Multilevel interference of a neutron wave

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We present an analytical and numerical analysis of neutron multilevel interference phenomena generated when a neutron passes through a series of N resonant coils operated at the successive conditions $\hbar(\omega_0+n\Delta\omega)=2\mu_n(B_0+n\Delta B)$ with $n=0,1,\ldots,N-1$. Each coil produces spin flip with probability ρ between 0 and 1; thus the number of waves for the neutron is doubled after each coil, finally giving 2^N interfering neutron waves. The phase difference between any pair is a multiple of a time dependent "phase quantum" $\Delta\Phi(t)$. The analysis predicts for each number N a highly regular pattern for the quantum mechanical probability to find the neutron spin in one specific state as a function of ρ and $\Delta\Phi$. These patterns evolve in time and show revivals after a time T determined by the step $\Delta\omega$ according to $T=2\pi/\Delta\omega$. For some adjustments of the system an analytical solution is obtained. Application of multilevel interference in high-resolution neutron modulated-intensity-by-zero-effort–type spectrometers is discussed.

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I. INTRODUCTION

Multipath interference in optics [1,2] and multimode interference in dynamical systems [3] has recently emerged as an extremely active field of research. Dynamical systems with a broad spectrum of excitations, when all the levels are populated, reveal rich interference patterns in both time and space [4,5]. Particularly, large scale interference leads to well-ordered long-range regularities (such as quantum revivals [5]) in the time-space probability distribution of the wave function. Therefore it is of great interest to prepare a wave packet in a controlled way and measure its multimode or multipath interference.

In our previous paper [6] we studied multipath interference of a neutron during passage through N resonant coils in a dc field B_0, each flipping the neutron spin with a probability ρ between 0 and 1, interspaced by regions of length L with a homogeneous magnetic field B_1. For this study we used Ramsey's resonance method of the "separated oscillating fields." The same configuration was described in Refs. [7,8] for two coils only. It was found that after the first resonant coil the neutron wave is split into two waves for the two different spin states. In the subsequent region with field B_1, these waves collect opposite phase shifts. In the next resonance coil each neutron wave is split again, thus making four waves. Hence, after N resonance coils we have 2^N interfering waves. Each pair contributes to a highly regular pattern for the quantum mechanical (QM) probability to find the neutron spin in a specific state (e.g., "up"), in a two-dimensional space subtended by the "axes" spin flip probability ρ and line integral $(B_1-B_0)L$. We derived an analytical expression for this probability as a function of these parameters and the number N. This expression was testified both by computer calculations and by neutron experiments. The experimental data were consistent with theory. We point out that the pattern in this 2D space is stationary, since all interfering waves correspond to states with the same energy and wave vector.

In the present paper we discuss again a set of N resonant coils in series, but now operated at successively increasing frequencies $\omega_0+i\Delta\omega$ $(i=0\cdots N)$. Again, the neutron wave is split into 2^N waves, however, each with different energy and wave vector. We will see that the phase difference between any pair of waves is a multiple of a phase quantum $\Delta\omega t$, i.e., depending on time. Moreover, an energy spectrum of equidistant levels occupied according to a binomial distribution is created. The resulting multilevel interference pattern is not stationary, but evolves in time, giving revivals on a time scale $T=2\pi/\Delta\omega$. The shape of the pattern is determined by the number of resonant coils N and the spin flip probability of one coil ρ.

This phenomenon has much in common with the neutron resonant spin echo (NRSE) method recently developed [9–11], based on earlier works on the resonant interaction of neutrons with time-dependent magnetic fields [12–14]. Thus, a high resolution spectrometer for quasielastic neutron scattering was proposed on the basis of the NRSE method with the resonant coils of different frequencies [15]. It has received the name modulation of intensity by zero effort (MIEZE) and produces a sinusoidal intensity modulation of the incoming beam. The first scattering experiment on MIEZE had been performed and proved the possibilities of this technique [16]. Furthermore, a combination of several MIEZE setups with one common detector position was proposed to get a periodic signal of arbitrary time shape [17]. In this scheme sharp signals well separated in time are possible. However, each MIEZE setup needs its own device for polarization analysis, limiting in practice their number to values around 5. In the present paper we lift out the analyzers and demonstrate that multilevel MIEZE may be realized in an easier way.

We give a theoretical treatment of multimode interference of neutron waves. The concept is developed in Sec. II and we describe how $2N$ neutron waves appear in an experiment with N resonant coils. We derive analytical expressions for the interference in the case of identical and nonidentical coils. Numerical calculations for the interference are given in Sec. III. Section IV presents both a short discussion and final conclusion.

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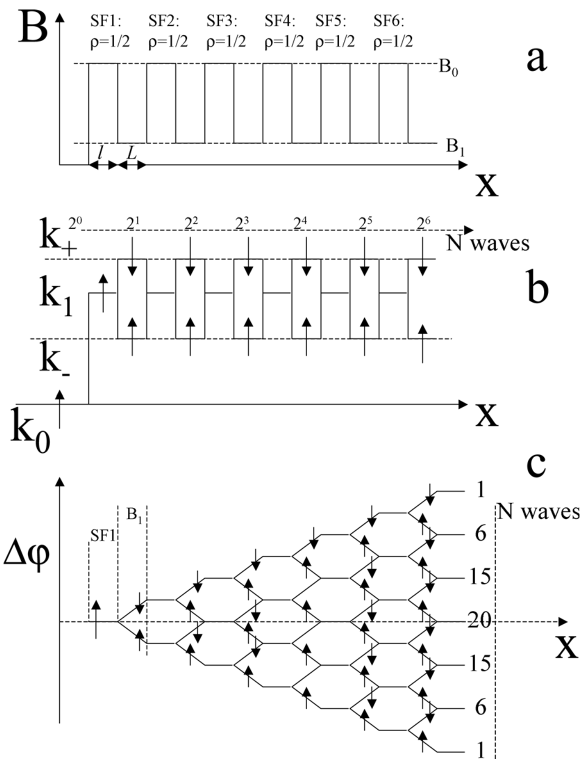

FIG. 1. (a) System with many ($N=6$) identical resonant coils with flip probability $0<\rho<1$ in dc fields B_0 separated by segments with field B_1, (b) (k,x) diagram of the wave vectors of the waves arising after the successive resonant coils due to the incomplete flip, (c) Diagram of the phases $\Delta\varphi$ of the waves produced in the successive resonance coils relative to a wave which would go at undisturbed level $k=k_1$ through the system.

II. NEUTRON MULTIWAVE RESONANCE INTERFERENCE

A. Identical resonant devices

The succession of magnetic fields B_0 and B_1, which is N times repeated, is shown in Fig. 1(a). rf coils in the path sections with field B_0 are operated at the resonance frequency ω_0. To understand interference between neutron waves in this configuration of fields we need to solve the Schrödinger equation, in which each rf coil is described as a (2×2) matrix $\hat{C}$ [given in Eq. (18) below] operating on the 2D complex spinor

$$\psi(\vec{r},t)=\begin{pmatrix}\alpha(t_1)\exp(ik_0x+\omega t)\\ \beta(t_1)\exp(ik_0x+\omega t)\end{pmatrix},\quad (\sqrt{\alpha^2+\beta^2}=1 \text{ at any time})$$

representing the spin state of the neutron at entrance time t into the coil after the neutron entered the first coil at t_1. Before presenting the mathematics of its solution in Sec. III, we give an intuitive description [7–12].

A valid solution is a plane neutron wave with wave number k_0, traveling along the x axis through the field configuration defined above. When the neutron enters the field B_0, the wave number k_0 changes to k_1. By energy conservation, the total energy $\hbar\omega$ does not change and the resulting wave number k_1 differs from k_0 in first approximation as

$$k_1=k_0+\mu_n B_0/(\hbar v), \tag{1}$$

where m_n, μ_n, and v are the mass, the magnetic moment, and the velocity of the neutron, respectively.

When spin flip occurs, a photon of energy $\hbar\omega_0$ is exchanged between the neutron state and the rf field, i.e., the neutron spin state with momentum k_1 gains or loses an amount of potential energy $\Delta E=2\mu_n B_0$. When the neutron passes the field boundary from B_0 to B_1, this potential energy is released as a kinetic energy change. We assume that the spin flip in the rf field was not complete but only partial, with a probability $\rho=1/2$, for all resonance coils. Then at this boundary the neutron wave is split into 2 plane waves with wave numbers k_+ and k_- corresponding to the spin states $\binom{1}{0}$ (up) and $\binom{0}{1}$ (down), respectively. Again, by energy conservation, the total energy corresponding to each state does not change at the transition from B_0 to B_1, so the wave numbers k^+ and k^- in the first approximation are

$$k_\pm=k_1\pm\frac{\mu_n(B_0-B_1)}{\hbar v}. \tag{2}$$

So, the initial neutron wave is split into a "nonflipped" and "flipped" part with wave vectors k_- and k_+ and energies $\hbar\omega$ and $\hbar(\omega+\omega_0)$. After the next coil each of these waves is split again into two waves with equal amplitudes—and so on. Thus, after N coils the initial wave is split into two groups of 2^{N-1} neutron waves with amplitudes $(1/2)^N$ of the initial wave. In the first group the neutron was flipped an odd number of times and therefore these waves have spin state "down." The energy of the states corresponding to these waves is $\hbar(\omega+\omega_0)$. They are located at the upper k level of diagram Fig. 1(b). In the other group the neutron was flipped an even number of times (or not flipped at all), so these waves have spin state "up." In the states corresponding to these waves the neutron has the same energy $\hbar\omega$ as initially. These waves are at the lower k level. In (k,x) space [Fig. 1(c)] we can follow the phase shifts $\Delta\varphi$ of the individual waves relative to the phase value $\varphi_0=k_1x$. Each wave in each group has its specific path in this diagram.

At any position after the system of resonance coils the phase difference for an arbitrary pair is $m\Delta\phi$, where $m=0,1,\ldots,N$ and

$$\Delta\phi=\int_0^l[k_+(x')-k_-(x')]dx' \tag{3}$$

is the line integral over one path section with field B_1. Thus, $\Delta\phi$ is a quantum of phase. The amplitude of the wave with a given phase shift $m\Delta\phi$ ($m=0,1\ldots,N$) is determined by three factors.

(i) The spin flip probability of one rf coil

$$\rho=\sin^2\left(\frac{2\mu_n}{\hbar}B_{\rm rf}\tau/2\right)\equiv\sin^2\xi \tag{4}$$

(so ρ depends on the amplitude of the rf field $B_{\rm rf}$ and the residence time τ, which is proportional to the neutron wavelength λ and the length of the rf coil l).

(ii) The number of flipping events m.

(iii) The number A_m of pathways in (k,x) space having this particular phase shift $m\Delta\phi$ after N coils (binomial distribution):

$$A_m = \frac{N!}{m!(N-m)!}.$$

Thus, the waves with the spin state "up" can be summarized as

$$\chi_1 = \sum_{m=1}^{N-1} \frac{A_m}{2^N} (\sin \xi)^{N-m} (\cos \xi)^m \exp(im\Delta\phi) \tag{5}$$

and the waves with the spin state "down" as

$$\chi_2 = \sum_{m=1}^{N-1} \frac{A_m}{2^N} (\sin \xi)^{m} (\cos \xi)^{N-m} \exp(-im\Delta\phi). \tag{6}$$

According to quantum mechanics the probability R for the neutron spin to collapse into the spin state "up" or "down" is equal to $|\chi_1|^2$ or $|\chi_2|^2$, respectively, with the polarization component P_{zz} along the z axis given as

$$P_{zz} = |\chi_1|^2 - |\chi_2|^2. \tag{7}$$

This problem was considered in detail in Sec. IV of Ref. [6] and the quantitative solution was obtained through the matrix method. The analytical expression for the probability R is

$$R = \rho \frac{\sin^2(N\gamma/2)}{\sin^2(\gamma/2)}, \tag{8}$$

where the angle γ is given by

$$\cos(\gamma/2) = \sqrt{1-\rho}\cos(\Delta\phi/2). \tag{9}$$

In this expression $\Delta\phi = (\mu_n/2\hbar)(B_1 - B_0)L/v$ is identical to the phase quantum Eq. (3).

B. Nonidentical resonance devices

Equations (8) and (9) imply that for identical resonance devices one obtains a stationary interference pattern in a 2D space subtended by the axes: field B_1 and spin flip probability ρ. However, in some cases, mentioned below, it is important that a time evolution occurs.

For this purpose, let us take N resonant coils, adjusted such that they have the successive resonance conditions fulfilled

$$\hbar(\omega_0 + i\Delta\omega) = 2\mu_n(B_0 + i\Delta B), \tag{10}$$

where i $(i=1,\ldots,N)$ is the number of the coil. So, the resonance frequency and magnetic field increase from one coil to the next by $\Delta\omega$ and ΔB, respectively [Fig. 2(a)]. Again we suppose that each coil has flip probability $\rho<1$, so a neutron wave entering the system is doubled after each coil and 2^N waves exist at the end of the system. As in the case of identical coils, each wave has its own path in the (k,x) diagram [Fig. 2(b)].

To find an analytical expression for the behavior of the neutron wave, similar to the case of identical coils in Eqs.

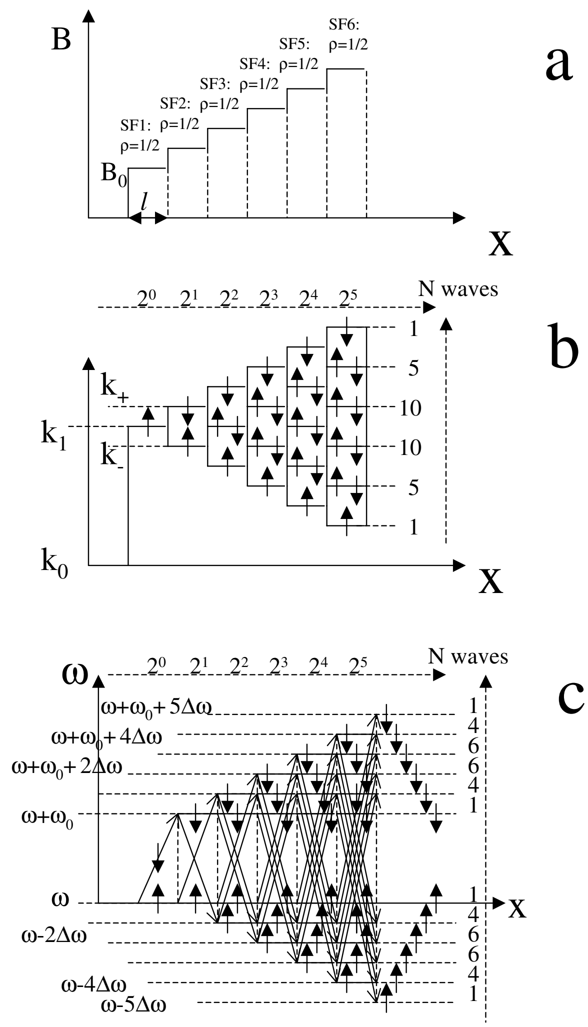

FIG. 2. (a) System with many nonidentical resonant coils with successively increasing dc fields $B_0+n\Delta B$ $(n=1\ldots n-1)$ giving resonance conditions $(B_0+n\Delta B)=(\hbar/2\mu_n)(\omega_0+n\Delta\omega)$. (b) (k,x) diagram of the wave vectors and their numbers arising after the successive resonance coils. (c) Development of the multilevel (ω,x) diagram of the energies of the waves produced in the system.

(7)–(9), we must split the problem in two steps. First, the path through the system of coils $0<x<N(l+L)$. Here the neutron wave, with energy ω, is N times flipped and split into waves with wave vectors $k_0 - i\Delta k$ $(i=1,\ldots,N)$, and now also with different energies $(\omega+i\Delta\omega)$, illustrated in Fig. 2(c). Each wave acquires its specific phase as a result of its history through the system, which is a multiple of the phase shift inside one resonance coil $\Delta\phi=\Delta\omega\tau=2l\Delta k$. Here τ is the residence time in one coil. So we create a phenomenon of "multilevel" interference.

The second step is the space after the system of coils: $x>N(l+L)$. Here these waves interfere and produce a pattern evolving in time and space. We want to express the interference analytically in the same way as in the case of identical coils. However, the phase of the individual waves contains the phase history of the first step, which is different for all 2^N waves.

To eliminate the effect of these phase shifts we adjust the frequency step $\Delta\omega$ and the time τ such that we fulfill the condition

$$\Delta\phi = \Delta\omega\tau = 2\Delta k l = n2\pi, \tag{11}$$

where n is an integer number. Then, at the end of the system of coils the phase differences between all N waves are multiples of 2π. As in the previous case, the amplitude of the wave with a given energy and wave vector is determined by (i) the spin-flip probability of one coil $\rho=\sin^2\xi$, (ii) the number of flipping events m, and (iii) the number of waves A_m at a particular energy level (binomial distribution).

Under the condition (11) the waves with the spin state "up" after the system can be summarized as

$$\chi_1 = \sum_{m=1}^{N-1} \frac{A_m}{2^N}(\sin\xi)^{N-m}(\cos\xi)^m \exp[im\Delta\Phi(t,x)] \tag{12}$$

and the waves with the spin state "down" as

$$\chi_2 = \sum_{m=1}^{N-1} \frac{A_m}{2^N}(\sin\xi)^{m}(\cos\xi)^{N-m} \exp[-im\Delta\Phi(t,x)]. \tag{13}$$

In full analogy to the case of identical coils [Eq. (8)], the quantum mechanical probability R can be analytically expressed as

$$R = \rho\frac{\sin^2[(N\gamma(x,t)/2]}{\sin^2[\gamma(x,t)/2]}, \tag{14}$$

where ρ is the spin flip probability of the neutron in one coil [Eq. (4)] and the angle $\gamma(x,t)$ is given by

$$\cos[\gamma(x,t)/2] = \sqrt{1-\rho}\cos[\Delta\Phi(x,t)/2]. \tag{15}$$

The "phase quantum" $\Delta\Phi$ now depends on time t and coordinate x:

$$\Delta\Phi(t,x) = \Delta k(x-x_0) - \Delta\omega(t-t_0), \tag{16}$$

where x_0 is the coordinate of the exit of the system and t_0 the time when the neutron leaves it. For nonidentical resonant devices $\Delta\Phi$ is no longer a quantum of phase, but rather the evolution of time t. We see that the phase differences after the system increase both in time and as observed at increasing distance behind the system. We notice that the phase will reproduce after a "revival time" $t-t_0|_{\text{rev}}$ given by

$$t - t_0|_{\text{rev}} = \frac{2\pi}{\Delta\omega}.$$

This holds at any place behind the system at increasing time t.

III. NUMERICAL EXPERIMENT

In order to testify the theoretical consideration done in the previous section and to formally solve the Schrödinger equation for the full system of N (non)identical resonance coils, a computer calculation was performed. The solution for a neutron entering one coil at time t_1 and leaving it at $t_1+\tau$ (where $\tau=l/v$) can be written [18]

$$\Psi(t_1+\tau) = \hat{C}(t_1,\tau,\omega_0)\Psi(t_1), \tag{17}$$

where the initial spin state of the neutron is $\Psi(t_1) = \alpha(t)\binom{1}{0} + \beta(t)\binom{0}{1}$ with $\alpha=1$ and $\beta=0$. The matrix $\hat{C}(t_1,\tau,\omega_0)$ is a 2×2 matrix of the form

$$\hat{C}(t_1,\tau,\omega_0) = \begin{pmatrix} \cos(\xi)\exp(i\omega_0\tau/2) & -i\sin(\xi)\exp[i\omega_0(t_1+\tau/2)] \\ -i\sin(\xi)\exp[-i\omega_0(t_1+\tau/2)] & \cos(\xi)\exp(-i\omega_0\tau/2) \end{pmatrix}. \tag{18}$$

Here we remind the reader that $\xi=(2\mu_n/\hbar)B_{\text{rf}}\tau/2$.

Then after N resonance coils the neutron wave function can be written

$$\Psi(t_1+N\tau) = \hat{C}(t_N,\tau,\omega_0+N\Delta\omega)\hat{C}[t_{N-1},\tau,\omega_0+(N-1)\Delta\omega]\cdots \times\hat{C}(t_2,\tau,\omega_0+\Delta\omega)\hat{C}(t_1,\tau,\omega_0)\Psi(t_1), \tag{19}$$

where $t_i=t_1+(i-1)\tau$ $(i=1,2,\ldots,N)$. Thus, the calculation involves successive multiplication of the matrices $\hat{C}$ [Eq. (18)] describing the action of one resonant coil.

The polarization component P_i is found by evaluating the well known expression

$$P_i \equiv \langle\sigma_i\rangle = \langle\Psi^*(t_1+N\tau)|\sigma_i|\Psi(t_1+N\tau)\rangle, \tag{20}$$

where $i=x,y,z$ and σ_i are the corresponding Pauli matrices. From this we obtain the quantum-mechanical (QM) probability R according to $R=(1-P_{zz})/2$.

As seen from Eqs. (18) and (19), the pattern of the QM probability R depends on parameters of the system that one can vary.

(i) Obviously it is ruled by the number of resonant coils N. This is the first parameter.

(ii) The second one is ξ, which determines the spin flip probability of the resonant coil according to Eq. (4): $\rho=\sin^2\xi$. We vary ξ from 0 to 2π.

(iii) The third parameter is the frequency step $\Delta\omega$. It is important that its value, combined with the residence time τ, is adjusted such that we fulfill the condition $\Delta\omega\tau=n2\pi$ [Eq. (11)]. Then, the resultant patterns can be also described by Eqs. (14)–(16).

In Fig. 3 we show the QM probability R for systems of $N=2$, 6, and 10 nonidentical coils as a function of the phase

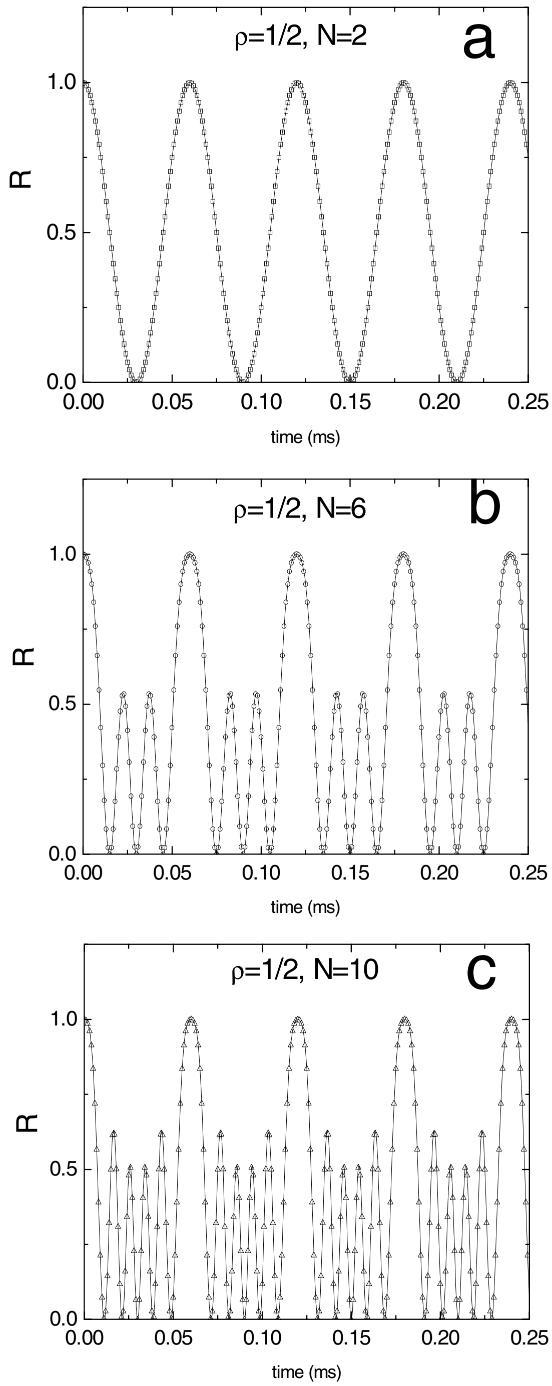

FIG. 3. The QM probability R [according to Eq. (14), lines] to measure the neutron spin state "up" as a function of time t at the exit of systems consisting of $N=2,6,10$ nonidentical resonance coils with spin flip probability $\rho=1/2$. The condition $\Delta\omega\tau=2\pi$ [Eq. (11)] is fulfilled. To define a time scale, the frequency step is set $\Delta\omega=200$ kHz. We notice a periodicity (revival time) equal to $2\pi/\Delta\omega=\pi\times 0.01$ ms. Squares: the same result obtained with the numerical approach [Eqs. (18)–(20)].

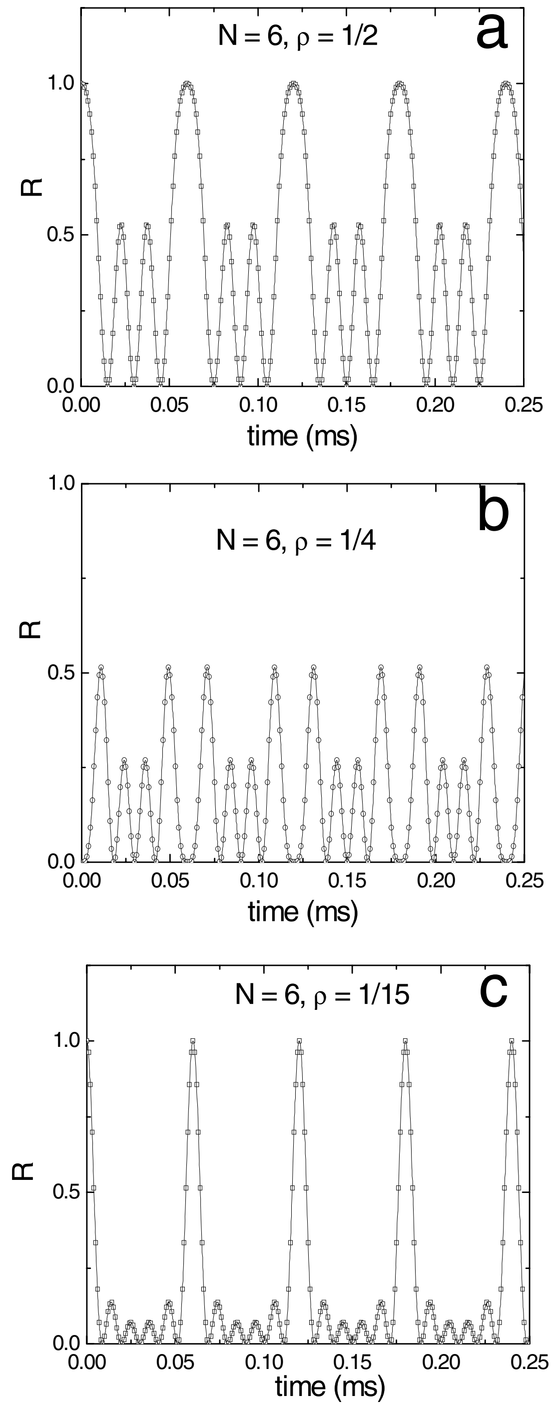

FIG. 4. The same as Fig. 3 for a system of $N=6$ nonidentical resonance coils for various values for the spin flip probability $\rho=\sin^2\xi$ with $\xi=i\pi/(2N)$, $i=1,2,3$. The condition $\Delta\omega\tau=2\pi$ is fulfilled.

$\Delta\Phi(x,t)$ [see Eq. (16)] at $x=x_0$ (just after the last coil), so the phase depends only on time t. For all N the parameter ξ was taken as $\pi/4$, so ρ becomes $1/2$. To define a specific time scale, we set the parameter $\Delta\omega=200$ kHz. For this value we can readily satisfy Eq. (11) for some wavelength in the thermal spectrum with a length of the rf coils equal to a few cm. As seen, the revival time comes at $2\pi/\Delta\omega=\pi 0.01$ ms for this choice. For $N=2$, with Eqs. (14)–(16) we get $R=\cos^2(\Delta\omega t/2)$. As N increases, we get sophisticated patterns with narrower main maxima and with secondary maxima. The points (squares) obtained by the computational technique fall on the lines on the basis of Eqs. (14)–(16).

Figure 4 shows the same as Fig. 3, now for fixed $N=6$ but various values for the parameter ξ in the spin flip probability $\rho=\sin^2\xi$: $\xi=i\pi/(2N)$ with $i=1,2,3$. Experimentally, these curves can be observed by taking appropriate values for the amplitude $B_{\rm rf}$ in the rf coils [see Eq. (4)]. Again, the points (squares) obtained by the computational technique fall on the lines on basis of Eqs. (14)–(16).

Our computational technique enables us also to investigate the QM probability R when the condition (11) is not fulfilled. Figure 5 shows R as a function of time t for $N=6$ and for the value of the frequency $\Delta\omega\tau=2\pi/j$ with $j=1,2,3,4,5$. It is seen that the function R is periodic in time with the period of $T=2\pi/j\Delta\omega$. The shape of the function within one period changes significantly with increase of j. Thus, it shows an arbitrary waving behavior with the shifted phase, which is difficult to describe in an analytic way in simple expressions.

IV. CONCLUDING REMARKS

In this paper we give, first, a theoretical description of polarized neutron multilevel experiments in a system of N nonidentical resonant coils with spin flipping probability between 0 and 1. A large number of neutron waves with different wave vectors and energies are obtained. These waves interfere and each pair contributes to highly regular patterns in quantum mechanical probability to find the neutron in a specific spin state. Behind the system this pattern evolves in time and in space. For the specific adjustment of the system of resonators ($\Delta\omega\tau=2\pi$) we derived an analytical expression for this probability, for arbitrary values of the flipping probability ρ of one resonator and a phase quantum equal to the line integral of the field between the resonators, which in practice becomes equal to time t. This expression was testified by computer calculations.

Secondly, such a system of resonators may be used for multilevel MIEZE but with no restrictions on the number of the resonators because one can use only one single analyzer at the exit of this device. For practical purposes it is not convenient to work with N resonators, whose frequencies increase from one to another by $\Delta\omega$. Then, the last Nth resonator will have (a rather high) frequency $\omega_0+N\Delta\omega$ with the corresponding dc field $B_0+N\Delta B$. It is also difficult to keep the difference in frequency between two neighboring coils equal to $\Delta\omega$. Fortunately for experimentalists, multilevel splitting will occur also when the resonant devices in the system are operated in the following way: all odd resonant devices at frequency ω_0 in the dc field $B_0=\omega_0(\hbar/2\mu_n)$ and all even resonant devices at $\omega_0+\Delta\omega$ in the dc field $B_0+\Delta B=(\omega_0+\Delta\omega)(\hbar/2\mu_n)$.

In this case the same analysis of Sec II B may be applied with exactly the same results as given by Eqs. (14)–(16). It will simplify significantly the experimental efforts to realize multilevel interference. Therefore we conclude that the problem of multilevel interference may be of interest both from theoretical and experimental points of view.

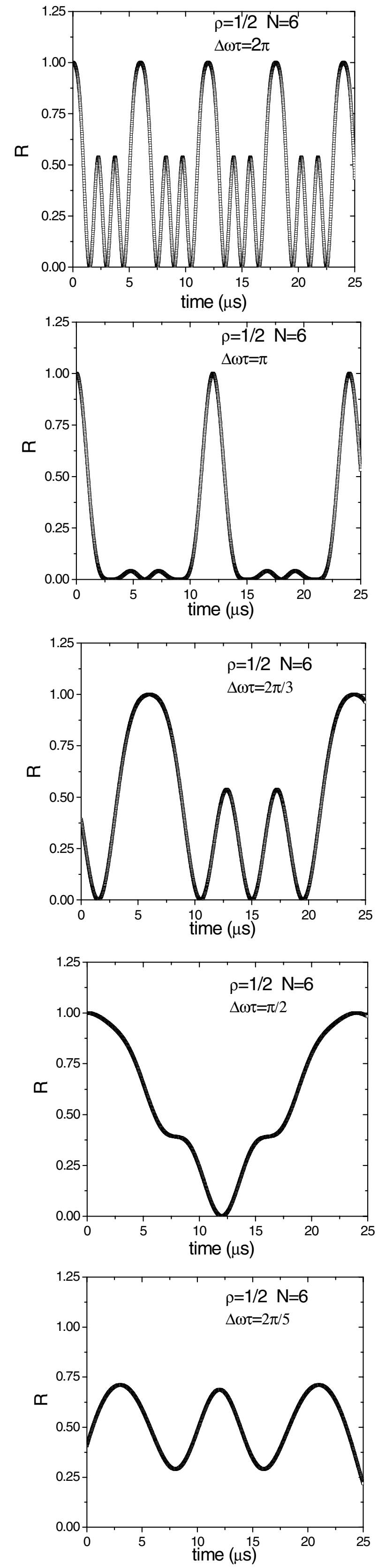

FIG. 5. Dependence of the QM probability R on the time t for systems of resonance coils with number of coils $N=6$ and for spin flip probability ρ in the resonance coils equal to $1/2$, providing the condition $\Delta\omega\tau=2\pi/j$ fulfilled with $j=1,2,3,4,5$.

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