{m,q}}\alpha_{m+q}, \tag{6}$$

where $\phi_{m,q}$ is the phase-mismatch angle,

$$\phi_{m,q}=\frac{l}{2}(k_{p,m+q}-k_{s,m}-k_{s,q}), \tag{7}$$

which can be computed using a Taylor expansion around $2\omega_0$ for the pump wave vectors $k_{p,m}$ and around ω_0 for the signal wave vectors $k_{s,q}$:

$$\phi_{m,q}\simeq\beta_1(m+q)+\beta_{2p}(m+q)^2-\beta_{2s}(m^2+q^2), \tag{8}$$

where $\beta_1=\frac{1}{2}\Omega(k_p'-k_s')l$, $\beta_{2p}=\frac{1}{4}\Omega^2k_p''l$, and $\beta_{2s}=\frac{1}{4}\Omega^2k_s''l$. k' and k'' are the first and second derivatives of the wave vector with respect to frequency. Finally $\hat{s}_{\text{in},m}$ are the input signal field operators at frequency $\omega_{s,m}$ transmitted through the coupling mirror. When the input is the vacuum state, which we consider here, their only non-null correlations are

$$\langle\hat{s}_{\text{in},m_1}(t_1)\hat{s}_{\text{in},m_2}^{\dagger}(t_2)\rangle=\delta_{m_1,m_2}\delta(t_1-t_2). \tag{9}$$

In order to get Eq. (3), we assumed, as usual, that $\mathcal{E}_{s,m}\simeq\mathcal{E}_{s,0}$ for all m and we neglected the dispersion of the nonlinear susceptibility. Therefore the present approach is not valid for ultrashort pulses, the spectrum of which extends over the whole visible region.

Let us first determine the average values of the generated fields. They are determined by the "classical" counterpart of Eq. (3), removing the input noise terms and replacing the operators by complex numbers. The solution of these equations is of the form $s_m(t)=S_{k,m}e^{\lambda_k t}$, where k is an index labeling the different solutions. The parameters $S_{k,m}$ and λ_k obey the following eigenvalue equation:

$$\lambda_k S_{k,m}=-\gamma_s S_{k,m}+\gamma_s\sigma\sum_q\mathcal{L}_{m,q}S_{k,q}^{*}. \tag{10}$$

As matrix $\mathcal{L}$ is both self-adjoint and real [$\mathcal{L}_{m,q}=\mathcal{L}_{q,m}$ real; see Eqs. (6)–(8)], its eigenvalues Λ_k and eigenvectors $\vec{L}_k$, of components $L_{k,m}$, are all real. As γ_s and σ are also real, there exist two sets of solutions of Eqs. (10), which we will call $S_{k,m}^{(+)}$ and $S_{k,m}^{(-)}$. The first set is given by $S_{k,m}^{(+)}=L_{k,m}$, and the second one is $S_{k,m}^{(-)}=iL_{k,m}$, with corresponding eigenvalues

$$\lambda_k^{(\pm)}=\gamma_s(-1\pm\sigma\Lambda_k). \tag{11}$$

Let us now label by index $k=0$ the solution whose eigenvalue Λ_k is maximum in absolute value: $|\Lambda_0|=\max\{|\Lambda_k|\}$. When $\sigma|\Lambda_0|$ is smaller than 1, all the rates $\lambda_k^{\pm}$ are negative, which implies that the null solution for the steady-state signal field is stable. For the simplicity of notation, we will take Λ_0 positive in the following [19]. The SPOPO reaches its oscillation threshold when σ takes the value $1/\Lambda_0$—i.e., for a pump power $P=P_{\text{thr}}$ equal to

$$P_{\text{thr}}=P_0/\Lambda_0^2. \tag{12}$$

The exact value of Λ_0, and therefore of the SPOPO threshold, depends on the exact shape of the phase-matching curve and on the exact spectrum of the pump laser [18]. In the most favorable situation, the theoretical SPOPO threshold can be extremely low, of the order of the single-mode threshold divided by the number of pump modes.

Let us now define the normalized amplitude pumping rate r by $r=\sqrt{P/P_{\text{thr}}}=\sigma\Lambda_0$. We will call "eigenspectrum" the set of $S_{k,m}$ values for a given k, which corresponds physically to the different spectral components of the signal field, and the critical eigenspectrum $S_{0,m}^{(+)}$ the one associated with $\lambda_0^{(+)}$, which changes sign at threshold. Above threshold, this critical mode will be the "lasing" one—i.e., the one having a nonzero mean amplitude when $r>1$. Let us note that the eigenspectrum in quadrature with respect to the critical one,

$S_0^{(-)}=iS_0^{(+)}$, has an associated eigenvalue $\lambda_0^{(-)}=-2\gamma_s$ at threshold. Furthermore, Eq. (11) implies that all the damping rates $\lambda_k^{(\pm)}$ are comprised below threshold between $-2\gamma_s$ and 0 and that, whatever the pump intensity, all the eigenvalues $\lambda_k^{(\pm)}(r)$ lie between $\lambda_0^{(+)}(r)$ and $\lambda_0^{(-)}(r)$. These properties will be useful for the study of squeezing.

We can now determine the squeezing properties of the signal field in a SPOPO below threshold. This is done by using the SPOPO linearized quantum equations. Let us introduce the operator $\hat{S}_{\text{in},k}(t)$ by

$$\hat{S}_{\text{in},k}(t)=\sum_m L_{k,m}\hat{s}_{\text{in},m}(t). \tag{13}$$

As $\Sigma_m|L_{k,m}|^2=1$, one has $[S_{\text{in},k}(t),S^\dagger_{\text{in},k'}(t')]=\delta(t-t')\delta_{k,k'}$: $\hat{S}_{\text{in},k}$ is the annihilation operator of a combination of modes of different frequencies, which are the eigenmodes of the linearized evolution equation (3). The corresponding creation operator applied to the vacuum state creates a photon in a single mode, which can be labeled as a "supermode," which globally describes a frequency comb. Defining in an analogous way as in Eq. (13) the intracavity operator $\hat{S}_k(t)$, one can then write

$$\frac{d}{dt}\hat{S}_k=-\gamma_s\hat{S}_k+\gamma_s\sigma\Lambda_k\hat{S}_k^\dagger+\sqrt{2\gamma_s}\hat{S}_{\text{in},k}. \tag{14}$$

Let us now define quadrature Hermitian operators $\hat{S}_k^{(\pm)}$ by

$$\hat{S}_k^{(+)}=\hat{S}_k+\hat{S}_k^\dagger, \tag{15}$$

$$\hat{S}_k^{(-)}=-i(\hat{S}_k-\hat{S}_k^\dagger), \tag{16}$$

which obey the following equation:

$$\frac{d}{dt}\hat{S}_k^{(\pm)}=\lambda_k^{(\pm)}\hat{S}_k^{(\pm)}+\sqrt{2\gamma_s}\hat{S}_{\text{in},k}^{(\pm)}, \tag{17}$$

with $\lambda_k^{(\pm)}$ given by Eq. (11). These relations enable us to determine the intracavity quadrature operators in the Fourier domain $\tilde{S}_k^{(\pm)}(\omega)$:

$$i\omega\tilde{S}_k^{(\pm)}(\omega)=\lambda_k^{(\pm)}\tilde{S}^{\pm}(\omega)+\sqrt{2\gamma_s}\tilde{S}_{\text{in},k}^{(\pm)}(\omega). \tag{18}$$

Finally, the usual input-output relation on the coupling mirror

$$\tilde{s}_{\text{out},m}(\omega)=-\tilde{s}_{\text{in},m}(\omega)+\sqrt{2\gamma_s}\tilde{s}_m(\omega), \tag{19}$$

extends by linearity to any supermode operator as the mirror is assumed to have a transmission independent of the mode frequency. One then obtains the following expression for the quadrature component in Fourier space of any signal supermode:

$$\tilde{S}_{\text{out},k}^{(\pm)}(\omega)=\frac{\gamma_s(1\pm r\Lambda_k/\Lambda_0)-i\omega}{\gamma_s(-1\pm r\Lambda_k/\Lambda_0)+i\omega}\tilde{S}_{\text{in},k}^{(\pm)}(\omega). \tag{20}$$

These expressions are particularly simple for the critical-mode quadrature components ($k=0$):

$$\tilde{S}_{\text{out},0}^{(\pm)}(\omega)=\frac{\gamma_s(1\pm r)-i\omega}{-\gamma_s(1\mp r)+i\omega}\tilde{S}_{\text{in},0}^{(\pm)}(\omega). \tag{21}$$

The variance of these operators can be measured using the usual balanced homodyne detection scheme: the local oscillator is in the present case a coherent mode-locked multimode field $E_{\text{L}}(t)$ having the same repetition rate as the pump laser:

$$E_{\text{L}}(t)=i\epsilon_L\sum_m e_m e^{-i\omega_{s,m}t}+\text{c.c.}, \tag{22}$$

where $\Sigma_m|e_m|^2=1$ and ϵ_L is the local oscillator field total amplitude factor. Assuming that the photodetectors measure the intensity of the Fourier components of the photocurrent averaged over many successive pulses, the balanced homodyne detection scheme measures the variance of the fluctuations of the projection of the output field on the local oscillator mode when the mean field generated by the OPO is zero, which is the case below threshold. As a result, when the coefficients e_m of the local oscillator field spectral decomposition are equal to the coefficients $L_{k,m}$ of the kth supermode, one measures the two following variances, depending on the local oscillator phase:

$$V_k^-(\omega)=\frac{\gamma_s^2(1-r\Lambda_k/\Lambda_0)^2+\omega^2}{\gamma_s^2(1+r\Lambda_k/\Lambda_0)^2+\omega^2}, \tag{23}$$

$$V_k^+(\omega)=V_k^-(\omega)^{-1}. \tag{24}$$

Equations (23) and (24) show that the device produces, as expected, a minimum uncertainty state and that quantum noise reduction below the standard quantum limit (equal here to 1) is achieved for any supermode characterized by a nonzero Λ_k value and that the smallest fluctuations are obtained close to threshold and at zero Fourier frequency:

$$(V_k)_{\min}=\left(\frac{\Lambda_0-|\Lambda_k|}{\Lambda_0+|\Lambda_k|}\right)^2. \tag{25}$$

In particular, if one uses as the local oscillator the critical mode $k=0$, identical to the one oscillating just above the threshold $r=1$, one then gets perfect squeezing just below threshold and at zero noise frequency, just like in the cw single-mode case. But modes of $k\neq 0$ may be also significantly squeezed, provided that $|\Lambda_k/\Lambda_0|$ is not much different from 1. This occurs in particular when the pump spectrum width is smaller than the phase-matching bandwidth [18]. For example, if the pump spectrum is 3 times smaller than the phase-matching bandwidth and using a Gaussian shape for the spectra [18], the value of V_k for the second mode ($k=1$) is 0.1 which is still significantly squeezed. Our multimode approach of the problem has therefore allowed us to extract from all the possible linear combinations of signal modes the ones in which the quantum properties are concentrated [20].

In conclusion, we have studied the quantum behavior of a degenerate synchronously pumped OPO, which seems at first sight a highly multimode system, since it involves roughly

10^5 different usual single-frequency modes for a 100-fs pulse. We have shown that its properties are more easily understood if one considers the "supermodes," linear combinations of all these modes that are eigenmodes of the SPOPO set of evolution equations and describe in a global way the frequency comb—or, equivalently, the train of pulses—generated by the SPOPO. The supermode of minimum threshold plays a particular role, as it is the one which turns out to be perfectly squeezed at threshold and will oscillate above threshold, but all the supermodes have nonclassical character and can be significantly squeezed. The present paper gives a first example of the high interest of studying frequency combs at the quantum level, as they merge the advantages of two already well-known nonclassical states of light: the cw light beams, with their high degree of coherence and reproducibility, and the single pulses of light, with their high peak power enhancing the nonlinear effects necessary to produce pure quantum effects.

Laboratoire Kastler Brossel, of the Ecole Normale Supérieure and the Université Pierre et Marie Curie-Paris6, is UMR8552 of the Centre National de la Recherche Scientifique. G.J.d.V. has been financially supported by Grant No. PR2005-0246 of the Secretaría de Estado de Universidades e Investigación del Ministerio de Educación y Ciencia (Spain).

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[19] Should $\Lambda_0 < 0$, then the null eigenvalue at threshold would be $\lambda_0^{(-)}$ instead of $\lambda_0^{(+)}$ and the following analysis should be accordingly modified.
[20] Our result is different from the "frequency comb of squeezing" reported in [21], which deals with single-mode cw OPO's and shows that squeezing is effective in a comb of noise frequencies.
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