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Generation of a high-visibility four-photon entangled state and realization of a four-party quantum communication complexity scenario

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We obtain a four-photon polarization-entangled state with a visibility as high as $(95.35\pm0.45)\%$ directly from a single down-conversion source. A success probability of $(81.54\pm1.38)\%$ is observed by applying this entangled state to realize a four-party quantum communication complexity scenario, which comfortably surpasses the classical limit of 50%. As a comparison, two Einstein-Podolsky-Rosen pairs are shown to implement the scenario with a success probability of $(73.89\pm1.33)\%$. This four-photon state can also be used to fulfill decoherence-free quantum information processing and other advanced quantum communication schemes.

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I. INTRODUCTION

Entanglement is one of the most important and interesting characteristics of quantum mechanics. Entangled states of two or more particles not only play a central role in the discussion of quantum mechanics versus local realism [1], but also enable one to perform certain communication and computational tasks with efficiency not achievable on the basis of laws of classical physics [2].

Many experiments employing type-II spontaneous parametric down-conversion (SPDC) processes have been reported to realize multiphoton entangled states, including a four-photon Greenberger-Horne-Zeilinger (GHZ) state with a visibility of $(79\pm6)\%$ [3], a four-photon decoherence-free state of visibility $(79.3\pm1.4)\%$ [4], and a four-photon cluster state with a fidelity of $(74.1\pm1.3)\%$ [5]. Yet in those schemes interference occurs pairwise between processes where the photon pair is created at distances $\pm x$ from the middle of the crystal after compensating the longitudinal walk-off [6]. Therefore, only the photon pair generated at the middle of the crystal can be completely compensated, which may limit the purity of the state when the crystal is pumped by a femtosecond pulse. What is more, in most of the previous experiments Hong-Ou-Mandel-type interferences are adopted to generate the multiphoton entangled state, which may markedly decrease the visibility.

On the other hand, communication complexity problems (CCP's) have been greatly investigated in the past few years [7–12]. Consider that a number of widely separated parties each possess some data as input and they want to perform a distributed computation with all the inputs. Because no party has access to all of the data, in general no party can complete the computation singlehandedly. The communication complexity of a problem is defined to be the minimal communication cost incurred in performing the distributed computation [7,8]. There are two types of CCP's [13]: the first one is to transmit the minimal amount of bits for all the parties to determine the correct value of the function with certainty; the second is to get the highest probability for all the parties to arrive at the correct value of the function if the number of transmitted bits is fixed.

Quantum communication complexity as an impressive branch of quantum information applies entangled states to reduce communication complexity. The quantum advantage can be clearly shown in the second type of CCP's.

In quantum information, CCP's which show a quantum advantage have been related to Bell's inequality [13]. Furthermore, it is suggested that a suitably chosen set of CCP's can be the basis of an information-theoretic axiomatization of quantum mechanics [12].

In this paper, we show that a polarization-entangled state observed behind a single-pulsed type-I SPDC source can reach a visibility as high as $(95.35\pm0.45)\%$. We use this state to realize a four-party quantum communication complexity scenario (QCCS) with a success probability of $(81.54\pm1.38)\%$, which is much higher than the classical limit of 50%. This is equal to showing that our state violates a kind of Bell's inequality [13] and may help to understand the information-theoretic axiomatization of quantum mechanics [12].

The rest of this study is organized as follows. In Sec. II we describe the experimental details of the generation and characterization of the four-photon polarization-entangled state. Section III gives the theoretical and experimental results of the four-party QCCS. Finally, a short conclusion is given in Sec. IV.

II. GENERATION AND CHARACTERIZATION OF THE FOUR-PHOTON POLARIZATION-ENTANGLED STATE

There is a reasonable probability of simultaneously producing four photons in a single, strongly pulsed SPDC source. In our experiment, we use two identically cut type-I beta-barium-borate (BBO) crystals ($8.0\times8.0\times0.6$ mm^3, $\theta_{pm}=30.35°$) with their optic axes aligned in mutually perpendicular planes [14]. Frequency-doubled ultraviolet (UV) pluses (390 nm center wavelength, ~200 fs pulse duration, 76 MHz repetition rate, ~500 mW average power) from a mode-locked Ti:sapphire laser are used to pass through the

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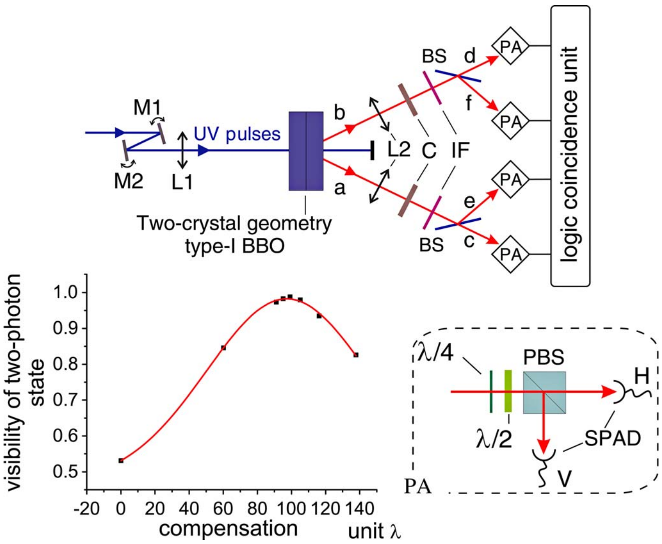

FIG. 1. (Color online) Experimental setup. The UV pulse is focused by a convex lens with a focal length of 50 cm ($L1$) and the direction of the UV beam is controlled by two mirrors $M1$ and $M2$. The four photons are emitted into two spatial modes a and b. After being collected by a convex lens with a focal length of 30 cm ($L2$) in each mode, the four photons pass through quartz plates (C) to compensate the birefringence in BBO. Then, they are distributed into the four modes c, d, e, and f by two 50-50 beam splitters (BS) behind interference filters (IF, $\Delta\lambda=3$ nm, $\lambda=780$ nm). In order to analyze the four-photon state and to realize the QCCS, polarization analysis (PA) in various bases is performed for each mode using quarter-wave plates ($\lambda/4$) and half-wave plates ($\lambda/2$) in front of polarizing beam splitters (PBS) and single-photon avalanche detectors (SPAD). The inset shows the visibility of two-photon entangled state versus compensation (100 mW pump). The solid line is a Gaussian function fitting (unit $\lambda=780$ nm).

two-crystal geometry BBO. Behind two 50-50 beam splitters, the four photons with distinct spatial modes are coupled into single-mode optical fibers (Fig. 1).

According to the Schrödinger equation, the four-photon state can be obtained as

$$|\Psi^4\rangle = (a_H^\dagger b_H^\dagger + a_V^\dagger b_V^\dagger)^2|0\rangle = (a_H^{\dagger 2} b_H^{\dagger 2} + a_V^{\dagger 2} b_V^{\dagger 2} + 2a_H^\dagger b_H^\dagger a_V^\dagger b_V^\dagger)|0\rangle, \quad (1)$$

where $a_H^\dagger$ is the creation operator of a photon with horizontal polarization in mode a, etc.

For simplicity, we assume that at the beam splitters a is transformed into $\frac{1}{\sqrt{2}}(c+e)$ and b into $\frac{1}{\sqrt{2}}(d+f)$ [15], where c, d and e, f denote the transmitted and reflected modes, respectively. We then expand Eq. (1) and keep only those terms which lead to four-photon coincidence behind the two beam splitters—i.e., only those terms for which there is one photon in each of the modes. As a result, this four-photon state can be written as

$$|\Psi^4\rangle = |HHHH\rangle_{cdef} + |VVVV\rangle_{cdef} + \frac{1}{2}(|HHVV\rangle_{cdef} + |HVVH\rangle_{cdef} + |VHHV\rangle_{cdef} + |VVHH\rangle_{cdef}), \quad (2)$$

where the four entries in the state vectors indicate horizontal (H) or vertical (V) polarizations of the photons in modes c, d, e, and f.

This state can be seen as a superposition of a four-photon GHZ state and a product of two Einstein-Podolsky-Rosen (EPR) pairs (normalized)

$$|\Psi^4\rangle = \sqrt{\frac{2}{3}}|\mathrm{GHZ}\rangle_{cedf} + \sqrt{\frac{1}{3}}|\mathrm{EPR}\rangle_{ce} \otimes |\mathrm{EPR}\rangle_{df}, \quad (3)$$

where $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHHH\rangle + |VVVV\rangle)$ is the GHZ state and $|\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)$ is the EPR state $|\Psi^+\rangle$.

Many efforts have been made to keep our experimental system stable for several days. An air conditioner is used to keep the room temperature to the order of ± 1 °C. To avoid damage to the second-harmonic generation BBO and the SPDC BBO, we pump N_2 around them. Moreover, by using a motion controller system (Newport, NSC200) to tilt two mirrors $M1$ and $M2$ (in Fig. 1) with the feedback of two charge coupled devices (not shown) and twofold coincidences of two paths (such as modes c and d), we manage to maintain the position of the pump beam.

To obtain a high-purity four-photon entangled state, the birefringence between horizontal and vertical photons in the two-crystal geometry BBO has been compensated for by quartz plates. Due to the symmetries of the two-crystal geometry BBO and the polarization of SPDC photons from type-I BBO, this birefringence can be compensated for completely in theory. The inset of Fig. 1 shows that the coher-

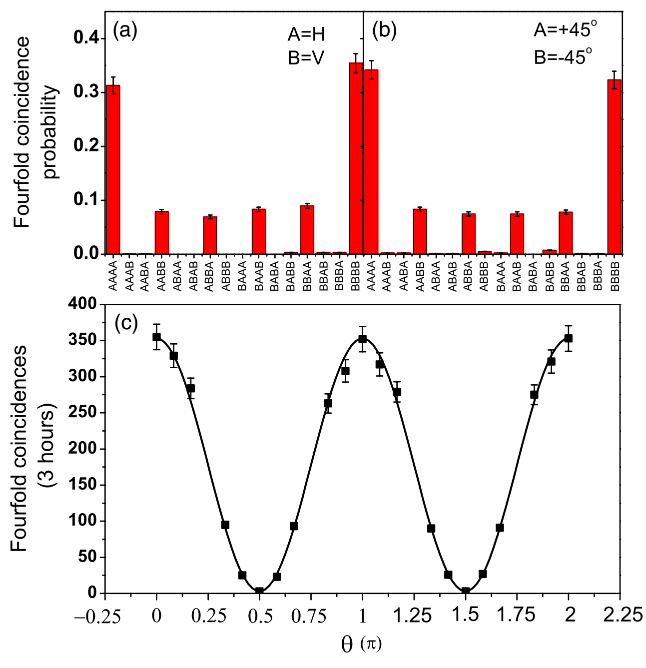

FIG. 2. (Color online) Fourfold-coincidence probabilities corresponding to different measurement basis settings. (a) the H/V basis and (b) the $+/-$ basis. (c) Four-photon interference curve. We vary the detection basis in mode f, while keeping modes c, d, and e in the $+(+45°)$ basis. θ represent the angle between the linear polarization detection basis and the $+$ basis in mode f. The solid line shows a sinusoidal fit to the experimental results with a visibility of (98.45 ± 0.15)%. Uncertainties due to counting statistics.

ence between horizontal and vertical photons is recovered perfectly while the compensation of optical path difference is about 99.1λ.

Figure 2(a) and 2(b) show the 16 possible fourfold-coincidence probabilities for detecting one photon in each mode with the four polarization analyzers oriented along the H/V basis and $+/-$ basis [±45° linear polarizations—i.e., $\frac{1}{\sqrt{2}}(H\pm V)$], respectively [4]. The integration time is 3 h per column. One can find two types of coincidences: the GHZ part and the fourfold coincidences due to the EPR pairs with average rates lower by a factor of 4, which is in very good agreement with the state in Eq. (3).

Figure 2(c) shows one of the four-photon interference curves of the entangled state. More strictly, we may use the correlation function to characterize the entangled state. The experimental value of the correlation function is obtained from the 16 four-photon coincidence rates with [4]

$$E(\phi_c,\phi_e,\phi_d,\phi_f) = \sum_{l_c,l_e,l_d,l_f=\pm 1} l_c l_e l_d l_f \times P_{l_c,l_e,l_d,l_f}(\phi_c,\phi_e,\phi_d,\phi_f), \tag{4}$$

where l_x, ϕ_x are corresponding to the eigenvectors $|l_x,\phi_x\rangle = \frac{1}{\sqrt{2}}(|V\rangle_x + l_x e^{-i\phi_x}|H\rangle_x)$ with eigenvalues $l_x=\pm 1$ for polarization measurements performed by the observation stations in the four modes $(x=c,e,d,f)$ and P_{l_c,l_e,l_d,l_f} are the four-photon probabilities. Theoretically [15], when $\phi_c=\phi_e=\phi_d=\phi_f=0$, the correlation function reaches its maximal value, which is equal to the visibility of the curve of E versus one of the angles, such as ϕ_c, with other angles $\phi_e=\phi_d=\phi_f=0$. From the data of Fig. 2(b), we obtain $V=(95.35\pm 0.45)\%$, compared to the theoretical result $V=100\%$ for a pure state.

In our experiment, the errors are mainly due to the uncertainty of the generation of photon pairs. The coincidence counts of photons obey a Poisson distribution and the standard deviation of each measurement is given by $\sqrt{N}$, where N is the total coincidence counts. The maximum four-photon coincidence count we get is about 400; as a result, the standard deviation is about 20 which is just 5% of the total counts. As many efforts have been made to keep our system stable, the influences of other error sources such as the drifts of the single-fiber coupling and laser system have been depressed far below 5% of the total counts in each measurement and may be negligible.

During the experiment, a different setup is attempted to achieve a better result. For example, if the lens with a focal length of 50 cm is replaced by a lens of 30 cm to focus the pump pulse onto the crystal, the four-photon coincidences will be 4 times brighter; however, the four-photon visibility will decrease to about 80%.

III. EXPERIMENTAL REALIZATION OF A FOUR-PARTY QCCS

Now, we use this entangled state to realize a four-party QCCS.

Suppose there are four parties A, B, C, and D receiving X, Y, Z, and K, respectively, where $X, Y, Z, K \in U \in \{0,1\}^2$, and they are promised that

$$(X+Y+Z+K)\bmod 2 = 0. \tag{5}$$

The common goal is for each party to get the correct value of the Boolean function

$$F(X,Y,Z,K) = \frac{1}{2}[(X+Y+Z+K)\bmod 4]. \tag{6}$$

X, Y, Z, and K can be represented in binary notation as x_1x_0, y_1y_0, z_1z_0, and k_1k_0. According to Eq. (5), $x_0y_0z_0k_0$ is one of the eight combinations 0000, 0011, 0101, 0110, 1001, 1010, 1100, and 1111.

We then rewrite Eq. (6) as

$$F = x_1 \oplus y_1 \oplus z_1 \oplus k_1 \oplus F_0(x_0,y_0,z_0,k_0), \tag{7}$$

where

$$F_0(x_0,y_0,z_0,k_0) = \frac{1}{2}[(x_0+y_0+z_0+k_0)\bmod 4]. \tag{8}$$

As a result, if $x_0y_0z_0k_0=0000$ or 1111, $F_0=0$; otherwise, $F_0=1$.

Obviously, if these four parties are restricted to broadcast one bit, respectively, they have a 50% probability to get the correct value of F in the classical situation [10]. On the other hand, if they share the four-photon entangled state we have

TABLE I. The input of $x_0y_0z_0k_0$, the corresponding local rotations, the components of $|\Psi^4\rangle'$ for successful communication, the result of F_0, and the corresponding theoretical probability (Theor. prob.) and experimental probability (Expt. prob.) to get the correct value of F.

$x_0y_0z_0k_0$	Local rotations	$\|\Psi^4\rangle'$				F_0	Theor. prob.	Expt. prob.
0000	$R(x)\otimes R(x)\otimes R(x)\otimes R(x)$	$\|0000\rangle$	$\|0011\rangle$	$\|0101\rangle$	$\|0110\rangle$	0	100%	97.68% ±0.23%
1111	$R(y)\otimes R(y)\otimes R(y)\otimes R(y)$	$\|1001\rangle$	$\|1010\rangle$	$\|1100\rangle$	$\|1111\rangle$	0	100%	96.32% ±0.35%
0011	$R(x)\otimes R(x)\otimes R(y)\otimes R(y)$					1	83.33%	80.49% ±1.57%
1100	$R(y)\otimes R(y)\otimes R(x)\otimes R(x)$	$\|0001\rangle$	$\|0010\rangle$	$\|0100\rangle$	$\|0111\rangle$	1	83.33%	82.63% ±1.44%
0110	$R(x)\otimes R(y)\otimes R(y)\otimes R(x)$	$\|1000\rangle$	$\|1011\rangle$	$\|1101\rangle$	$\|1110\rangle$	1	83.33%	79.06% ±1.66%
1001	$R(y)\otimes R(x)\otimes R(x)\otimes R(y)$					1	83.33%	84.13% ±1.34%
0101	$R(x)\otimes R(y)\otimes R(x)\otimes R(y)$	$\|0001\rangle$	$\|0010\rangle$	$\|0100\rangle$	$\|0111\rangle$	1	66.67%	67.18% ±2.20%
1010	$R(y)\otimes R(x)\otimes R(y)\otimes R(x)$	$\|1000\rangle$	$\|1011\rangle$	$\|1101\rangle$	$\|1110\rangle$	1	66.67%	64.82% ±2.28%

prepared initially, the probability they get the correct value of F can reach 83.33%, as shown below.

Each of the four parties A, B, C, and D share one photon of the state

$$|\Psi^4\rangle = \frac{1}{\sqrt{3}}(|0000\rangle + |1111\rangle) + \frac{1}{2\sqrt{3}}(|0011\rangle + |1001\rangle + |0110\rangle + |1100\rangle), \tag{9}$$

where 0 and 1 represent H and V in Eq. (2), respectively (we have omitted the subscripts c, d, e, and f for simplicity).

If x_0 $(y_0,z_0,k_0)=0$, then A (B,C,D) applies rotation

$$R(x) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

on his own photon with half wave plate and quarter-wave plate; if x_0 $(y_0,z_0,k_0)=1$, then A (B,C,D) applies rotation

$$R(y) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

on his own photon. Then each of the four parties measures the photon under $0/1(H/V)$ basis and gets the result of a, b, c, d. Due to the entanglement of the state they share, initially, A, B, C, and D only have to broadcast the four bits $x_1\oplus a$, $y_1\oplus b$, $z_1\oplus c$, and $k_1\oplus d$, respectively. Then they have on average 83.33% probability to get the correct value of F as

$$F = x_1 \oplus a \oplus y_1 \oplus b \oplus z_1 \oplus c \oplus k_1 \oplus d, \tag{10}$$

that is,

$$F_0 = a \oplus b \oplus c \oplus d. \tag{11}$$

First, in the case $x_0y_0z_0k_0=0000$, local rotations $R(x)\otimes R(x)\otimes R(x)\otimes R(x)$ do not change the four-photon state—i.e., $|\Psi^4\rangle'=|\Psi^4\rangle$, where $|\Psi^4\rangle'$ is the state after local rotations.

Consequently, the success probability is 100%.

Next, in the case where $x_0y_0z_0k_0=0011$, after the rotation $R(x)\otimes R(x)\otimes R(y)\otimes R(y)$ the entangled state becomes

$$|\Psi^4\rangle' = \frac{\sqrt{3}i}{4}(|0001\rangle + |0010\rangle + |1101\rangle + |1110\rangle) + \frac{\sqrt{3}}{12}(|0100\rangle - |0111\rangle + |1000\rangle - |1011\rangle) + \frac{\sqrt{3}i}{12}(|0000\rangle + |0011\rangle - |1100\rangle - |1111\rangle) + \frac{\sqrt{3}}{12}(|0101\rangle - |0110\rangle + |1010\rangle - |1001\rangle). \tag{12}$$

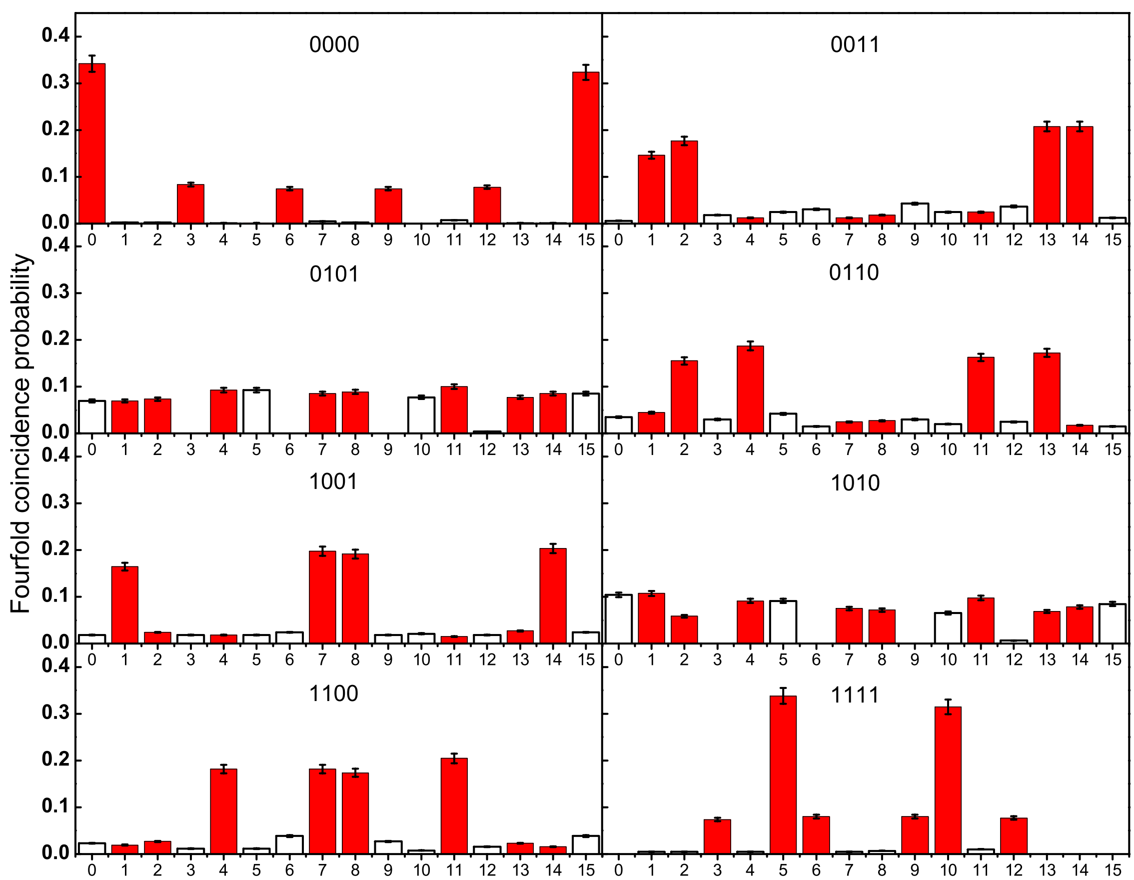

FIG. 3. (Color online) Fourfold-coincidence probabilities for the four-party QCCS. Each frame represents a kind of combination of $x_0y_0z_0k_0$, denoted by 0000, 0011, etc. The x axis of each frame $(0,1,\ldots,15)$ represents the 16 different $0/1$ (H/V) basis settings in binary representation—e.g., $9=1001$. The solid and open columns denote the probabilities of getting the correct and wrong values of F, respectively.

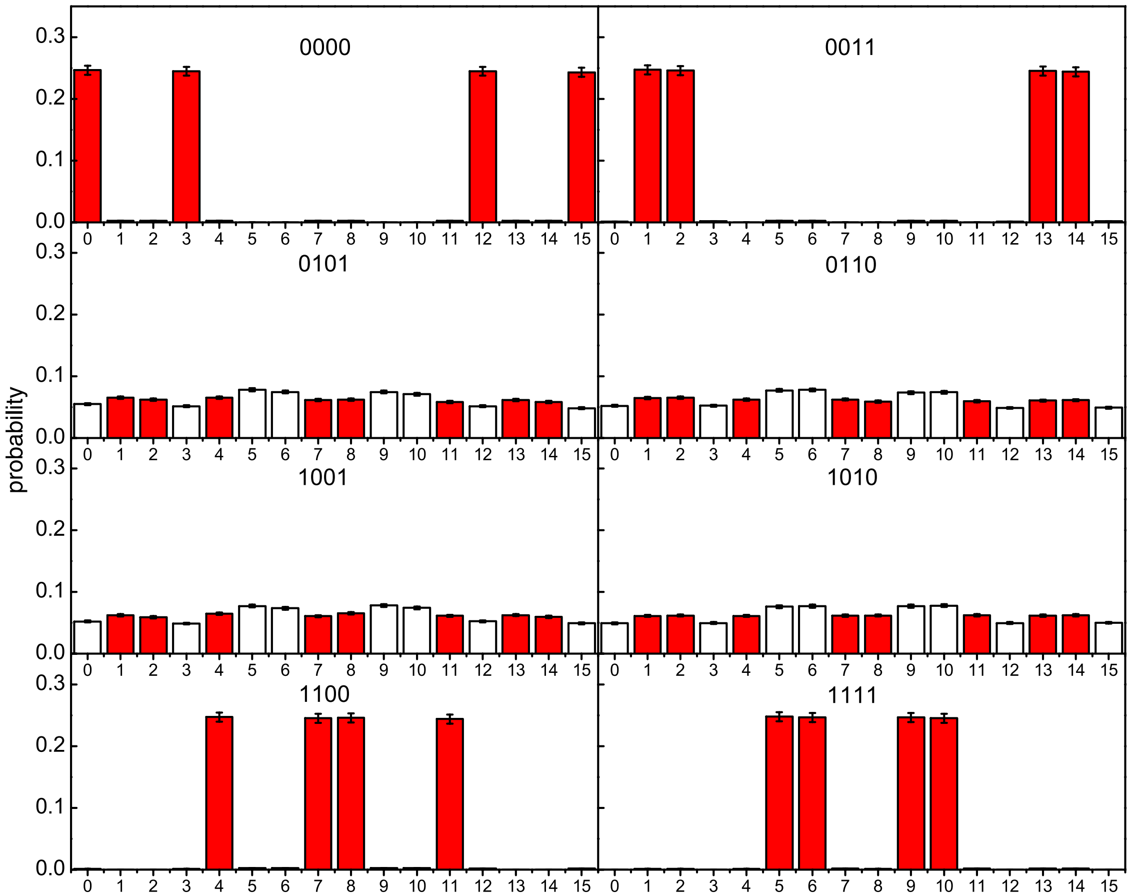

FIG. 4. (Color online) Probability distribution for the four-party QCCS when the four parties share two identical EPR states.

After broadcasting $x_1 \oplus a$, $y_1 \oplus b$, $z_1 \oplus c$, and $k_1 \oplus d$, respectively, the four parties have 83.33% probability to get the correct value of f.

The remaining cases can be similarly analyzed, as shown in Table I.

Figure 3 illustrates the experimental result in detail. It is shown that the average probability for the four parties to get the correct value of F in our experiment is $(81.54 \pm 1.38)\%$, which greatly surpasses the classical limit of 50%. This result proves that the state we have prepared violates a kind of Bell's inequality [13].

To illustrate that there is genuine four-photon entanglement in the state we have prepared, we further consider another case, where (A,B) and (C,D) share two EPR states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, respectively. It can be deduced that the probability for the parties to get the correct value of F is 75% in this case.

In experiment, the EPR state is generated by 100-mW UV pulses. We manipulate (A,B) and (C,D) independently with twofold coincidences and combine their results to get the probability distribution for the four-party QCCS.

Figure 4 shows the experimental result when these four parties share two EPR states. The average success probability we obtain is $(73.89 \pm 1.33)\%$. Compared to the experimental result of the entangled state we have prepared, we can see that it is the genuine four-photon entangled part [$|\mathrm{GHZ}\rangle$ in Eq. (3)] making the success probability reach $(81.54 \pm 1.38)\%$.

IV. CONCLUSION

Due to the complete compensation of the temporal difference of SPDC photons, we generate a high-visibility four-photon entangled state by employing two-geometry type-I BBO. Without interferometric setups we demonstrate the nonlocality of our state by realizing a four-party QCCS, which may also help one to understand the information-theoretic axiomatization of quantum mechanics. The experimental results have proved the utility of our source with high visibility and ease of operation in multiparty quantum communication complexity. This source can also be used to fulfill decoherence-free quantum information processing [16] and other advanced quantum communication schemes.

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