Quantum phase gate in an optical cavity with atomic cloud

Yun-Feng Xiao, Xu-Bo Zou,* Zheng-Fu Han, and Guang-Can Guo
Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China
(Received 21 March 2006; published 12 October 2006)

We propose a scheme for implementing a two-qubit quantum phase gate for intracavity fields. In the scheme, two qubits are encoded in zero- and one-photon Fock states of two intracavity modes, and a four-level N-type atomic ensemble trapped in a cavity mediates the conditional phase gate within a given interaction time. We also discuss the influence of the atomic spontaneous emission and the decay of the cavity modes on the photon loss and gate fidelity, showing the scheme is within the current experiment technology.

DOI: 10.1103/PhysRevA.74.044303 PACS number(s): 03.67.Lx, 42.50.Dv

Cavity quantum electrodynamics (QED) offers an almost ideal system for the generation of entangled states and the implementation of quantum information processing [1]. In the context of cavity QED, numerous theoretical schemes for generating entangled states of many atoms and nonclassical states of cavity fields have been proposed [2], which led to experimental realization of the Einstein-Podolsky-Rosen state [3] of two atoms, Greenberger-Horne-Zeilinger state [4] of three parties (two atoms plus one cavity mode), Schrödinger cat state [5], and Fock state [6] of a single-mode cavity field. Most of these schemes are based on the interaction of atoms and a single-mode cavity field. An experiment was reported for preparing two modes of a superconducting cavity in a maximally entangled state by using a sequence of interactions of an atom with two cavity modes [7]. This experiment opened up a new possibility for quantum state engineering and quantum information processing using multiple modes in a superconducting cavity. In Ref. [8], a scheme was proposed to realize a quantum phase gate of two intracavity modes in which a single detuned atom with cascade configuration mediated the interaction between them. Recently, Garcia-Maraver *et al.* [9] showed a single V-type atom can also be used to realize a two-qubit phase gate between a vacuum and a single-photon state of two intracavity modes.

Most of the schemes mentioned above are based on the interaction of single atoms and a cavity field, and operated in the strict strong-coupling regime. Recently, it was shown that the strict strong-coupling condition can be relatively relaxed if the atomic ensemble was placed in an optical cavity due to the collective enhancement of an atom-photon interaction. Several experiments have been reported for continuous variable entanglement using cold atoms [10] and the photon statistics of the light emitted from an atomic ensemble into a single cavity mode [11]. Theoretical schemes have also been proposed for generating entangled spin squeezed states of a large number of atoms [12], entangled atomic ensembles state [13], and realizing two-mode field squeezing using atomic ensemble as medium [14], and carrying out quantum phase-gate operation for two traveling single photons using an M-type atomic ensemble trapped in a gas cell [15], and implementing the teleportation of an atomic ensemble quantum state [16].

*Electronic address: xbz@ustc.edu.cn

In this paper, we take an alternative approach to implement a two-qubit quantum phase gate of two intracavity modes in which a cloud of atoms is used to enhance a photon-photon interaction. The motivation of our scheme is at least twofold. First, it relaxes the requirement of strict strong-coupling regime and still obtains high fidelity and a low error rate. Second, a Lamb-Dicke limit is no longer required since not single atoms, but atomic cloud mediates the coherent interaction between photons.

Our model consists of an ensemble of N identical N-type atoms inside an optical cavity, as sketched in Fig. 1(a). The energy-level structure is shown in Fig. 1(b), including two excited states $|2\rangle$ and $|4\rangle$, and two stable ground states $|1\rangle$ and $|3\rangle$. Each atom interacts with two polarized quantized cavity modes and a classical laser field. The Hamiltonian is written as

$$H = H_0 + H_{\text{int}}, \tag{1}$$

with

$$H_0 = \omega_h a_h^\dagger a_h + \omega_v a_v^\dagger a_v + \sum_{k=1}^{N}\sum_{\alpha=1}^{4} \omega_\alpha |\alpha\rangle_{kk}\langle\alpha|, \tag{2}$$

and

$$H_{\text{int}} = \sum_{k=1}^{N} \{g_h(\vec{r}_k)a_h|2\rangle_{kk}\langle 1| + g_v(\vec{r}_k)a_v|4\rangle_{kk}\langle 3| + \Omega(\vec{r}_k) \times |2\rangle_{kk}\langle 3|e^{-i\omega_L t} + \text{H.c.}\}, \tag{3}$$

where subscript k represents the kth atom. a_h and a_v are the annihilation operators associated with two cavity modes,

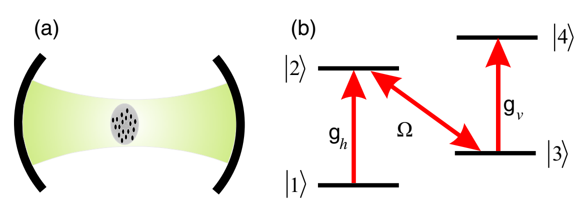

FIG. 1. (Color online) (a) Schematic illustration of an atomic ensemble trapped in a high-Q optical cavity. (b) Energy-level diagram of the trapped N-type atoms, and the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|3\rangle \leftrightarrow |4\rangle$ are resonantly coupled to the cavity modes a_h and a_v, respectively. The transition $|2\rangle \leftrightarrow |3\rangle$ is associated with a strong classical laser field.

with frequencies ω_h and ω_v, respectively. The atomic states $|\alpha\rangle$ $(\alpha=1,2,3,4)$ have Bohr frequencies ω_α. The atomic transitions $|1\rangle\leftrightarrow|2\rangle$ and $|3\rangle\leftrightarrow|4\rangle$ are resonantly coupled with the cavity modes a_h and a_v, with the coupling constants $g_h(\vec{r_k})$ and $g_v(\vec{r_k})$, respectively. While the atomic transition $|2\rangle\leftrightarrow|3\rangle$ is resonantly driven by a classical laser field with the frequency ω_L and coupling strength $\Omega(\vec{r_k})$. H.c. stands for the Hermitian conjugate.

We offer some brief remarks about coupling constants $g_h(\vec{r})$, $g_v(\vec{r})$, and $\Omega(\vec{r})$ before we obtain the ultimate expression of effective Hamiltonian. In a *FP*-type cavity the coupling factor $g_{h(v)}(\vec{r})$ can be expressed by $g_{h(v)}\chi_{h(v)}(\vec{r})$, where $\chi_{h(v)}(\vec{r})$ is the mode function described by $\chi_{h(v)}(\vec{r})=\sin(k_{h(v)}z)\exp[-(x^2+y^2)/w_{0,h(v)}^2]$ [17]. Here $w_{0,h(v)}$ and $k_{h(v)}=2\pi/\lambda_{h,v}$ are, respectively, the waist and the wave vector of the Gaussian cavity mode $a_{h(v)}$, and $\vec{r_k}(x,y,z)$ describes the kth atomic location; z is assumed to be along the axis of the cavity. For the sake of convenience, we denote $g_h(\vec{r_k})=g_{hk}$, $g_v(\vec{r_k})=g_{vk}$ and $\Omega(\vec{r_k})=\Omega_k$. Then the interaction Hamiltonian in the interaction picture can be written as

$$H_I=\sum_{k=1}^{N}\{g_{hk}a_h|2\rangle_{kk}\langle 1|+g_{vk}a_v|4\rangle_{kk}\langle 3|+\Omega_k|2\rangle_{kk}\langle 3|+\text{H.c.}\}. \tag{4}$$

In general, the Hamiltonian H_I is difficult to treat exactly because there exist the classical driving terms. In order to obtain physical insight into the dynamics of such a physical system, some approximations are necessary. To demonstrate how the system dynamics is modified by the strong classical field, we introduce the atomic dressed basis $|\pm\rangle_k=\frac{1}{\sqrt{2}}(|2\rangle_k\pm|3\rangle_k)$, then H_I can be rewritten as

$$H_I=\sum_{k=1}^{N}\Bigg\{\Omega_k(|+\rangle_{kk}\langle+|-|-\rangle_{kk}\langle-|)+\Bigg[\frac{g_{hk}}{\sqrt{2}}a_h(|+\rangle+|-\rangle)_{kk}\langle 1|+\frac{g_{vk}}{\sqrt{2}}a_v|4\rangle_{kk}(\langle+|-\langle-|)+\text{H.c.}\Bigg]\Bigg\}. \tag{5}$$

In order to further simplify the dynamics of the system, we switch to the interaction picture with respect to the $\sum_{k=1}^{N}\Omega_k(|+\rangle_{kk}\langle+|-|-\rangle_{kk}\langle-|)$. In the strong laser regime, i.e., with the choice of $|\Omega_k|\gg|g_{hk}|,|g_{vk}|$, neglecting the effect of rapidly oscillating terms and using the time-averaging method of Refs. [8,18,19], we can further reduce H_I to an effective interaction Hamiltonian

$$H_{\text{eff}}=\sum_{k=1}^{N}\Bigg\{\frac{1}{2\Omega_k}(g_{hk}^2a_ha_h^\dagger-g_{vk}^2a_v^\dagger a_v)\times(|+\rangle_{kk}\langle+|-|-\rangle_{kk}\langle-|)+\left(\frac{g_{hk}g_{vk}}{\Omega_k}a_ha_v|4\rangle_{kk}\langle 1|+\text{H.c.}\right)\Bigg\}. \tag{6}$$

In the following we consider the temporal evolution under different initial states in the form of $|N\text{ atoms}\rangle\otimes|\text{two cavity modes}\rangle$.

(i) If the initial state of the system is prepared as $\Pi_k|1\rangle_k\otimes|h\rangle|v\rangle$, which means all atoms initially occupy the ground state $|1\rangle$ and there are two polarized photons h and v in the cavity. Then H_{eff} is reduced to the form

$$H_{\text{eff}}^{(1)}=\sum_k(\lambda_ka_ha_v|4\rangle_{kk}\langle 1|+\text{H.c.}) \tag{7}$$

with the effective coupling constant $\lambda_k=g_{hk}g_{vk}/\Omega_k$.

The time evolution of this system is spanned in the space $\{|\phi_0\rangle=\Pi_k|1\rangle_k\otimes|h\rangle|v\rangle,|\phi_k\rangle=|4\rangle_k\Pi_{j\neq k}^{N}|1\rangle_j\otimes|\text{vac}\rangle|\text{vac}\rangle\}$. Governed by $H_{\text{eff}}^{(1)}$, the system state is described by $|\Psi(t)\rangle=c_0(t)|\phi_0\rangle+\Sigma_kc_k(t)|\phi_k\rangle$. According to Schrödinger equation $i\partial_t|\Psi(t)\rangle=H_{\text{eff}}^{(1)}|\Psi(t)\rangle$, we have

$$dc_0(t)/dt=-i\sum_{k=1}^{N}\lambda_kc_k, \tag{8a}$$

$$dc_k(t)/dt=-i\lambda_kc_0. \tag{8b}$$

Considering the initial conditions we obtain

$$c_0(t)=\cos Gt \tag{9}$$

with $G=\sqrt{\Sigma_{k=1}^{N}\lambda_k^2}$.

(ii) If the system state is initially in the state $\Pi_k|1\rangle_k\otimes|h\rangle|\text{vac}\rangle$, by letting $g_v=0$, the effective Hamiltonian in the case of strong laser regime reads

$$H_{\text{eff}}^{(2)}=0. \tag{10}$$

(iii) For the initial state $\Pi_k|1\rangle_k\otimes|\text{vac}\rangle|v\rangle$ or $\Pi_i|1\rangle_i\otimes|\text{vac}\rangle|\text{vac}\rangle$, obviously, there is no interaction between the cavity modes and the atoms, so

$$H_{\text{eff}}^{(3)}=H_{\text{eff}}^{(4)}=0. \tag{11}$$

Therefore, with the choice $t_I=\pi/G$, which can be controlled by the classical laser field, we easily obtain

$$\Pi_k|1\rangle_k\otimes|h\rangle|v\rangle\rightarrow-\Pi_k|1\rangle_k\otimes|h\rangle|v\rangle, \tag{12a}$$

$$\Pi_k|1\rangle_k\otimes|h\rangle|\text{vac}\rangle\rightarrow\Pi_k|1\rangle_k\otimes|h\rangle|\text{vac}\rangle, \tag{12b}$$

$$\Pi_k|1\rangle_k\otimes|\text{vac}\rangle|v\rangle\rightarrow\Pi_k|1\rangle_k\otimes|\text{vac}\rangle|v\rangle, \tag{12c}$$

$$\Pi_k|1\rangle_k\otimes|\text{vac}\rangle|\text{vac}\rangle\rightarrow\Pi_k|1\rangle_k\otimes|\text{vac}\rangle|\text{vac}\rangle. \tag{12d}$$

This is a phase gate between two intracavity modes.

In order to demonstrate the feasibility of the present scheme, we analyze the photon loss during the gate operation and the fidelity of the phase gate. The decoherence mechanisms arise through three dominant channels: (i) qubit dipoles decay at the rate γ_s, cavity decay with the rates κ_h and κ_v, and atomic collisions. A single trajectory in the quantum jump mode [20] is well suitable for evaluating the effects on the gate fidelity. We suppose that the photon decays from the cavity are continuously monitored, and that the single trajectory is specified by the evolution of the system conditioned

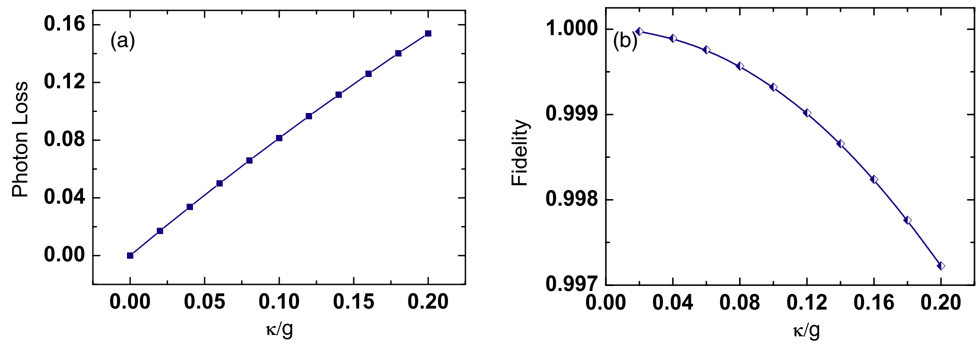

FIG. 2. (Color online) Photon loss and fidelity of the phase gate vs κ/g in (a) and (b), respectively. Other common parameters: $\kappa=\gamma_s$, $g_{hk}=g_{vk}=g$, $N=10^4$, $\Omega_k=20g$.

to no-photon detection. The conditional dynamics satisfies the non-Hermitian Hamiltonian

$$H^{(\prime)} = H_I - \frac{i}{2}\sum_{j=2,4}\sum_k \gamma_{s,j}|j\rangle_{kk}\langle j| - \frac{i}{2}\sum_{p=h,v}\kappa_p a_p^\dagger a_p, \quad (13)$$

where we omit the double excitation effect since the atoms in the states $|1\rangle$ and $|3\rangle$ cannot be excited again in the case with a single-photon input, respectively.

For simplicity, we assume $\kappa_h=\kappa_v=\kappa$, $\gamma_{s,2}=\gamma_{s,4}=\gamma_s$, $\kappa=\gamma_s$, $g_{hk}=g_{vk}=g$, and $\Omega_k=\Omega$ in the following numerical simulations, so it can be seen $G=\sqrt{N}g^2/\Omega$. The initial atom-cavity state is prepared in $|\Psi(0)\rangle=\Pi_k|1\rangle_k\otimes(|h\rangle|v\rangle+|h\rangle|\text{vac}\rangle+|\text{vac}\rangle|v\rangle+|\text{vac}\rangle|\text{vac}\rangle)/2$. After a phase gate, the conditional state can be described by $|\Psi(t_I)\rangle=\Pi_k|1\rangle_k\otimes(\beta_{h,v}|h\rangle|v\rangle+\beta_{h,0}|h\rangle|\text{vac}\rangle+\beta_{0,v}|\text{vac}\rangle|v\rangle+\beta_{0,0}|\text{vac}\rangle|\text{vac}\rangle)/2$ with the choice of $t_I=\pi\Omega/(\sqrt{N}g^2)$. Note here $|\Psi(t_I)\rangle$ has not been normalized, so the photon loss can be calculated by $P_{\text{loss}}=1-\langle\Psi(t_I)||\Psi(t_I)\rangle$. Figure 2(a) plots the numerical calculation of the photon loss after a full gate operation for the initial state $|\Psi(0)\rangle$. The photon loss is proportional to κ/g due to the assumption $\kappa=\gamma_s$.

The gate fidelity, which is an efficient measure of the distance between the quantum logic gates, can be defined as $F\equiv|\langle\Psi_{\text{out}}||\Psi_{\text{ideal}}\rangle|^2$. Here $|\Psi_{\text{out}}\rangle$ is the normalized conditional output state $|\Psi(t_I)\rangle/\langle\Psi(t_I)||\Psi(t_I)\rangle$ after the actual phase gate, and $|\Psi_{\text{ideal}}\rangle$ is the ideal output state $\Pi_k|1\rangle_k\otimes(-|h\rangle|v\rangle+|h\rangle|\text{vac}\rangle+|\text{vac}\rangle|v\rangle+|\text{vac}\rangle|\text{vac}\rangle)/2$. As shown in Fig. 2(b), the gate fidelity reaches very high even under a slightly strong-coupling condition, and it monotonically decreases when κ/g_0 grows.

We turn to discuss the preparation of the initial cavity field states. The problem can be switched to prepare the initial two-photon polarized state and inject them into the cavity. Up to the present, single photons have been produced successfully in strong-coupling cavity QED [21], which are suitable for quantum information processing. In principle, any two-photon polarized state within the state space $\{|h\rangle|v\rangle, |h\rangle|h\rangle, |v\rangle|v\rangle, |v\rangle|h\rangle\}$, can be expediently prepared. To implement the phase gate, the above-mentioned scheme can be used. However, the photon state should be first translated into the corresponding state space $\{|h\rangle|v\rangle, |h\rangle|\text{vac}\rangle, |\text{vac}\rangle|v\rangle, |\text{vac}\rangle|\text{vac}\rangle\}$, which can be carried out using simple linear optical elements. The following key is how to inject photon qubits from the outside into an optical cavity with high efficiency. Photon injection is one of critical steps for distributed quantum computation and quantum network [8,22]. Recently, Fattal, Beausoleil, and Yamamoto [23] proposed a significant scheme to achieve photon injection from the outside to the inside of an optical cavity and then store it in the atomic internal state even with highly imperfect hardware. However, more theoretical suggestions and experimental realization still provide a challenge.

Moreover, visible photons decay too fast in an optical cavity. As a result, conventional schemes in which a single atom interacts with cavity modes, usually need to be operated in the strict strong-coupling regime and Lamb-Dicke limit. However, in the case of atomic ensemble, the operation of the above phase gate is accelerated due to the collective enhancement of atom-photon interaction (approximately, the time changes from π/g to π/G). This allows the full gate operations in the presence of the decoherence mechanisms, including the atomic spontaneous emission and cavity decay. Hence, compared with those single-atom schemes, our scheme relaxes the requirements of the strong-coupling condition and Lamb-Dicke limit, and can be operated in the weak-coupling regime in principle. Nevertheless, this relaxation cannot be exaggerated with no limit. A slightly strong-coupling regime (g can be on the same order of magnitude of κ and γ_s) is usually required caused by some practical ingredients. For instance, on one hand, to avoid collisions the atom number is limited by the cavity mode volume; on the other hand, to obtain a large coupling factor g, a small cavity mode volume is necessary. In the following, we will address these parameters in a real system.

Finally, we discuss some experimental parameters which possibly lead to the imperfection of the gate operation. We consider a low-density vapor of ^{85}Rb in an optical FP cavity. The atomic configuration can be chosen from the hyperfine states of ^{85}Rb. For instance, the two lower levels $|1\rangle$ and $|3\rangle$ are the $F=2,3$ hyperfine states of the $5S_{1/2}$ electronic ground state, while $|2\rangle$ and $|4\rangle$ are the $F=3,4$ hyperfine states of the $5P_{3/2}$ electronic excited state, respectively. The three important cavity QED parameters describing the cavity-atom interaction system have been obtained as $(g,\kappa,\gamma_s)/2\pi=(16,1.4,3)$ MHz [24]. The waist (w) of the cavity modes and the waist (d) of the homogeneous classical laser beam are about 35 and 50 μm [24], respectively, so an approximate interaction volume of 5×10^4 μm^3 can be obtained. Choose atom number $N\sim10^4$, corresponding to an atom number density of 0.2 μm^{-3}, which is enough to prevent coherence losses due to the atomic collisions. A cloud of atoms is released from a magneto-optical trap and falls through a cavity with a velocity of $v_m=2$ m/s. The interaction time of the atom cloud with the TEM_{00} mode of the cavity amounts to about 17.5 μs, which is much lager than the gate operation time t_I, about $\pi\Omega/(\sqrt{N}g^2)\sim6$ ns. In addition, we should switch off the external field (generally, the coupling strength Ω is one order of magnitude higher than g for the sake of the validity of the effective Hamiltionian H_{eff} and enough coupling strength $G=\sqrt{N}g^2/\Omega$) before and after the gate operations, so the atomic spontaneous emission possibly plays an important role due to the resonant interaction between the horizontally polarized cavity mode and the atomic transition $|1\rangle\leftrightarrow|2\rangle$. To overcome this imperfection, we can introduce another external classical laser and an auxiliary atomic ground state $|\mathrm{aux}\rangle$, which is beyond the above-mentioned state space of system evolution [25]. For instance, the atoms can be transferred into the stable ground state $|\mathrm{aux}\rangle$ from $|1\rangle$ by operating the additional external classical lasers before and after the gate operations [25]. In this case atomic spontaneous emission no longer plays a significant role due to the neglectable population of the excited states $|2\rangle$ and $|4\rangle$.

In summary, we have proposed a scheme to carry out a two-qubit phase gate of intracavity modes by using the four-level N-type atomic ensemble as a coherent interaction medium. We also discuss the influence of the atomic spontaneous emission and the decay of the cavity modes on the photon loss and gate fidelity. It is shown that the quantum phase gate has ultrahigh fidelity and small error rate without the strict strong-coupling condition and Lamb-Dicke limit.

ACKNOWLEDGMENTS

We acknowledge the fruitful discussions with Dr. Xiang-Fa Zhou. This work was funded by the Innovation funds and "Hundreds of Talents" program from Chinese Academy of Sciences, and National Natural Science Foundation of China (Grant No. 10674128).

[1] J. M. Raimond *et al.*, Rev. Mod. Phys. **73**, 565 (2001).
[2] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45**, 5193 (1992); B. M. Garraway, B. Sherman, H. Moya-Cessa, P. L. Knight, and G. Kurizki, *ibid.* **49**, 535 (1994); K. Vogel, V. M. Akulin, and W. P. Schleich, Phys. Rev. Lett. **71**, 1816 (1993); C. K. Law and J. H. Eberly, *ibid.* **76**, 1055 (1996).
[3] E. Hagley *et al.*, Phys. Rev. Lett. **79**, 1 (1997); S. Osnaghi *et al.*, *ibid.* **87**, 037902 (2001).
[4] A. Rauschenbeutel *et al.*, Science **288**, 2024 (2000).
[5] M. Brune *et al.*, Phys. Rev. Lett. **77**, 4887 (1996).
[6] S. Brattke, B. T. H. Varcoe, and H. Walther, Phys. Rev. Lett. **86**, 3534 (2001); P. Bertet *et al.*, *ibid.* **88**, 143601 (2002).
[7] A. Rauschenbeutel *et al.*, Phys. Rev. A **64**, 050301(R) (2001).
[8] M. S. Zubairy, M. Kim, and M. O. Scully, Phys. Rev. A **68**, 033820 (2003).
[9] R. Garcia-Maraver *et al.*, Phys. Rev. A **70**, 062324 (2004).
[10] V. Josse, A. Dantan, A. Bramati, M. Pinard, and E. Giacobino, Phys. Rev. Lett. **92**, 123601 (2004).
[11] M. Hennrich, A. Kuhn, and G. Rempe, Phys. Rev. Lett. **94**, 053604 (2005).
[12] A. SondbergSorensen and K. Mølmer, Phys. Rev. A **66**, 022314 (2002).
[13] M. D. Lukin, S. F. Yelin, and M. Fleischhauer, Phys. Rev. Lett. **84**, 4232 (2000).
[14] R. Guzman, J. C. Retamal, E. Solano, and N. Zagury, Phys. Rev. Lett. **96**, 010502 (2006); A. S. Parkins, E. Solano, and J. I. Cirac, *ibid.* **96**, 053602 (2006).
[15] C. Ottaviani, S. Rebic, D. Vitali, and P. Tombesi, Phys. Rev. A **73**, 010301(R) (2006).
[16] A. Dantan, N. Treps, A. Bramati, and M. Pinard, Phys. Rev. Lett. **94**, 050502 (2005).
[17] L.-M. Duan, A. Kuzmich, and H. J. Kimble, Phys. Rev. A **67**, 032305 (2003).
[18] The effective Hamiltonian is derived using techniques developed in D. F. V. James, Fortschr. Phys. **48**, 823 (2000) (Appendix I).
[19] X. Zou, Y. Dong, and G. Guo, Phys. Rev. A **73**, 025802 (2006).
[20] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998); A. S. Sørensen and K. Mølmer, Phys. Rev. Lett. **91**, 097905 (2003); L.-M. Duan and H. J. Kimble, *ibid.* **90**, 253601 (2003); C. Simon and W. T. M. Irvine, *ibid.* **91**, 110405 (2003); D. E. Browne, M. B. Plenio, and S. F. Huelga, *ibid.* **91**, 067901 (2003).
[21] J. McKeever *et al.*, Science **303**, 1992 (2004); A. Kuhn, M. Hennrich, and G. Rempe, Phys. Rev. Lett. **89**, 067901 (2002).
[22] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**, 3221 (1997); J. Hong and H.-W. Lee, *ibid.* **89**, 237901 (2002); T. W. Chen, C. K. Law, and P. T. Leung, Phys. Rev. A **68**, 052312 (2003); A. Biswas and G. S. Agarwal, Phys. Rev. A **69**, 062306 (2004), and references therein.
[23] D. Fattal, R. G. Beausoleil, and Y. Yamamoto, eprint quant-ph/0606204.
[24] M. Hennrich, T. Legero, A. Kuhn, and G. Rempe, Phys. Rev. Lett. **85**, 4872 (2000); P. Maunz *et al.*, *ibid.* **94**, 033002 (2005).
[25] Two classical laser pulses can be used to change the occupation between two atomic ground states. For example, see Y.-F. Xiao, Z.-F. Han, J. Gao, and G.-C. Guo, J. Phys. B **39**, 485 (2006).