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(Received 2 September 2006; published 10 November 2006)

We derive a system of nonpolynomial Schrödinger equations for one-dimensional wave functions of two components in a binary self-attractive Bose-Einstein condensate loaded in a cigar-shaped trap. The system is obtained by means of the variational approximation, starting from the coupled three-dimensional (3D) Gross-Pitaevskii equations and assuming, as usual, the factorization of 3D wave functions. The system can be obtained in a tractable form under a natural condition of symmetry between the two species. A family of vector (two-component) soliton solutions is constructed. Collisions between orthogonal solitons (ones belonging to the different components) are investigated by means of simulations. The collisions are essentially inelastic. They result in strong excitation of intrinsic vibrations in the solitons, and create a small orthogonal component (“shadow”) in each colliding soliton. The collision may initiate collapse, which depends on the mass and velocities of the solitons.

DOI: [10.1103/PhysRevA.74.053610](https://doi.org/10.1103/PhysRevA.74.053610)

PACS number(s): 03.75.Ss, 03.75.Hh, 64.75.+g

I. INTRODUCTION

Bose-Einstein condensates (BECs) with attractive interactions between atoms can form stable wave packets in nearly one-dimensional (1D) “cigar-shaped” traps, which provide for tight confinement in two transverse directions, while leaving the condensate almost free along the longitudinal axis. This trapping geometry made it possible to create stable bright solitons [1] and trains of such solitons [2] in the ⁷Li condensate, in which the interaction between atoms was made weakly attractive by means of the Feshbach-resonance technique. In the ⁸⁵Rb condensate trapped under similar conditions, stronger attraction between atoms leads to controllable collapse and creation of nearly 3D solitons [3].

This experimentally relevant situation is described by effective 1D equations which may be derived from the full 3D Gross-Pitaevskii equation (GPE) under various conditions and by means of different approximations [4–8]. In some cases, the deviation of the effective equation from a straightforward 1D variant of the GPE amounts to keeping an extra self-attractive *quintic* term in the equation, which may be sufficient to essentially alter properties of the corresponding solitons [6,9,10]. A more consistent derivation, that starts with the factorization of the 3D wave function into the product of a transverse one (it actually represents the ground state of the 2D harmonic oscillator) and arbitrary slowly varying longitudinal (one-dimensional) wave function, and then uses the variational approximation [11], leads to a more sophisticated but also more accurate *nonpolynomial Schrödinger equation* (NPSE) for the longitudinal wave function [5,12]. The above-mentioned simplified equation featuring the combination of cubic and quintic terms can be then obtained by an expansion of the NPSE for the case of a relatively weak nonlinearity [10]. The ratio of the coefficients in front of the cubic and quintic terms in the model derived in Ref. [6] is not the same as follows from the expansion of the NPSE, which is explained by a coarser character of the approxima-

tion used in that work (the approximation did not allow a deviation of the nonlinearity from the cubic-quintic form).

A physically significant generalization of the above-mentioned equations is a system of two nonlinearly coupled equations for a binary BEC, which can be created in the experiment by means of the *sympathetic-cooling* technique [13]. Accordingly, a relevant problem is to derive a system of effective 1D equations for a mixture of two BEC species in the cigar-shaped trap, starting from the two coupled GPEs in the 3D space. In this work, we aim to derive such a system in the form of coupled NPSEs, using a generalized version of the method elaborated in Refs. [5,12,14].

The paper is organized as follows. The derivation of the coupled NPSE system, which is based on the variational approximation, is presented in Sec. II. In the most general case, it leads to a cumbersome system. However, we demonstrate that, under a natural condition of the symmetry between the two species, the equations may be reduced to a tractable closed system of two NPSEs for longitudinal wave functions of the two components. Then, in Sec. III, we consider solutions for vector solitons (i.e., two-component ones) generated by this system; the solutions are found in an implicit analytical form up to a point where they cease to exist due to collapse.

A natural application of the thus derived NPSE system is to consider collisions between two *orthogonal* solitons, which belong to the two different components. This analysis, based on numerical simulations, is reported in Sec. IV. The collisions are inelastic, which is manifested in the excitation of intrinsic oscillations in the solitons after the collision, and generation of small “shadows” in them (each soliton captures and keeps a small share of atoms from the other species). The strongest manifestation of the inelasticity, as we demonstrate in Sec. IV, is a possibility to initiate collapse by the collision between two orthogonal solitons (which depends on their relative velocity). The paper is concluded by Sec. V.

II. COUPLED NONPOLYNOMIAL SCHRÖDINGER EQUATIONS

The system of 3D GPEs for a dilute binary condensate, confined in the transverse direction by a strong harmonic potential with frequency ω_{\perp} and in the axial direction by a generic weak potential $V(z)$, can be derived from the Lagrangian density,

$$L = \sum_{k=1,2} \psi_k^* \left[i\partial_t + \frac{1}{2}\nabla^2 - \frac{1}{2}(x^2 + y^2) - V(z) - \pi g_k |\psi_k|^2 \right] \psi_k - 2\pi g_{12} |\psi_1|^2 |\psi_2|^2. \quad (1)$$

Here $\psi_k(\mathbf{r}, t)$ is the macroscopic wave function of the k th species, which is subjected to the normalization condition,

$$\int \int \int |\psi_k(x, y, z)|^2 dx dy dz = N_k, \quad (2)$$

where N_k is the number of atoms in the k th species, and

$$g_k \equiv 2a_k/a_{\perp}, \quad g_{12} \equiv 2a_{12}/a_{\perp} \quad (3)$$

are strengths of the intra- and interspecies interactions, where a_k and a_{12} are the scattering lengths, and $a_{\perp} = \sqrt{\hbar/(m\omega_{\perp})}$ is the transverse harmonic-confinement length. Here we consider the binary condensate with attraction between atoms, which implies that both $a_{1,2}$ and a_{12} are *negative*. In the Lagrangian density, lengths, time, and energy are written in units a_{\perp} , ω_{\perp}^{-1} , and $\hbar\omega_{\perp}$, respectively.

The ordinary variational procedure applied to Eq. (1) gives rise to the coupled 3D GPEs,

$$i\partial_t \psi_k = \left[-\frac{1}{2}\nabla^2 + \frac{1}{2}(x^2 + y^2) + V(z) + 2\pi g_k |\psi_k|^2 + 2\pi g_{12} |\psi_{3-k}|^2 \right] \psi_k, \quad k = 1, 2. \quad (4)$$

Our objective here is to derive a system of effective 1D NPSEs, following the lines of the derivation of the NPSE for the single-component condensate developed in Ref. [5] [its generalization for an axially nonuniform trapping potential, with $\omega_{\perp} = \omega_{\perp}(z)$, was reported in Ref. [8]]. Using the cylindrical coordinates (r, θ) in the transverse plane (x, y) , we adopt the usual ansatz for the wave functions strongly localized in this plane, and weakly confined in the axial direction z :

$$\psi_k(r, z, t) = \frac{1}{\sqrt{\pi\sigma_k(z, t)}} \exp\left\{-\frac{r^2}{2\sigma_k(z, t)^2}\right\} f_k(z, t), \quad (5)$$

where real $\sigma_k(z, t)$ and complex $f_k(z, t)$ are dynamical fields, the latter ones obeying normalization $\int_{-\infty}^{+\infty} |f_k(z)|^2 dz = N_k$, as it follows from Eqs. (2) and (5).

Inserting this ansatz in Lagrangian density (1), performing the integration in the transverse plane, and neglecting derivatives of $\sigma(z, t)$ (for the same reasons as in Refs. [5,8]), one can derive the following effective Lagrangian:

$$\bar{L} = \sum_{k=1,2} f_k^* \left[i\partial_t + \frac{1}{2}\partial_z^2 - \frac{1}{2}\left(\frac{1}{\sigma_k^2} + \sigma_k^2\right) - V(z) - \frac{1}{2}\frac{g_k}{\sigma_k^2} |f_k|^2 \right] f_k - 2\frac{g_{12}|f_1|^2|f_2|^2}{(\sigma_1^2 + \sigma_2^2)}.$$

This Lagrangian gives rise to a system of four Euler-Lagrange equations, obtained by varying \bar{L} with respect to f_k^* and σ_k :

$$i\partial_t f_k = \left[-\frac{1}{2}\partial_z^2 + V(z) + \frac{1}{2}\left(\frac{1}{\sigma_k^2} + \sigma_k^2\right) + \frac{g_k}{\sigma_k^2} |f_k|^2 + 2\frac{g_{12}}{(\sigma_1^2 + \sigma_2^2)} |f_{3-k}|^2 \right] f_k, \quad (6)$$

$$\sigma_k^4 = 1 + g_k |f_k|^2 + 4g_{12} |f_{3-k}|^2 \frac{\sigma_k^4}{(\sigma_1^2 + \sigma_2^2)^2}, \quad (7)$$

with $k=1, 2$ in both Eqs. (6) and (7). This is a full system of the coupled NPSEs describing the two-component nearly 1D BEC.

Note that the ansatz (5) is relevant when the transverse confinement size is much smaller than a characteristic axial length of a structure (in particular, soliton) to be obtained as a solution of the axial equation. Physically, this means that the quantum pressure in the transverse direction is much stronger than the nonlinear self-attraction in the condensate.

III. VECTOR BRIGHT SOLITONS

In the subsequent analysis of the coupled NPSEs, we focus on the most natural symmetric case, when the (negative) effective nonlinearity coefficients accounting for the intra- and interspecies interactions are equal, namely,

$$g_{12} = g_1 = g_2 \equiv g. \quad (8)$$

As follows from Eqs. (3), this relation takes place, in particular, when the all scattering lengths are equal. In the symmetric case, we assume that numbers of atoms in the two species are equal too. We note that, for the self-repulsive binary BEC, with $g > 0$ (recall here we are going to consider the case of $g < 0$), Eq. (8) may pose a formal problem, as it precisely corresponds to the miscibility-immiscibility threshold in the infinite system. Nevertheless, the problem does not really take place, as the pressure exerted by the external potential shifts the equilibrium towards the miscibility (see, e.g., Ref. [15]), hence the case corresponding to relation (8) in the repulsive binary BEC is not going to be a degenerate (i.e., structurally unstable) one.

If constraint (8) holds, Eqs. (7) take the form

$$\sigma_k^4 = 1 + g |f_k|^2 + 4g |f_{3-k}|^2 \frac{\sigma_k^4}{(\sigma_1^2 + \sigma_2^2)^2}, \quad (9)$$

and admit an *exact symmetric solution*:

$$\sigma_1^2 = \sigma_2^2 = \sqrt{1 + g(|f_1|^2 + |f_2|^2)} \equiv \sigma_0^2. \quad (10)$$

Of course, there remains a question if some additional *asym-*

metric solutions to Eqs. (9) may also exist. One may assume that an asymmetric solution, if any, branches off from the symmetric one through a *bifurcation*. Then, close to the bifurcation point, one will have $\sigma_{1,2}^2 = \sigma_0^2(1 + \delta_{1,2})$, with some infinitesimal $\delta_1 \neq \delta_2$ [σ_0 is the symmetric solution given by Eq. (10)]. Substituting this in Eqs. (9) and linearizing them in δ_1 and δ_2 , one arrives at a system

$$2\delta_k = F_k(\delta_k - \delta_{3-k}), \quad k = 1, 2, \quad (11)$$

$$F_k \equiv g|f_k|^2/\sigma_0^4. \quad (12)$$

The resolvability condition for linear system (11) (equating its determinant to zero) takes the following form, after simple calculations: $F_1 + F_2 = 2$. However, this condition *cannot hold* for the attractive BEC, with $g < 0$, because expressions F_1 and F_2 , as given by Eq. (12), are negative in this case. This means the bifurcation giving rise to asymmetric solutions is impossible in the attractive binary condensate [provided that constraint (8) is valid], which substantiates the use of symmetric solution (10).

The substitution of Eq. (10) in Eqs. (6) leads to closed-form equations for the complex amplitude functions f_1 and f_2 ,

$$i\frac{\partial f_k}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} + V(z) + g\frac{|f_1|^2 + |f_2|^2}{\sqrt{1 + g(|f_1|^2 + |f_2|^2)}} + \frac{1}{2}\left(\frac{1}{\sqrt{1 + g(|f_1|^2 + |f_2|^2)}} + \sqrt{1 + g(|f_1|^2 + |f_2|^2)}\right) \right] f_k. \quad (13)$$

Equations (13) reduce to the familiar integrable *Manakov's system* (MS) [17],

$$i\partial_t f_k = \left[-\frac{1}{2}\partial_z^2 + V(z) + g(|f_1|^2 + |f_2|^2) \right] f_k, \quad (14)$$

if $g(|f_1|^2 + |f_2|^2) \ll 1$. Only under this condition may the system be considered as truly one-dimensional, and only in this limit it is integrable. Nevertheless, in the general case Eqs. (13) share the “isotopic invariance” with the Manakov's system: the nonlinearity appears solely through the invariant combination $|f_1|^2 + |f_2|^2$. Due to this fact, Eqs. (13) conserve an additional dynamical invariant (“isotopic spin”), $S = \int_{-\infty}^{+\infty} [f_1(z)f_2^*(z) + f_1^*(z)f_2(z)] dz$, with an asterisk standing for the complex conjugate.

In the case of attraction, $g < 0$, *vector* (two-component) bright solitons are looked for as $f_k = \exp(-i\mu_k t)\Phi_k(z)$, where Φ_1 and Φ_2 are real localized functions obeying the following coupled equations:

$$\mu_k \Phi_k = \left[-\frac{1}{2}\Phi_k'' + V(z)\Phi_k + g\frac{\Phi_1^2 + \Phi_2^2}{\sqrt{1 + g(\Phi_1^2 + \Phi_2^2)}} + \frac{1}{2}\left(\frac{1}{\sqrt{1 + g(\Phi_1^2 + \Phi_2^2)}} + \sqrt{1 + g(\Phi_1^2 + \Phi_2^2)}\right) \right] \Phi_k. \quad (15)$$

Due to the isotopic invariance, the solitons with equal chemical potentials of their components are tantamount to the

single-component (scalar) one, with $\Phi_2 = 0$ and $\Phi_1 \equiv \Phi(z)$ being a solution of a single equation,

$$\left[-\frac{1}{2}\frac{d^2}{dz^2} + V(z) + \frac{1 + (3/2)g\Phi^2}{\sqrt{1 + g\Phi^2}} \right] \Phi = \mu\Phi. \quad (16)$$

If a soliton solution to Eq. (16) is found, the corresponding vector soliton can be constructed in an obvious way,

$$\begin{Bmatrix} f_1(z,t) \\ f_2(z,t) \end{Bmatrix} = \begin{Bmatrix} \cos \theta \\ \sin \theta \end{Bmatrix} \begin{Bmatrix} \exp(-i\mu t) \\ \exp(-i\mu t) \end{Bmatrix} \Phi(z), \quad (17)$$

with an arbitrary “isotopic angle,” $0 \leq \theta \leq \pi/2$. More general vector solitons, with different chemical potentials in their components, are possible too. However, in the symmetric case that we are dealing with here, it is obvious that the vector solitons with unequal chemical potentials cannot realize the ground state, therefore they are not considered here.

For $V(z) = 0$ and $g < 0$, a family of soliton solutions to Eq. (16) was constructed in Refs. [5,12]. In this case, setting $\Phi(z) = \sqrt{N}\phi(z)$, with $N_1 = N_2 \equiv N$, and

$$\gamma \equiv N|g|, \quad (18)$$

the field $\phi(z)$ and the chemical potential μ are given by implicit formulas,

$$z = \sqrt{\frac{1}{2(1-\mu)}} \operatorname{Arctanh}\left(\sqrt{\frac{\sqrt{1-\gamma\phi^2-\mu}}{1-\mu}}\right) - \frac{1}{\sqrt{2}}\sqrt{\frac{1}{1+\mu}} \tan^{-1}\left(\sqrt{\frac{\sqrt{1-\gamma\phi^2-\mu}}{1+\mu}}\right), \quad (19)$$

$$\gamma = \frac{2\sqrt{2}}{3}(2\mu+1)\sqrt{1-\mu}. \quad (20)$$

and the wave function satisfies the normalization condition $\int_{-\infty}^{+\infty} \phi(z)^2 dz = 1$. The family is then characterized by the dependence of γ on μ . The inverse of Eq. (20) demonstrates that, in terms of the $\mu(\gamma)$ dependence, there are two branches of the soliton family, but only one of them, that satisfies condition $d\mu/d\gamma < 0$ (which is nothing else but the known *Vakhitov-Kolokolov* stability criterion [20] in the present notation), is stable. In addition, there is a critical nonlinearity strength, $\gamma_c = 4/3$ (which corresponds to $\mu = 1/2$), above which the solution does not exist, because of the *longitudinal collapse* [5,16], which is a manifestation of the 3D collapse possible in the underlying system of GPEs, Eqs. (4). In the limit of weak nonlinearity, $\gamma \rightarrow 0$, Eq. (19) reduces to the ordinary soliton wave form, $\phi(z) = (\sqrt{\gamma/2})\operatorname{sech}(\gamma z/2)$ [5,14].

IV. COLLISIONS BETWEEN SOLITONS

A straightforward application of the system of NPSEs (13) is to study collisions between two *orthogonal* solitons, taken in the form of Eq. (17), with equal values of μ and isotopic angles $\theta = 0$ and $\theta = \pi/2$. Using the Galilean invariance of the equations, the velocities of the two solitons are taken to be $\pm v$. The corresponding initial condition, at $t = 0$, is

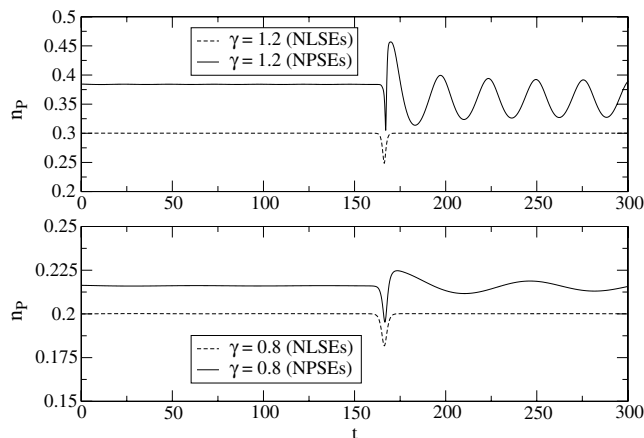


FIG. 1. Peak density n_p of the two colliding solitons as a function of time (t), in the nonintegrable system of NPSEs, Eqs. (13) and in the integrable Manakov’s system (alias NLSEs), Eqs. (14). In both cases, the initial velocity is $v=0.6$.

$$\begin{cases} \phi_1^{(0)}(z) \\ \phi_2^{(0)}(z) \end{cases} = \begin{cases} e^{ivz} \phi(z - z_0/2) \\ e^{-ivz} \phi(z + z_0/2) \end{cases}, \quad (21)$$

with large initial separation z_0 . To determine the time evolution of the fields $\phi_k(z, t)$, $k=1, 2$, we solved both nonlinear Schrödinger equations (NLSEs) and NPSEs, i.e., Eqs. (14) and (13) respectively, numerically, by using a two-component extension of a well-tested finite-difference code based on the Crank-Nicolson predictor-corrector algorithm [21]. In the MS [alias the NLSE system, Eqs. (14)], which is integrable, collisions are always elastic. However, since NPSEs (13) are not integrable, collisions described by these equations are expected to be *inelastic*. This expectation is borne out by Fig. 1, where we compare the collision outcomes in the MS and NPSEs for identical sets of parameters. The figure shows the peak densities n_p of both colliding solitons (which are equal, due to the symmetry of the configuration) as a function of time. The outcome does not depend on the initial separation z_0 between the solitons in Eq. (21), provided that it is large enough (we took $z_0=200$). After the collision the MS solitons remain undisturbed (dashed lines), while their NPSE counterparts come out from the collision with excited intrinsic oscillations (solid lines). This result not only shows that the collision in the NPSEs is inelastic, but also suggests that the solitons supported by this system, i.e., ones given by Eqs. (17), (19), and (20), unlike their counterparts in the integrable MS, feature an intrinsic mode, with a well-defined eigenfrequency ω . In fact, this mode was predicted in Ref. [12], by means of the variational approximation. It was shown that ω vanishes at $\gamma \rightarrow 0$, and it attains a maximum close to the above-mentioned collapse threshold, $\gamma = \gamma_c \equiv 4/3$.

As shown in Fig. 2, the amplitude and frequency (ω) of the oscillations excited by the collisions of solitons in the NPSE system grow with interaction strength γ . Contrary to that, the simulations demonstrate that the amplitude and frequency of the intrinsic oscillations *do not* depend on initial velocity v (see Fig. 5 below). The independence of ω on v is

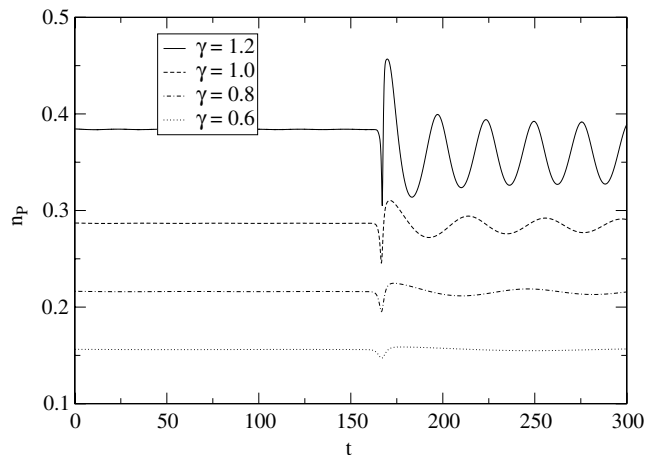


FIG. 2. Peak density n_p of the two colliding solitons as a function of time, found from numerical integration of the NPSEs, Eqs. (13), for different values of interaction strength γ . The initial velocity is $v=0.6$.

quite natural, as the frequency is determined solely by the internal structure of the soliton.

In Fig. 3 we compare the intrinsic frequency ω as found from the direct simulations of the NPSEs, Eqs. (13), to the frequency calculated by means of the variational approach of Ref. [12]. The figure shows that the variational approximation somewhat overestimates both the critical strength of longitudinal collapse γ_c and frequency ω . For both quantities, the relative error is about 15%.

Collisions between solitons in NPSEs give rise to another noteworthy effect: after the collision, a small part of the field, ϕ_1 , which originally belonged to the first soliton remains trapped in the second soliton, and vice versa, see Fig. 4. The effect is visible only for $\gamma > 1$, in the velocity interval of $0.4 < v < 0.7$ (outside this parameter range, the effects takes

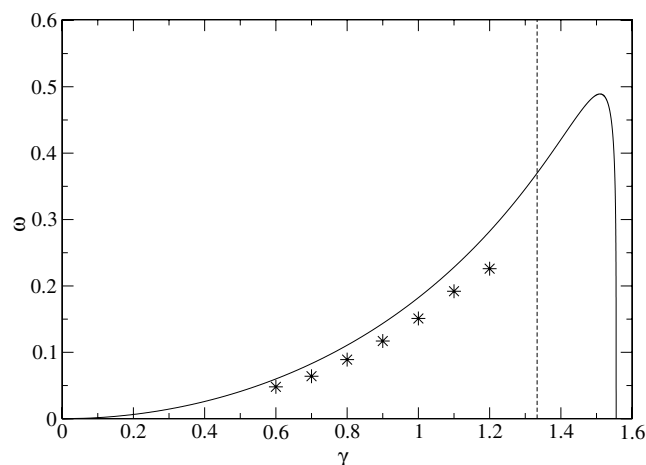


FIG. 3. Frequency ω of the intrinsic oscillations of the two solitons as a function of interaction strength γ . Stars: ω found from the numerical solution of NPSEs, Eqs. (13), as the frequency of intrinsic oscillations excited by the collision between two solitons. Solid line: a result of the variational approximation from Ref. [12]. The collapse point, found from simulations of Eqs. (13), is indicated by the dashed vertical line. The initial velocity is $v=0.8$.

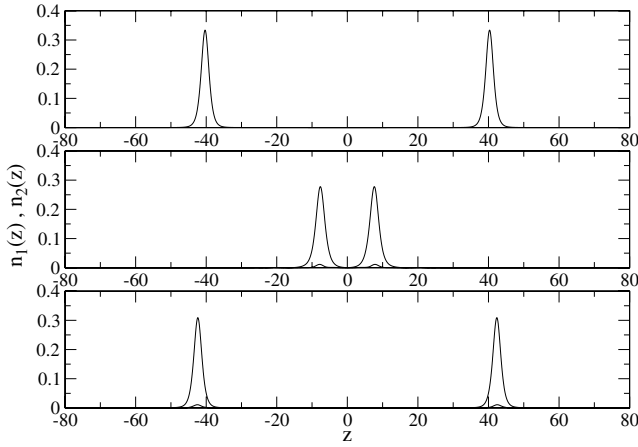


FIG. 4. The trapping effect in the collision of two solitons: small parts of fields $\phi_2(z, t)$ and $\phi_1(z, t)$ remain bound, respectively, in the first and second soliton after the collision. Top, central, and bottom panels display the density configurations, $n_{1,2}(z) \equiv |\phi_{1,2}(z)|^2$, at $t = 100, 180$, and 240 . The interaction strength is $\gamma = 1.2$, and the initial velocity is $v = 0.6$.

place too but is very weak). Note that a similar effect (“shadow formation”) was observed in the model describing the interaction of two polarizations of light in a nonlinear optical fiber, which was based on a nonintegrable system of two NLSEs with the cubic nonlinearity, see Refs. [18,19] and references therein. An explanation of the trapping effect is based on the fact that the soliton in each field $\phi_k(z, t)$ ($k = 1, 2$) supports not only the above-mentioned mode of intrinsic vibrations in the same field, but also an external eigenmode of small perturbations in the mate field, $\phi_{3-k}(z, t)$. The latter mode is excited as a result of the collision [18].

The collisions feature a trend to be more inelastic at smaller velocities, as illustrated by Fig. 5, which shows the maximum and minimum values of the peak density, $n_p^{(M)}$ and

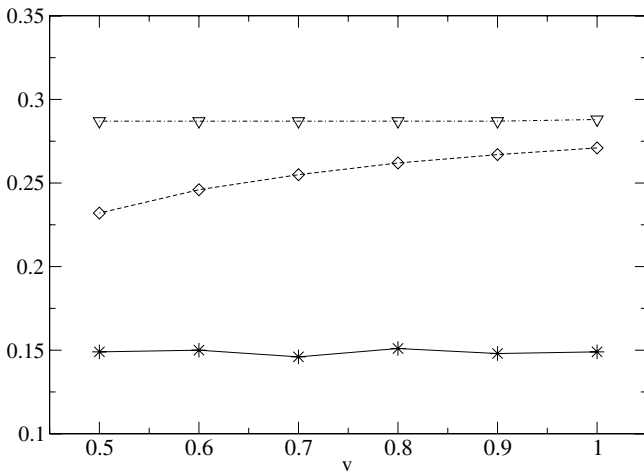


FIG. 5. Dependence on the collision velocity v of the maximum (triangles) and minimum (rhombuses) values of the peak density, $n_p^{(M)}$ and $n_p^{(m)}$, in solitons disturbed by the collision. The respective frequency of the intrinsic mode, ω , is shown by stars. The interaction strength is $\gamma = 1$.

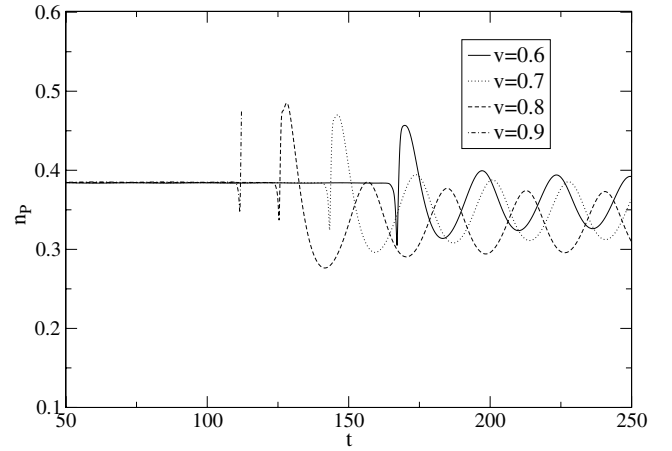


FIG. 6. Peak density n_p of the two colliding solitons as a function of time for different values of initial velocity v . The collapse is induced by the collision at $v = 0.9$ (the corresponding curve shoots up vertically at the collapse moment, $t = t_c \approx 110$). The interaction strength is $\gamma = 1.2$, and the initial separation is $z_0 = 200$.

$n_p^{(m)}$, in the oscillating solitons emerging from the collision. The figure shows that, while $n_p^{(M)}$ does not depend on initial velocity v , $n_p^{(m)}$ is smaller at smaller velocities, which implies stronger inelasticity. The results are shown in Fig. 5 for $\gamma = 1$, and similar trends are found for $\gamma \leq 1$. For completeness, in Fig. 5 we also plot frequency ω of the intrinsic mode excited by the collision, which confirms that ω does not depend on v .

An interesting issue is whether the collision may result in collapse. As follows from Eqs. (13), the collapse happens when condition $|g|(|f_1|^2 + |f_2|^2) = 1$ takes place at some point. If the first maximum of the peak density is achieved when the centers of the colliding solitons nearly coincide, this condition can be estimated as $n_p^{(M)} \approx 1/(2\gamma)$. While Fig. 5 shows that $n_p^{(M)}$ does not depend on collision velocity v at $\gamma \leq 1$, it depends on v at $\gamma > 1$, and the collapse can thus be reached by increasing v . In particular, Fig. 6 shows that, for $\gamma = 1.2$, the maximum value of peak density, $n_p(t)$, grows with v , and the collapse takes place at $v = 0.9$. Note that collapse induced by the collision between two solitons in a single NPSE was reported in Ref. [12], but in that case the onset of the collapse did not depend on the initial velocity.

V. CONCLUSIONS

In this work, we have derived a system of one-dimensional coupled nonpolynomial Schrödinger equations (NPSEs) for longitudinal wave functions of two components in a binary BEC, in the case of attraction between the atoms. The system was derived by means of the variational approximation, starting from the coupled 3D Gross-Pitaevskii equations for the two species and assuming (as usual) the factorization of 3D wave functions into products of the strongly confined transverse and slowly varying longitudinal ones.

The system was cast in a tractable form under a natural symmetry constraint. Then, a family of two-component (vector) soliton solutions was obtained, and collisions between orthogonal solitons (each belonging to one component only) were studied in detail by dint of systematic numerical simulations. It was found that the collisions are inelastic. They lead to strong excitation of intrinsic oscillations in the solitons emerging from the collision, and to formation of a small orthogonal component (“shadow”) in each soliton. Eventu-

ally, the collision may initiate collapse of the solitons, depending on their mass and velocities.

ACKNOWLEDGMENTS

L.S. thanks Alberto Parola and Luciano Reatto for many discussions and suggestions. The work of B.A.M. was supported, in a part, by the Israel Science Foundation through the Center-of-Excellence Grant No. 8006/03.

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