Timing of pair production in time-dependent force fields

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We examine the creation and annihilation dynamics for electron-positron pairs in a time-dependent but subcritical electric force using a simplified model system. Numerical and semianalytical solutions to computational quantum field theory show that despite the continuity of the quantum field operator in time, the actual number of created particles can change in a discontinuous way if the field changes abruptly. The number of permanently created particles after the pulse, however, increases continuously with the duration of the electric field pulse, suggesting a transition from an exclusive annihilation to a creation regime.

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Most predictions of traditional quantum field theory are based on the computation of various transition amplitudes and perturbative expansions of the interactions [1,2]. A prediction by this theory is the possibility of creating matter in the form of electron-positron pairs from the vacuum if a very strong (supercritical) field is present. This process has been observed experimentally in heavy-ion collisions and studied based on the strong-field approximation to quantum field theory [3]. In this case the creation process is possible when the sum of the Coulomb fields of the two colliding ions becomes supercritical. The same quantum transitions that lead to the creation of a particle pair from vacuum can also be obtained for a weaker (subcritical) field if the force is time dependent. For instance, at energies around 10 GeV/u, where electron-positron pairs can be produced in uranium-uranium collisions, the corresponding electromagnetic field can contain frequency components that exceed twice the rest mass energy of the electron [4,5] and for certain impact parameters the supercritical field has not been reached at all. In nuclear collisions with muon production [6] the static field cannot become supercritical because of the much heavier muon mass. For further reviews, see Refs. [7–9].

As the development of high-power laser sources is advancing it has been conjectured that light can be converted directly into matter in such a focused laser pulse. Recently the work by Blaschke *et al.* [10] suggested that even with optical lasers in the 10^{20} W/cm^2 range, about 5–10 pairs could be generated per laser pulse [11–13].

About four years ago, we have begun to examine the various pair-creation processes from a more fundamental point of view, by analyzing the dynamics via a fully temporally and spatially resolved perspective based on computational quantum field theory for simplified model systems. These studies are still in their cradle stage but the computer simulation of the matter creation process enabled us to revisit several controversial aspects about the interpretation of quantum field theory. These studies used one-dimensional systems to resolve the Klein paradox [14,15] and the relativistic localization problem [16] and explored the spatial dynamics of the creation of bound states in a supercritical environment [17].

As a microscopic view is necessary to understand the details of this fascinating matter creation process, several questions about the spatial and temporal correlation of the pair-creation process need to be addressed. Can one choose an arbitrarily short pulse duration to create a particle? Are the pairs created truly simultaneously or can one particle arise a little bit delayed from the other? These questions are challenging, as the definition of a particle and its creation and annihilation operators inside a spatially and temporally varying external field is nontrivial [3,18], but they need to be addressed in order to predict the properties of the particles when detected inside and outside the field.

In this paper we report on progress with regard to the time scales required for the temporary and permanent creation. We consider an electric field that is subcritical. As a static subcritical field by itself is too weak to produce pairs, this choice therefore automatically guarantees that a pair can be created only during those moments at which the force is time dependent. The total number of particles decreases with an increased duration of the turn-on and off times, as in the asymptotic limit of an adiabatic field no pairs can be created neither during nor after the subcritical force field. By appropriately choosing the durations of these time intervals, we can therefore control the precise moments when a pair is created. To examine the minimum required time scale we consider also an abruptly turned on field. The quantum field operator as a solution to the time-dependent Dirac equation must be a continuous function of time even in this limit. One could therefore (incorrectly) conjecture that a sudden creation of the electron would be prohibited by quantum field theory. A more careful analysis below suggests that—in principle—the number of temporarily created pairs can indeed change abruptly in time. However, this number varies continuously as a function of the total duration of the electric field and therefore requires a pulse of nonzero duration to create a detectable (permanent) particle outside the focus.

The time evolution of the quantum field operator for the electron-positron complex $\hat{\Psi}(t)$ can be obtained as a solution to the Dirac equation (in atomic units) $i(\partial/\partial t)\ \hat{\Psi}=H(t)\hat{\Psi}$, with the Hamiltonian $H(t)=c\alpha_z p_z+\beta c^2+V(t)$, where α_z is the z component of the 4×4 Pauli matrix, β is the diagonal matrix, and c is the speed of light [19]. The details of the

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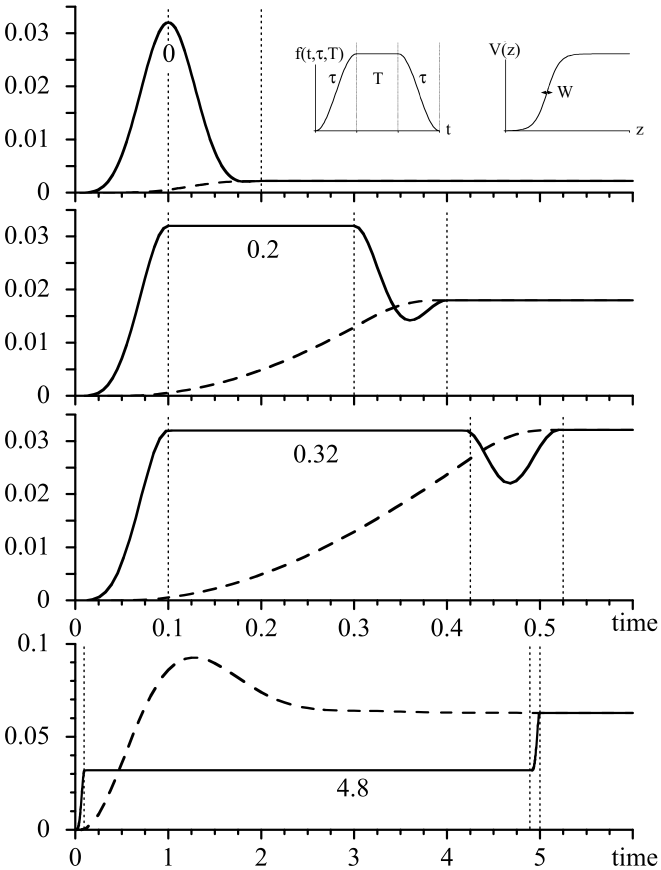

FIG. 1. The total number of electron-positron pairs $N(t)$ as a function of time for the interaction of an initial vacuum state with a subcritical potential $V(z)$, that is turned on from $t=0$ to $t=\tau$ and turned off from $t=\tau+T$ to $t=2\tau+T$. The time is in units of 1.29 $\times 10^{-21}$ s $(=1/c^2)$. The dashed line is the prediction based on the projection of $\hat{\Psi}(t)$ on the field-free electronic states, $\hat{\Psi}_e^{(p)}(t)$, denoted by $N^{(p)}(t)$. The inset on the top shows the pulse shape and spatial dependence of the potential ($V_0=2c^2-10^4$ a.u., $W=0.3/c$, $\tau=0.1/c^2$).

spatial representation of $V(t)$ are not too important, we have chosen a simple steplike potential [20,21] of the form $V(z,t)=V_0\,[\tanh(z/W)+1]/2\,f(t,\tau,T)$ that is characterized by the width W of the region where the corresponding force, proportional to the derivative $V'(z)$, is nonzero. We chose $W=0.3/c$ and $V_0=2c^2-10^4$ a.u. making $V(z,t)$ subcritical. The time-dependent part (shown in Fig. 1) is given by $f(t,\tau,T)\equiv\sin^2[t\pi/(2\tau)]\theta(t;0,\tau)+\theta(t;\tau,\tau+T)+\cos^2[(t-T-\tau)\pi/(2\tau)]\ \theta(t;\tau+T,T+2\tau)$ where $\theta(t;x_1,x_2)$ is the square pulse function which is 1 only if $x_1\leqslant t<x_2$ and zero otherwise. It naturally divides the time into four regions, $t<0$ and $2\tau+T\leqslant t$, for which there is no force, $0\leqslant t<\tau$, during which the potential is turned on, the plateau region of duration T, $\tau\leqslant t<\tau+T$, during which the force is constant, and the turn-off region $\tau+T\leqslant t<2\tau+T$. Consequently, the Dirac Hamiltonian, is time dependent only during the two time intervals of duration τ each.

To solve the Dirac equation numerically, the operator $\hat{\Psi}$ can be expanded as the sum (integral) over the Fermion creation and annihilation operators, $\hat{\Psi}(t)=\Sigma_p\hat{b}_p(t)|p\rangle+\Sigma_n\hat{d}_n^{\dagger}(t)|n\rangle$, where $|p\rangle$ and $|n\rangle$ are (four-component) positive and negative energy eigenstates of $H(t=0)$. We obtain as a solution

$$\hat{b}_p(t)=\Sigma_{p'}\hat{b}_{p'}(t=0)\langle p|U(t)|p'\rangle+\Sigma_{n'}\hat{d}_{n'}^{\dagger}(t=0)\langle p|U(t)|n'\rangle, \tag{1a}$$

$$\hat{d}_n^{\dagger}(t)=\Sigma_{p'}\hat{b}_{p'}(t=0)\langle n|U(t)|p'\rangle+\Sigma_{n'}\hat{d}_{n'}^{\dagger}(t=0)\langle n|U(t)|n'\rangle, \tag{1b}$$

where the coefficients are the matrix elements of the (time-ordered) unitary propagator $U(t)\equiv\exp[-i\int_0^t dt' H(t')]$ between the basis states. These matrix elements can be computed numerically using a space-time grid fast Fourier transform method described in Refs. [22,23].

In order to extract the electronic properties of $\hat{\Psi}$, the field operator can be projected onto the subspace of the corresponding upper-energy eigenstates of the (single-particle) Dirac Hamiltonian, $\hat{\Psi}_e(t)\equiv\Sigma_p|P(t)\rangle\langle P(t)|\ \hat{\Psi}(t)$. It is important to note that the states $|P(t)\rangle$ are not time-dependent solutions of the Dirac equation, but instantaneous energy eigenvectors of $H(t)$, i.e., they are obtained via $[c\alpha_z p_z+\beta c^2+V(t)]|P(t)\rangle=E_p(t)\ |P(t)\rangle$. Therefore—in contrast to time-integrated solutions—the states $|P(t)\rangle$ have no memory of the past time dependence of $H(t)$. These instantaneous states serve as a yardstick to decide which portion of $\hat{\Psi}(t)$ can be associated with electrons. In the special (and better known) case of a time-independent Hamiltonian for $V(z,t)=0$ this projection reduces to the so-called positive frequency part [18,24]. The time evolution of the density operator for the electrons is therefore given by the Fock space scalar product $\rho_e(t)\equiv\langle\text{vac}\|\hat{\Psi}_e(t)^{\dagger}\hat{\Psi}_e(t)\|\text{vac}\rangle$. The multiparticle state $\|\text{vac}\rangle$ is the Fock representation of the vacuum of the force-free Hamiltonian $c\alpha_z p_z+\beta c^2$, and serves as our initial state before any interaction $V(z,t)$ is turned on. The trace of this density operator represents the total number of created pairs, $N(t)=\text{tr}\ \rho_e(t)$. This quantity can be obtained from the matrix elements of $U(t)$ via

$$\begin{aligned} N(t)=\text{tr}\ \rho_e(t)&=\Sigma_P\langle P(t)|\rho_e(t)|P(t)\rangle \\ &=\Sigma_P\Sigma_n|\langle P|U|n\rangle|^2 \\ &=\Sigma_P\Sigma_n|\Sigma_p\langle P|p\rangle\langle p|U|n\rangle \\ &\quad+\Sigma_{n_i}\langle P|n_1\rangle\langle n_1|U|n\rangle|^2. \end{aligned} \tag{2}$$

For the limiting case of an abrupt turn-on ($\tau=0$), the time evolution of $\Psi(t)$ can be obtained semianalytically, and requires only the eigenvectors $|P\rangle$ and their eigenenergies obtained from $[c\alpha_z p_z+\beta c^2+V]\ |P\rangle=E_P|P\rangle$. The time evolution of $\hat{\Psi}(t)$ leads to $N=\Sigma_P\Sigma_n|\langle P|n\rangle|^2$.

In Fig. 1 we have graphed the time dependence of the number of created pairs for pulses of four durations T of the plateau. Consistent with our expectation, we see in each curve that the number of pairs can only change during those moments when the force is time dependent. If the turn-on duration were very short, the resulting number of particles would jump, associated mathematically with a discontinuity of the electronic projector $\Sigma_P|P(t)\rangle\langle P(t)|$ and $\hat{\Psi}_e(t)$, whereas $\hat{\Psi}(t)$ is continuous.

In the top graph of Fig. 1, we show that if the turn on is followed directly by the turn off ($T=0$), almost no permanent population remains after the field is turned off. The turn off immediately reverses the creation and the pairs can annihilate. There are two mechanisms responsible for this annihilation. First, the turn on has already created pairs that occupy certain states which—as a consequence of Pauli principle—block out [14,15,25–27] the creation of further pairs. Second, as the state is nearly invariant under velocity time reversal, the turn off must return the system to its original state, the field-free vacuum $\|\text{vac}\rangle$.

The second graph is obtained for a nonzero plateau time ($T=0.2/c^2 \approx 2.6\times 10^{-22}$ s) and shows that there is now a permanent electron population that survives the turn off. This irreversible creation of pairs is associated with the fact that the particles have enough time to leave the force region, they "roll down" the very force field that created them. With the exception of errants, discussed in Ref. [28], the electrons and positrons are ejected in opposite directions by the force field and can no longer recombine as they are too far apart from each other. Consequently only a small portion that is still close to the gradient region of the potential can be annihilated during the turn-off interval.

The third graph was chosen for a special time duration $T=0.32/c^2$ to show that—at least in principle—the creation and annihilation can exactly cancel during turn off. The fourth graph corresponds to a very long time interval ($T=4.8/c^2$) when all of the originally created particles had enough time to leave the focus and the turn off can create another burst of particles of equal number as the turn on. This turn-off burst is similar to the Haan effect [29,30], initially observed in strong-field photoionization of hydrogen and later also predicted for two-electron systems [31]. The Haan effect is associated with the field dressing of atomic continuum states.

The data have suggested that in order for the electrons to become permanent and detectable after the pulse, they must be able to survive the turn off, which requires a certain minimum amount of time. To explore this particular time scale, we analyze in Fig. 2 the number of pairs after the potential is turned off, $S(T)\equiv N(t=T+2\tau)$, as a function of the pulse duration of the field T. To obtain the best contrast in comparison to $N(t)$ in Fig. 1, we choose zero turn on and off times ($\tau=0$). We see that in contrast to the discontinuous time dependence of the actual electron number $N(t)$, the survival probability as a function of the interaction duration T is continuous in time and shows that the temporary electrons require a minimum time of the order of $1/c^2$ to become permanent. We should point out, however, that temporary electrons are real and should not be confused with virtual particles that are sometime introduced to visualize quantum fluctuations.

If $\hat{\Psi}(t)$ is continuous in time, how can it predict at all correctly the permanent creation of pairs, if the force is time dependent during only two infinitely short moments? The answer is the ghost states that were introduced in Ref. [32]. To illustrate the occurrence of these states we could project $\hat{\Psi}(t)$ onto the field-free states with positive energy, defined as $(c\alpha_z p_z+\beta c^2)|p\rangle = \sqrt{(c^4+c^2p^2)}|p\rangle$, to define an electronic operator as $\hat{\Psi}_e^{(p)}(t)\equiv \Sigma_p |p\rangle\langle p|\hat{\Psi}(t)$. This quantity is continuous in time and was used in several previous works [14–17,21,28,32]. All quantities obtained from $\hat{\Psi}_e^{(p)}(t)$ during the interaction need to be interpreted as those properties that the "physical" particles would take if the interaction were turned off instantaneously. One could view this particular projection as an alternative definition of what one could call a particle during the interaction time. The physically reasonable time dependence of $N(t)$ based on $\hat{\Psi}_e(t)$, however, makes $\hat{\Psi}_e(t)$ a more plausible description for a particle *inside* the interaction region.

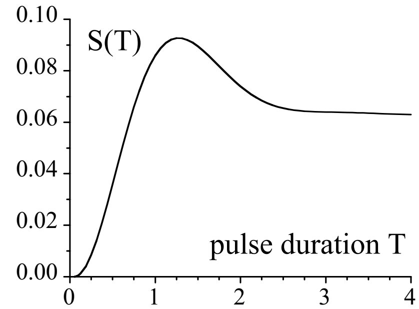

FIG. 2. The total number of electron-positron pairs $S(T)$ as a function of the duration of the electric field pulse T. The pulse duration T is in units of 1.29×10^{-21} s ($=1/c^2$). For simplicity the field was turned on and off abruptly ($V_0=2c^2-10^4$ a.u., $W=0.3/c$, $\tau=0$).

Using the operator $\hat{\Psi}_e^{(p)}(t)$ for the corresponding density operator, we obtain similarly as above

$$\rho_e^{(p)}(t) = \langle \text{vac}\|\hat{\Psi}_e^{(p)}(t)^\dagger \hat{\Psi}_e^{(p)}(t)\|\text{vac}\rangle,$$

whose trace we define as

$$N^{(p)}(t) = \text{tr}_p\, \rho_e^{(p)}(t) = \sum\nolimits_p \langle p|\rho_e^{(p)}(t)|p\rangle = \sum\nolimits_p \sum\nolimits_n |\langle p|U|n\rangle|^2$$

The numerical prediction of $N^{(p)}(t)$ was superimposed as the dashed lines in Fig. 1. It is clear that $N^{(p)}(t)$ agrees precisely with the pair-creation probability $N(t)$ before and after the potential is present and is identical to the predictions of the in- or out-formalism of the S-matrix theory [2,3]. However, during the interaction $N^{(p)}(t)$ is different from $N(t)$. It is interesting to note that as a consequence of the required continuity of $\Psi_e^{(p)}(t)$, this quantity grows much slower, but then it even "overshoots" and approaches a value that is twice that of $N(t)$. The spatial (for $T=4.8/c^2$) manifestation of this overshoot effect are the ghost states [32] that are localized at those spatial regions where the potential's gradient is nonzero. If the potential is turned off, these ghost states become real electrons, corresponding to the creation of permanent particles during the turn-off time.

In summary, we have shown that a suitable projection of the quantum field operator $\hat{\Psi}(t)$ based on the subspace of instantaneous electronic energy eigenstates can provide a consistent and intuitive picture of pair-creation and annihilation processes triggered by a time-dependent and spatially dependent force. This approach can now be used to compute for the first time spatial, momentum and energy densities of created particles inside and outside the force field and to help to tackle other challenges outlined earlier.

As mentioned in the Introduction, computational quantum field theory is only in its cradle stage and relies presently on several simplifying assumptions to overcome several conceptual as well as computational difficulties. For example, the calculations are restricted to only one spatial dimension and the effect of the magnetic field is not taken into account. Furthermore, a truly abruptly turned-off field would violate causality and the very concept of an external force relies on the strong field approximation. Also the Coulombic fermion-fermion interaction (based on photon exchange) is not included in the present approach. To overcome this restriction the (second-quantized) vector potential of the photon field would have to be included as a fully dynamically varying variable requiring the very challenging solution of the coupled Dirac-Maxwell equations.

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